Options with regime change in syndicated lending: A formal model of risk-shifting

Andrew Clare[e], Carlos Manuel Pinheiro[a,b,c,d], Alberto Franco Pozzolo[f,g], João Miguel Reis[a,b,*]

[a] *Universidade Europeia.*
[b] *Business Research Unit (BRU-IUL), Lisboa, Portugal.*
[c] *ISCAL - Instituto de Contabilidade e Admnistração de Lisboa*
[d] *Center for Transdisciplinary Development Studies (CETRAD)*
[e] *Bayes Business School*
[f] *Roma Tre Univerity*
[g] *Centro Studi Luca d'Angliano*

Abstract

We present a novel framework for modeling credit risk frictions in syndicated lending through options, focusing extending a regime change model. Departing from the seminal Merton model, we utilize an approach based on barrier option formulae to analyse the effects of borrower risk-shifting behavior, which can occur via the pursuit of riskier projects or increased payouts to shareholders. To this effect, we introduce a bond with regime change possibilities when a barrier is crossed. Risk-shifting is identified when the value of the loan crosses a predefined threshold relative to its initial borrowing value. This framework facilitates the examination of risk-shifting dynamics, shedding light on the lead bank's role within the syndicate and its monitoring efforts over the borrower. Our sensitivity analysis, conducted on stylized syndicated loans, reveals that lead lenders have incentives to deter risk-shifting by the borrower, particularly through dividend distributions, as such behavior may jeopardize future collaborations. Additionally, our findings framework indicate that the vigilance of the lead bank in monitoring syndicated loans tends to increase with loan maturity and volatility. Our model should be of interest both to academics and practitioners alike in identifying credit risk frictions in syndicated lending.

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*Corresponding author. Tel.: +351 963398489.
Email address: joao.reis@universidadeeuropeia.pt (Andrew Clare)

1. Introduction

Syndicated lending gained a prominent role in the last decades. Both in private and public debt, its amounts have been massive. The loans ballooned from $339 million of loans in 1988 to, on the onset of the great financial crisis, a historic high of $2.2 trillion in 2007.

A loan syndicate consists of a group of financial institutions that provides financing to a single borrower. The loan is administered by a common lead bank and governed by a common document (or set of documents) among the lenders. Despite syndicate members being referred to by different titles, such as lead and participant, each lender holds a common loan agreement with the borrower.

Typically, the lead bank plays a more active role in analyzing the borrower's credit worthiness and in monitoring its financial and operating activities. Consequently, typically, the lead has a more proximate relationship with the borrower than the participant banks.

This setup leads to frictions within a lending syndicate. Given the different roles of the lead bank and the participant banks, there are agency and informational frictions within a lending syndicate. We examine the lead bank's heightened retention of loan shares, analyzing the frictions often stemming from this phenomenon.

In syndicated lending two problems arise. The first, the moral hazard, an ex-post issue which arises from the (mis)alignment of monitoring incentives. The lead bank assumes monitoring responsibilities, while other participants adopt a more passive stance, potentially diminishing the lead lender's incentive for active monitoring of the loan

The second, the adverse selection problem is an ex-ante problem that results from the private information about the borrower, which is collected by the lead bank before the syndication process. If this information cannot be credibly communicated to the participants, it leads to a "lemons" market problem, as the lead bank has an incentive to peddle bad quality loans to the loan participants.

We focus in the first kind of problem, whose nature has been notably explored in Jensen and Meckling (1976), Pennacchi (1988) and Gorton and Pennacchi (1995).

By focusing on the monitoring of the loans by the lead bank, a key component is the value that banks attribute to their relationship with the borrowers. While, for instance, James (1987) and Lummer and McConnell (1989) establish the value of lending relationships to borrowers, here, we focus on the value of that relationship on the lenders perspective.

Since early, for instance in Leland and Pyle (1977), the theories of financial intermediation emphasize the role of information about the borrowers. Diamond (1991) and Rajan and Zingales (1995) look into the role of monitoring which is used to establish a reputation by the lenders. As lending relationships involve repeated interactions between the borrower and the lender, it can be argued that it reduces the cost of providing future loans. As pointed out in Petersen and Rajan (1994), there is value in the relationships between borrowers and lenders, with the time spent in a relationship increasing the purchase of financial services and a concentration of the borrowing with the lender.

In this paper, we bridge the syndicated lending with the possibility of risk-shifting. In order to link the allocation of the loan among the syndicate, we highlight the possibilities of risk-shifting as a source of the dynamics. The riskiness of the loan is highlighted as a factor in determining the loan shares. In the setup we propose, the risk-shifting, and whether occurs is a key element. We focus on the lead lender having an incentive to allow the borrower to engage in risk-shifting, in detriment of the participant lenders.

The aim of our study is to build a structural model where, upon the firm value decreasing in value, there is the possibility of the risk-shifting occurring. In the studied risk-shifting possibility, we allow the value of the volatility and the dividend yield (total payout to debt and equity in the case of credit risk) to change once a lower barrier is crossed. This is going to be done through a regime change. The valuation of bonds under this possibility is presented.

Therefore, in addition to contributing to the literature of risk-shifting, we present a novel regime change framework for options. In framework developed here, for a given contract, upon a value of the underlying asset being crossed by the asset value, we allow the drift parameters to change.

This is achieved through a barrier option framework, where the option which changes its drift parameters is decomposed into a knock-in barrier option and knock-out barrier option, with the latter representing the drift after the risk-shifting occurs. The second barrier option, which is ruled by one drift before being knocked-in and by another upon doing so is calculated with the Stopping Time (ST), as presented by Dias et al. (2015). This approach is originally developed to obtain the price of various barrier options results in the Jump to Default CEV model (JDCEV) of Carr and Linetsky (2006).

Studying these dynamics, where the value of the relationship between the lead lender and the borrower is crucial, and can affect the share of the syndicated loan owned by the former. Our results suggest a result in different directions when it comes to the volatility of the borrowing firm. While the initial volatility tends to increase the share to be held by the lead lender in order to ensure the syndicated loan, the change (increase) in volatility that follows the risk-shifting reduces this share. The increase in the payments to shareholders upon the risk-shifting occurring also reduce the required share. In addition, as the enforcement of discipline by the lead lender over the borrower becomes more difficult, the required loan share also increases.

In Chapter 2 we go over the literature related to our study. In Chapter 3 we apply the setup to a syndicated debt setup. In Chapter 4 we present various numerical analysis on the previously developed setup.In Chapter 5 the final conclusions are displayed. In the Appendix we present the bond with a regime which is used during the rest of the paper.

2. Literature Review

Bharath et al. (2007), in the context of loans in general, find that strong past lending relationships are significantly relevant to determine future lending (and investment banking business), with a greater likelihood of winning future business being a significant benefit to a lender with a relationship. It is found that a bank with a prior lending relationship with the borrower has a more than 40% chance of winning future loans, while a bank which does not have such a relationship only has a 3% probability. This result is more pronounced for borrowers with greater information asymmetries, such as small and non-rated borrowers.

In this vein, Dass et al. (2011) explore the syndicated loans, regarding conflicts among the lead lender and the participants by considering that the lead has a benefit of not enforcing the monitoring mechanisms. Focusing on the role of covenants that minimize these conflicts among the lenders, they find that a lesser use of covenants corresponds to the cases where the lead bank owns a greater share of the loan.

Dennis and Mullineaux (2000) highlight to dimension of transparency, as in the cases of firms which are listed or whose debt is rated, the amount which is syndicated to the participants is greater. The reputation of the lead, measured by business repetition between the lead and the participants, also contributes to greater shares of the loan being syndicated to the participants.

Sufi (2007) shows that due to the moral hazard in lead's monitoring of the borrower, the lead holds a larger share of the syndicated loan. The effect is stronger when there is greater information asymmetry.

Gustafson et al. (2021) present a recent extensive study on syndicated loans. They highlight the importance of the lead, as a significant proportion of the variation in the monitoring can be attributed to it. They find a positive association between lead share of the loan and monitoring, while the relation between the lead share and monitoring becomes stronger as borrowers become riskier. Regarding covenants, they find that active monitoring goes along covenant-based monitoring, when it provides information that can be used to force compliance.

The risk-shifting[1] problem can be traced back to seminal works of Galai and Masulis (1976) — formulating the equity value of a firm as a call option which benefits from the asset's volatility —, and Jensen and Meckling (1976) — who developed the concept of agency costs.

In the classic risk-shifting theory, companies engage in extracting value from debt-holders, to benefit of the shareholders, with the latter being able to nominate the management of the firm. Several authors have addressed how to mitigate that issue. For instance, Smith and Warner (1979) explore different kinds of covenants, finding that those on dividend and financing policies involve lower costs than the ones on investment policy. Barnea et al. (1980) discuss the role of call provisions and debt maturity to deal with agency costs. Green (1984) points to the usefulness of convertible debt and warrants to control risk incentives. John and John (1993) find that managerial compensation has a role in minimizing the agency costs of debt.

As for a theoretical studies on the nature of the issue, we can highlight several works. Leland (1998) challenges several conventional wisdom assumptions by finding results such as risk-shifting occurring even when there are no agency costs and that optimal leverage will be reduced when asset substitution is possible. With a different result, Ericsson (2000)

[1]Risk-shifting in lending refers to the situation in which borrowers take on riskier investments once they have secured financing from lender, thus shifting risk to the latter. This is similar to the problem of asset-substitution, where we can have the case of low-risk debt being turned into a high-risk debt by the decisions of the firm management.

finds that if asset substitution was excluded, a typical firm could take on an additional 20% leverage. Eisdorfer (2010) provides a study which finds situations where risk-shifting behavior is not necessarily associated with an increase in risk, but still imposes costs on debt-holders.

The endogeniety of actions that represent risk-shifting make its observation in real data harder. There are even results as for instance Gilje (2016), who finds that when firms approach financial risk, they reduce, not increase, investment risk. As explanations for doing so, the role of shorter debt maturities and financial covenants is highlighted.

Despite these challenges, many recent empirical results show risk-shifting occurrences. In the case of financially distressed firms, using 40 years of empirical data, Eisdorfer (2008) finds that the value of debt in distressed firms is reduced by approximately 6.4% due to over-investment during high volatility periods. In the case of banks, Srivastav et al. (2014) observe that CEOs with high inside debt are associated with more conservative payout policies, that is, less eager to engage in risk-shifting. Favara et al. (2017) find the presence of risk-shifting by comparing the behavior of firms in countries with different strictness of bankruptcy laws — the imperfect enforcement of debt contracts induces firms to take on less risk than they would do if those contracts were to be perfectly enforced.

This calls for formal modelization in order to unveil the possible dynamics in a syndicated loan. To study this, we use a setup based on structural credit risk models to study the value of loans in a syndicate. Thus, we depart from the classic Merton (1974) model, based on a put option formulae, to structure our analysis. This represents a seminal work in structural credit risk modelization, whose simplicity allows for various degrees of flexibility, which have spanned various models through its extensions. For instance, Black and Cox (1976) extend the Merton Model within the framework of barrier options and the Longstaff and Schwartz (1995) include the possibility of floating interest rates.

3. The syndicated debt model

A syndicated loan is issued by a group of banks, each holding a fraction of the loan. Among the lenders, the lead stands in a particular position. He tends to set up the loan terms, find other loan participants and collect information on the borrower.

In a traditional bank loan, as one would approach with the Merton model, the dynamics are determined by the characteristics of the borrower. However, in syndicated loans, new factors are introduced. In particular, in this paper, we emphasize the relationship that the leads holds with the borrower.

This dynamic leads to issues such as information asymmetry and moral hazard. The main dynamic we explore here is the latter. We take into consideration that the lead lender values his relation with the borrower firm, and thus they have an incentive to allow its management firm to engage in risk-shifting.

In this analysis, the relation between the lead lender and the borrower is crucial. To prevent the risk-shifting, the lead lender is in charge of monitoring the borrower on behalf of the whole syndicate, although they have an incentive not to enforce decisions and allow the borrower to engage in risk-shifting. Such will happen when the lead lender sees the gains of allowing the risk-shifting to be greater than losses of not doing so.

Through an extension to the Merton model, we will allow the risk-shifting on the part the borrower to occur from either the management of the firm increasing the volatility of the enterprise, increasing the dividend payments, or engaging in both the alternatives simultaneously.

3.1. Risk-shifting dynamics

In order to model the syndicated loan, we use bond payoffs where there is the possibility of the borrower engaging in risk-shifting, we use a option with regime switching as defined above. As the firm value drifts towards lower values, the default becomes a stronger possibility, thus affecting the bond value.

Therefore, given the expected lower values for the equity, shareholders have an extra incentive to engage in risk-shifting, that is, change how the firm behaves in order to increase their value, while reducing the market value of the debt, thus harming the position of the shareholders.

This kind of behavior was described various studies. For instance, regarding high volatility periods, as a result of over-investment, Eisdorfer (2008) finds that the value of debt in distressed firms is reduced by approximately 6.4%. Srivastav et al. (2014), by observing the CEOs inside debt, observe that banks divert cash to shareholders, thus increasing the value of the debts. Favara et al. (2017), through a panel with different countries that hold different bankruptcy laws, find that the prospect of bankruptcy enforcement induces firms to take on more risk.

Here, we assume that this is reflected in two non-exclusive ways. The first is by the firm management adopting projects with a higher risk, that is, by increasing the volatility of the enterprise, σ. The second is by increasing the total payout to shareholders, q. We assume that there are no payments to debt holders for the duration of the loan, thus q corresponds solely to dividends. Note that, regarding the equity value, as presented in equation (A.6), while the increases in volatility increase it, the increases in the dividend payments reduce its value. In the second case, the risk-shifting is beneficial to the shareholders, but by absorbing value from the firm, not by increasing the equity value.

As for when the risk-shifting occurs, it assumed that it does so when the firm value crosses a lower threshold, which is measured in relation the the firm value at the beginning. That is, when the firm value goes below a barrier $L = lV_0$ the shareholders choose to engage in risk-shifting, changing the process of the underlying asset, as defined in equation (A.7), from the first set of parameters to second. That is, at time $\tau_L := \inf\{u > t_0 : V_u = L\}$, with $L = lV_0$, the process parameters of the V_t process go from (r_1, q_1, σ_1) to (r_2, q_2, σ_2). In this setting, the risk-free interest rate is not changed by the risk-shifting, thus $r_1 = r_2$ for all cases. The other two parameters may present different values with the regime change.

Therefore, by adapting the put that represents the discount, now under the possibility of risk-shifting, which we denominate $G_{rs}(V_0, D, L, T; 1)$, for the bond from equation (A.5) with a put as presented by equation (A.10), we are able to adapt the original Merton model, obtaining a bond where, upon crossing a lower barrier, the borrower engages in risk-shifting. Therefore, the value of a bond with the possibility of risk-shifting by the borrower is given by

$$B_{rs}(V_0, D, L, T) = De^{rT} - \underbrace{P_{rc}(V_0, D, L, T; 1)}_{G_{rs}(V_0, D, L, T; 1)} \tag{1}$$

with the debt discount including the possibility of risk-shifting.

The regime change dynamic can be visualized in Figure 1 simulations. After crossing the barrier of 80, the path of the price becomes ruled by a new regime, with a higher volatility process. In the case the barrier is not crossed, the regime is never changed. The price paths after crossing the barrier are represented by a dashed line. In terms of risk-shifting, as the firm value decreases in price, the new volatility represents the higher

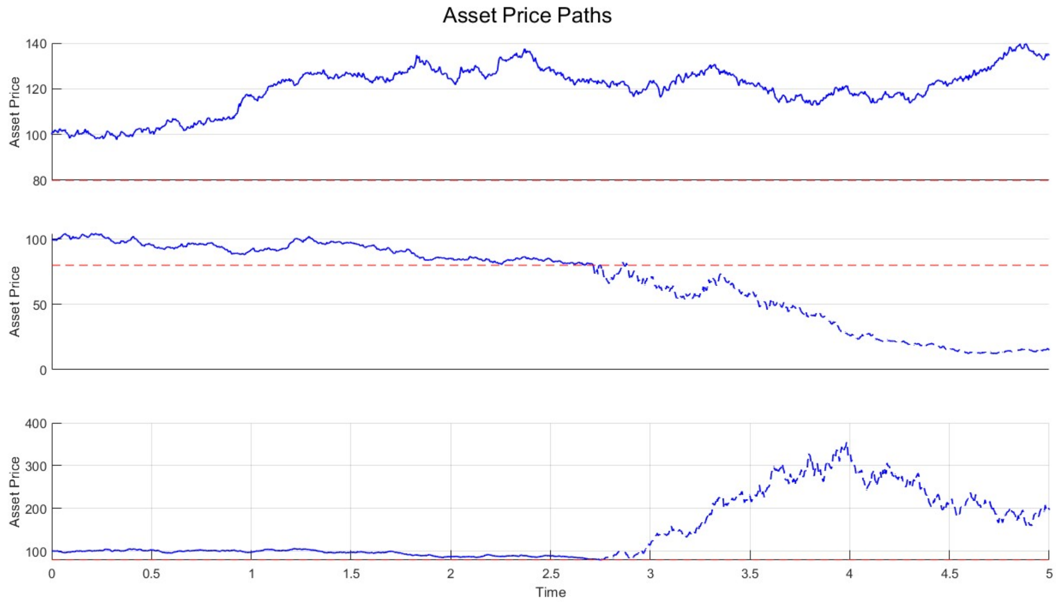

Figure 1: Path examples under regime change trough risk-shifting

risk in the enterprise. Here the spirit is similar to that of the Heston (1993) and CEV model presented in Cox (1975). While in these two seminal models the volatility changes gradually and we allow it (and the other parameters) to change once a barrier is crossed.

In a similar fashion to a bond, we can also consider the value of the equity with the regime change. As it will be argued in chapter 3.2, the payoff of the equity for the lead lender will only occur if the barrier is crossed and the regime is changed. Therefore, we have

$$E_{rs}(V_0, D, L_{NM}, T) = P_{ki}(V_0, K, L_{NM}, T; -1) \tag{2}$$

with $\phi = -1$ indicating that the above represents a call option value.

3.2. The syndicated loan share

Here, we provide the dynamics that rule the loan share to be held by the lead lender. As for the assumptions of this model, first, as in the Merton model, the default occurs at the maturity date, time-T, if the value of the assets of the firm, V_T, is below the nominal value of the debt, D. Plus, we separate the loan into two lenders, the lead borrower, and, without loss of generality, we assume there is only one participant lender, as the key factor are the decisions taken by the lead. We assume that the decision of monitoring or not monitoring the debt is taken at time-0, when the loan is issued. Furthermore, the lead lender is able to enforce control over the decision of the borrower to risk-shift, with a binary decision of either allowing or not allowing the risk-shifting to occur, while also including the possibility of an imperfect control. We also assume the absence of covenants that could affect the dynamics of risk-shifting - the key factor is the enforcement or not by the lead borrower.

Each corresponding share of the bond issuance is given by α_l and α_p, thus $\alpha_l + \alpha_p = 1$. As we shall see, the shares held by when the loan is issued are those that determine that decision of monitoring or not monitoring the debt.

Overall, the lead lender will chose to monitor when the value of his position from monitoring is greater than the value of not doing so. This will be represented by the condition that follows:

$$\begin{aligned} \alpha_l B_{rs}(V_0, D, L_M, T) - (1-\alpha_l)\lambda G_{rs}(V_0, D, L_M, T) > \\ \alpha_l B_{rs}(V_0, D, L_{NM}, T) + \gamma E_{rs}(V_0, D, L_{NM}, T) - (1-\alpha_l)\lambda G_{rs}(V_0, D, L_{NM}, T). \end{aligned} \tag{3}$$

From which we can isolate for α_l value that ensures monitoring, which will hereafter be name α_l^*. Thus we have

$$\alpha_l^* > \frac{\gamma E_{rs}(V_0, D, L_{NM}, T) + \lambda(G_{rs}(V_0, D, L_M, T) - G_{rs}(V_0, D, L_{NM}, T))}{B_{rs}(V_0, D, L_M, T) - B_{rs}(V_0, D, L_{NM}, T) + \lambda(G_{rs}(V_0, D, L_M, T) - G_{rs}(V_0, D, L_{NM}, T))}, \tag{4}$$

So, the above expression, for a set of parameters, yields the minimum share to be held by the lead lender in order for them to monitor the debt. Given the shares go from 0% to 100%, $0 \leq \alpha_l^* \leq 1$.

The result above comes from comparing the two possibilities, assuming the payoff has three components. First, the value of the bond, weighted by their share in the loan syndicate. This bond, which is subject to risk-shifting, will be represented by $B_{rs}(V_0, D, L_{NM}, T)$, where L_{NM} corresponds to the barrier which, upon being crossed triggers risk-shifting by the borrower firm. Second, the benefit provided by his relation with the borrower, which we assume that is zero in the case the lead does not allow the borrower to engage in risk-shifting. Third, a penalty in the case of default by the borrower, which we will link to the syndicate loan participants. When lead lender allows risk-shifting to occur, the payoff will be

$$\alpha_l B_{rs}(V_0, D, L_{NM}, T) + \beta - \kappa, \tag{5}$$

with β representing the benefit to the lender from allowing the risk-shifting to occur, while κ is the penalty to the lead lender linked to the loan participants.

As for the form β takes, we propose it to be in the form of a share of the borrowing firm's equity, which only materializes in the cases where the risk-shifting actually occurs. If the lead lender allows risk-shifting at the loan issuance but it does not occur, then the value of the benefit is zero. This will follow from how equity is modeled through a options framework, as presented in equation (A.6).

With this, we are considering that, if the risk-shifting is allowed and occurs, the relationship between the borrower and the lead lender is strengthened, and this is reflected in future business opportunities (from lending or other services) for the lead lender, which in turn will be linked to the equity value of the firm - the more valuable the firm becomes in the future, the greater will be those opportunities.

This can be modeled by considering the fraction of that equity value, γ, in the cases where the risk-shifting occurs, that is, when the lower barrier L is breached. The γ parameter will for now be presented as constant, although it will be argued that it can be viewed as a function of various parameters.

Following the setup used for the risk-shifting, before the lower barrier is crossed, the V_t process is given by the firm regime, and afterwards it is given by the second, as stated in equation (A.7). Thus, we have equity in the form of a down-and-in call such that

$$\beta = \gamma E_{rs}(V_0, D, L_{NM}, T) \tag{6}$$

with the value of the knock-in call, where the risk-shifting occurs when the option is activated, as represented by equation (2).

In the case of the κ, we intend to contemplate the damage to the lead lender upon default by the borrower. For instance, Gopalan et al. (2007) find that upon default in a syndicated loan, there is a drop in the lead lender level of activity in the syndicated market, which is followed by shifting lending to less risky endeavors. In addition Gopalan et al. (2011) confirm the hypothesis that upon the borrowers default, the lead lender faces a loss in reputation, which harms future activity through assuming higher loan shares, although the result is less pronounced when the loan arranger holds a dominant market position. In order to model this dynamic in dynamic which is proportional to the size of the loan we propose it to be in form of the lead lender providing a guarantee on the participant's share of the loan. That is, if default occurs, weather the lead lender did or did not enforce the monitoring in terms of risk-shifting, they are responsible for a part loan losses taken by the loan participant. This corresponds to the value of discount on the loan held by participant lender. That fraction will be denominated by λ. Therefore, at the bond issuance, we have

$$\kappa = (1 - \alpha_l)\lambda G_{rs}(V_0, D, L_{NM}, T), \tag{7}$$

where $(1 - \alpha_l) = \alpha_p$, the fraction of the loan held by the participants, from which the lead lender is responsible for the fraction λ. Given the evaluated bond is composed by a risk-free bond minus the debt discount, when the lead lender provides part of the discount, they are providing a guarantee on the loan. In the case where $\lambda = 1$, the lead lender is providing a full guarantee to the loan participant, making the latter's investment fully risk-free.

In the opposite case, when the debt being monitored, the benefit will always be zero, $\beta = 0$. We admit various levels of effectiveness of the monitoring. If there is a perfect monitoring, the lead lender is always capable of preventing risk-shifting, therefore the loan value will be given as in the Merton model. If the monitoring is not perfect, we assume that there can still be risk-shifting, but the barrier at which it occurs is lower than it would otherwise be, if there was no monitoring. Therefore, the value of the bond is be given by $B_{rs}(V_0, D, L_M, T)$, such that L_M is the lower barrier which triggers the risk-shifting. Given that now there is monitoring, this barrier will be lower than without the monitoring, $L_M < L_{NM}$ — the likelihood of risk-shifting is smaller. In the case where the monitoring is perfect, $L_M = 0$, there is no possibility of risk-shifting and the bond is given by $B_0(V_0, D, T)$, as in the Merton baseline case.

Therefore, for the case of monitoring, we rewrite equation (5) as

$$\alpha_l B_{rs}(V_0, D, L_M, T) - \kappa, \tag{8}$$

while κ, which weights the guarantee amount, is also given in terms of a bond discount with the lower barrier which corresponds to the monitoring case:

$$\kappa = (1 - \alpha_l)\lambda B_{rs}(V_0, D, L_M, T). \tag{9}$$

Therefore, by combining equations (5), (6) and (7) for the value of not monitoring, and equations (8) and (9) for the value of monitoring.

As it can be observed in equation (4), higher shares of the loan being held by the lead lender, α_l, and bigger guarantees on the participant's loan, λ, have ambiguous effects on both sides of the inequality, given $B_{rs}(V_0, D, L_M, T) > B_{rs}(V_0, D, L_{NM}, T)$ and $G_{rs}(V_0, D, L_M, T) < G_{rs}(V_0, D, L_{NM}, T)$. A bigger stake on the future business of the borrower firm, γ, only increases the value of not monitoring. With the previous inequality we are able to determine when the lead lender will have an incentive to monitor the debt.

We also need to consider the conditions for the participant lender. We assume they have no benefit or stake with the borrower. Therefore, if the debt is not monitored, their payoff is

$$(1 - \alpha_l)B_{rs}(V_0, D, L_{NM}, T) + (1 - \alpha_l)\lambda G_{rs}(V_0, D, L_{NM}, T) \tag{10}$$

and if the debt is monitored,

$$(1 - \alpha_l)B_{rs}(V_0, D, L_M, T) + (1 - \alpha_l)\lambda G_{rs}(V_0, D, L_M, T). \tag{11}$$

If we assume that the participant lender faces the choice between investing in a standard loan without any guarantee and no risk-shifting possibility, evaluated by a bond as presented by the Merton mode, and the syndicated loan, with the possibility of risk-shifting, if they expect the loan to be monitored, then they will prefer the syndicated loan in the cases where

$$B_0(V_0, D, T) < (1 - \alpha_l)B_{rs}(V_0, D, L_{NM}, T) + (1 - \alpha_l)\lambda G_{rs}(V_0, D, L_{NM}, T). \quad (12)$$

If they expect the loan not to be monitored, they will prefer the standard bond, without risk-shifting nor guarantee if,

$$(1 - \alpha_l)B_{rs}(V_0, D, L_M, T) + (1 - \alpha_l)\lambda G_{rs}(V_0, D, L_M, T) < B_0(V_0, D, T). \quad (13)$$

If both cases presented above are not verified, then the decision of the lead lender to monitor or not to monitor is rendered irrelevant. If the first condition is not satisfied, the loan participant will not be part of the loan syndicate, as they will prefer the alternative bond, regardless of the monitoring. If the second condition is not verified, the loan participant will always prefer to be included in the load syndicate, regardless of the loan not being not monitored. Therefore, the study over equation (4) is only relevant when both equation (12) and equation (13) are respected. Thus, in order to study the value of the loan share to held by the lead lender, the amount of the loan guarantee, λ, must be in a value such that

$$\begin{aligned}(1 - \alpha_l)B_{rs}(V_0, D, L_M, T) + (1 - \alpha_l)\lambda G_{rs}(V_0, D, L_M, T) < B_0(V_0, D, T) <\\ (1 - \alpha_l)B_{rs}(V_0, D, L_{NM}, T) + (1 - \alpha_l)\lambda G_{rs}(V_0, D, L_{NM}, T).\end{aligned} \quad (14)$$

4. Numerical Analysis

4.1. Bond value under the possibility of risk-shifting

First, we evaluate the bonds under the possibility of risk-shifting. As for the base parameters, the value of the firm, V_0, is 1 and the face value of the bond, D is 0.75, that is, the initial leverage ratio is 0.75. The barrier at which the regime change is triggered, L, is at the value of 80% of the face firm value, that is, $l = 0.8$. The risk free rate, r is 2%

and the (initial) total payout to shareholders (dividends), q_1, is 2%. The initial volatility, σ_1 is 15%. As for the first possibility of a regime change, the volatility increases by 10% to 25%, the latter being the value of σ_2. As for the second possibility, the total payout to shareholders (dividends) increases by 4% to 6%, thus $q_2 = 6\%$. The amount of the changes in the parameters will be designated by $\Delta\sigma = 10\%$ and $\Delta q = 4\%$. Regarding the maturity, the values will range until 15 years and $T = 15$. Upon default, the bond fully recovers the remaining value, that is, $\phi_{dwl} = 1$.

In Table 1 we compute the spreads for the bond, using the usual formula, now with the bond under regime in the place of the evaluated asset:

$$\mathcal{S} = -\frac{1}{T - t_0} \log\left(\frac{B_{rs}(V_0, D, L, T)}{De^{-r(T-t_0)}}\right).$$

It can be observed, as expected, that as the initial volatility, σ, and regime change parameters, $\Delta\sigma$ and Δq, are increased, the spreads of the bonds increase (corresponding to a decrease in the value of the bonds). As for the parameter L, as the value decreases, so do the spreads. As we decrease the barrier that must be breached in order to trigger the risk-shifting, the less likely is for it to be crossed and for the risk-shifting to decrease the value of the bonds.

In Figure 2, we compare three cases. The first with the classical Merton model as in equation (A.5), in the second we have the possibility of the regime change as defined in equation (A.10) for the volatility increasing from 15% to 25% and in the third we present regime change, again as defined in equation (A.10) for the dividend payments increasing from 2% to 6%. As we can observe, the possibilities of risk-shifting reduce the value of the bonds, as, when triggered, the new regime increases the value of the discount. Although, there is a difference. While the increases in volatility have a stronger impact over shorter maturities, the increases in dividends are more intense reducing the value of the bonds for longer maturities. We can attribute this to the different nature of the changes. While the increased volatility can occasionally increase the value of the firm, thereby reducing the probability of default, the increases in the dividends always reduce the value of the firm and increase the probability of default.

In Figure 3, we compare the baseline Merton case with various parameters for the barrier at which the regime change is triggered. In this case, when the risk-shifting

Table 1: Spreads values for bonds with the possibility of risk-shifting for different parameters

		$\Delta\sigma = 0.1$, $\Delta q = 0$	$\Delta\sigma = 0$, $\Delta q = 0.04$	$\Delta\sigma = 0.05$, $\Delta q = 0.02$	$\Delta\sigma = 0.10$, $\Delta q = 0.02$	$\Delta\sigma = 0.05$, $\Delta q = 0.04$	$\Delta\sigma = 0.1$, $\Delta q = 0.04$
$\sigma = 0.15$	0.9	1.9816	2.5687	2.1482	2.6833	3.0103	3.4976
	0.8	1.7318	2.1980	1.8701	2.2519	2.4961	2.8285
	0.7	1.4821	1.8497	1.5986	1.8451	2.0259	2.2304
$\sigma = 0.25$	0.9	3.5615	4.0210	3.6968	4.3729	4.6278	5.2730
	0.8	3.3611	3.7863	3.4919	4.0464	4.2720	4.7862
	0.7	3.1469	3.5407	3.2749	3.7082	3.9079	4.2977
$\sigma = 0.35$	0.9	5.3137	5.6020	5.3826	6.1764	6.3359	7.1077
	0.8	5.1407	5.4314	5.2174	5.9071	6.0615	6.7192
	0.7	4.9517	5.2465	5.0374	5.6187	5.7692	6.3121

This table studies the spreads of the bonds in relation to the risk-free rate with the possibility of risk-shifting using various parameters. When the parameters are not stated, the baseline values are used.

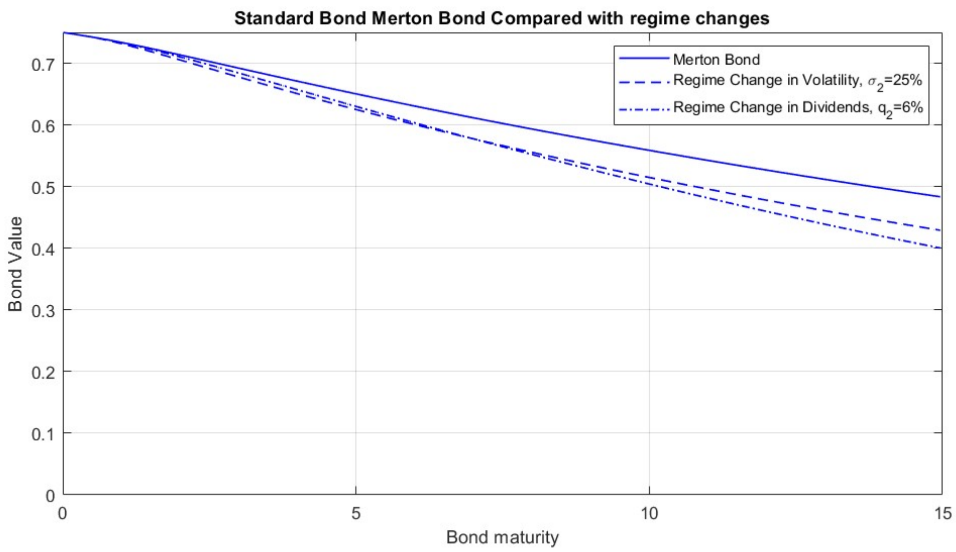

Figure 2: Risk-shifting from $\sigma_2 = 15\%$ to $\sigma_2 = 25\%$ and from $q_2 = 2\%$ to $q_2 = 6\%$

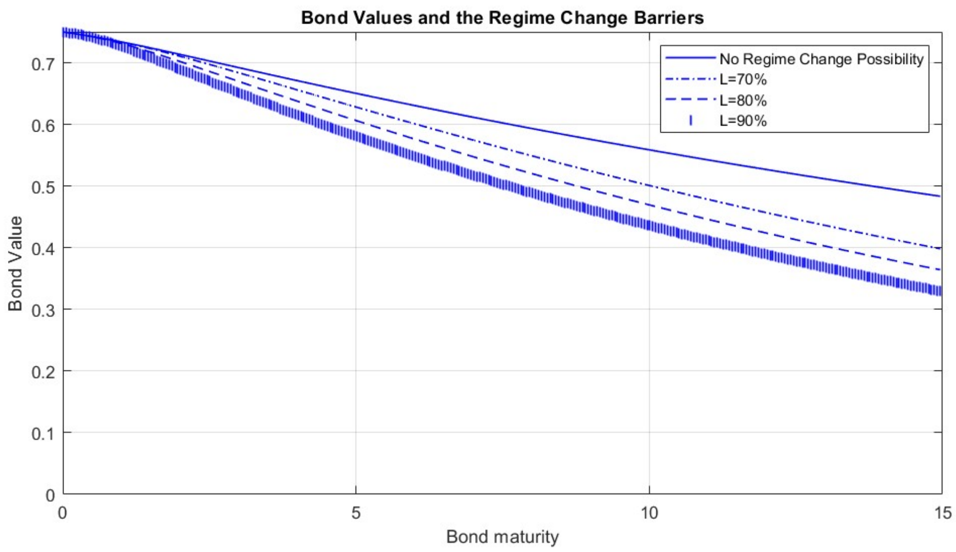

Figure 3: Risk-shifting in both σ and q, from $\sigma_2 = 15\%$ and $q_2 = 2\%$ to $\sigma_2 = 25\%$ and $q_2 = 6\%$

occurs, it increases both the value of the volatility, from 15% to to 25%, and of the dividend payments, from 2% to 6%, in other words, $\Delta\sigma = 10\%$ and $\Delta q = 4\%$. As expected, the barriers reduce the value of the bonds. In addition, higher values for the barriers result in greater reductions in the value of the bonds, as the risk-shifting becomes easier to occur, as the firm value needs to be reduced in a lesser amount.

4.2. Loan syndicate dynamics

Now, with the bond values which correspond to possibility of risk-shifting, we apply the methodologies to study a model on the syndicated loan market, where the main emphasis is the study of how it is affected by parameters such as the volatility and the maturity.

We focus our analysis on the role of the moral hazard. In order for the participant lender to agree to accept the loan syndicate, they will observe the share of the loan held by the lead lender. Based on that information, on the issuance date, they will decide to either or not to participate in the loan syndicate. As it will be observed, factors such as the change in volatility, the change in the payoff to shareholders and the capacity to enforce discipline over the borrower affect, not always in the same direction, the required share to held by the lead lender.

The inequality presented equation (12) will be determinant decision factor, with the loan participant only accepting values higher or equal to the presented α_l, while respecting the conditions regarding λ presented in equation (14).

As for the baseline parameters to be used, we will have that the initial firm value, V_0 is 1, the amount of debt is $D = 0.5$. The maturity of the syndicated loan is $T = 15$, that is, 15 years, the risk-free interest rate $r = 2\%$, the initial dividend yield is $q_1 = 2\%$, the initial value of the volatility is $\sigma_1 = 0.15$, and the bond, upon default, fully recovers the remaining value, that is, $\phi_{dwl} = 1$. The lower barrier which triggers the risk-shifting, when there is no debt monitoring is placed at 80% of the initial firm value, which translates into $L_{NM} = 0.80$. If the lead lender monitors the debt, the amount in relation to the initial value at which the risk-shifting is triggered is at 30%, that is $L_M = 30\%$. Upon the risk-shifting occurring, the dividend yield is increased to $q_2 = 4\%$, an increase of 2%, that is, the change is set $\Delta q = 2\%$ and the volatility is so to $\sigma_2 = 0.25$, an increase of 10%, that is, $\Delta\sigma = 10\%$. As for the share of the participant's loan the baseline amount is $\lambda = 10\%$ and the parameter which represents the interest of the lead borrower when the risk-shifting occurs will be $\gamma = 5\%$ for the baseline case.

Before investigating the values for α_l which will assure monitoring, we will look at both sides of the inequality presented in equation (3). Thus, with the data presented in Figure 4, we observe, for the lead borrower, the difference among the payoffs of monitoring and not monitoring the debt. The payoff difference corresponds to the left-hand-side of equation (3) minus its right-hand-side. The γ parameter takes three different values, 0, 0.05 and 0.10 and the maturity is set to 5 years, $T = 5$ while the remainder of the parameters take the baseline values.

As it can be observed, in all the cases, as α increases, the relative benefit of monitoring is greater. The smaller are the values of the γ, the lower is the needed value for the α to ensure monitoring. That is, the less the lead lender benefits from allowing the borrower to engage in risk-shifting, the lesser is the share of the loan that needs to be held by them in order to ensure the loan participant that the monitoring of the loan will occur. In the case where $\gamma = 0$, the lead lender always monitors the loan, as the benefit from doing so is always greater than the payoff of not monitoring. In the case of $\gamma = 0.10$, the situation is reversed, as the lead will never monitor the loan, given his benefit from not doing so is always greater, therefore leading to a negative difference. In the intermediate case of $\gamma = 0.05$, in the lower range of α_l values, the lead will not monitor the loan, although, around the share of the loan close to 58% they will start doing so. After that point, which is given by equation (4), the lead will prefer to monitor the loan, as the higher α_l values

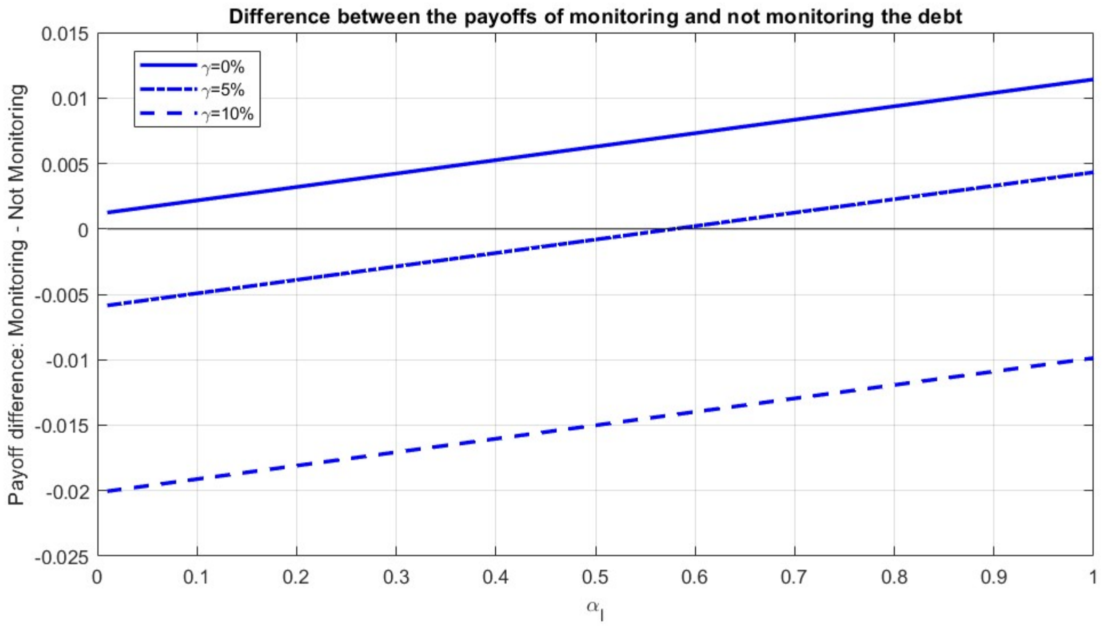

Figure 4: Payoff in terms of α_l with different values for γ

will make it so that they will prefer to do the monitoring.

Now, we will explore the dynamics of the γ in relation to the initial volatility. The minimum value for α_l that assures monitoring will be the key element, as obtained in equation (4). Using the baseline parameters for different range of γ values, $\gamma = 0.05$, $\gamma = 0.10$, $\gamma = 0.15$ and $\gamma = 0.20$, we obtain Figure 5.

The first conclusion that emerges is that, given the higher valuation of the benefit, higher values for the γ correspond to higher values for α_l^*. The required share of the loan to be held by the lead lender, in order for them to prevent risk-shifting, is increased. Second, for each level of γ, as we increase the initial volatility, the required α_l^* is increased, that is $\frac{\partial \alpha_l^*}{\partial \sigma_1} > 0$. Third, α_l^* is convex in terms of σ_1, that is, $\frac{\partial^2 \alpha_l^*}{\partial \sigma_1^2} > 0$. Therefore, as the initial volatility increases, the required increases for the α_l^* are greater. Fourth, the higher is the γ parameter, the stronger is the effect from the previous conclusion — higher levels of the benefit, which is to be received in the case the risk-shifting occurs where the debt is not monitored, enhance the impact of the initial volatility over amount that should be held by the lead lender.

Then, we study the effects of the changed parameters to the levels of α_l^*. Using the baseline values with three different values for the change in the dividend payments Δq, 0.02, 0.03 and 0.04 we obtain Figure 6. In a similar exercise with the baseline values, we use $\Delta\sigma$ with the three different values, 0.02, 0.03 and 0.04, thus obtaining Figure 7. In

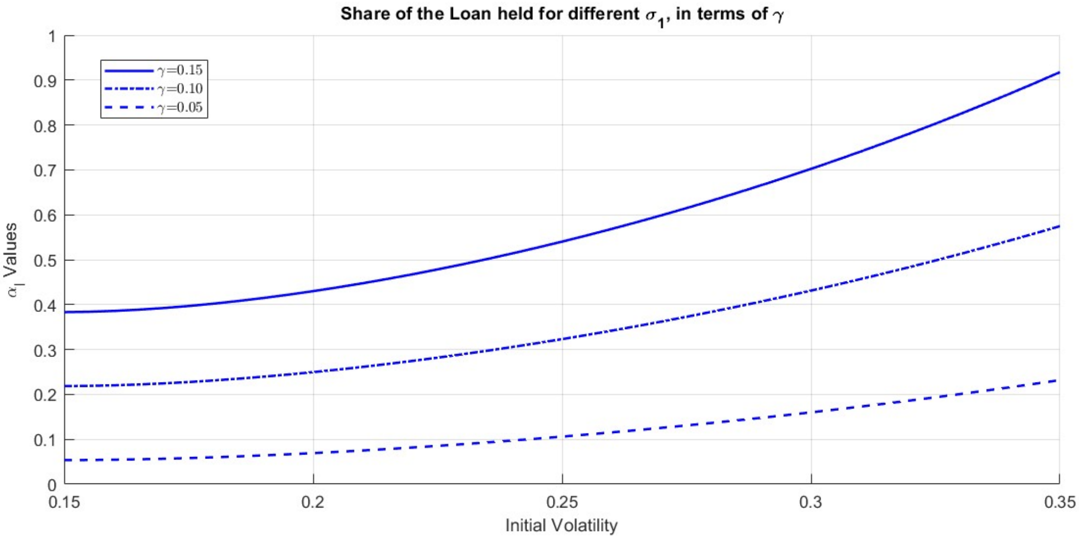

Figure 5: Payoff in terms of α_l with different values for γ

both figures, the conclusions are similar. While α_l^* is still increasing in terms of the initial volatility, it is decreasing in terms of the regime change parameters. That is, the bigger the change in the firm's drift parameters originated by the risk-shifting, the lower will be the share held by the lead lender in order to prevent them from allowing the risk-shifting. Although the behavior is the opposite of what it is in relation to the initial volatility, it is aligned with the portfolio position of the lead lender. The stronger is the risk-shifting, the greater will be the loss in the lead lender's bond portfolio, thus magnifying their loss. The greater are the losses to the lead lender, the less needs to be his share to ensure his alignment with enforcing discipline on the borrowing firm. In this framework, the increased value of the benefit in terms of the equity when the volatility is increased tends to not be enough to counterweight the loss in value of the lead lender's bond position.

This highlights how we should regard risk in a syndicated loan, where we assume the possibility of risk-shifting and loan monitoring. While the initial volatility always affects the value of the portfolio, with and without risk-shifting, the parameters after the regime change only affect the first possibility. The different dimensions of risk affect the loan differently.

In addition, we highlight the difference in behaviors regarding $\Delta\sigma$ and in Δq. In the case $\Delta\sigma > 0$, increasing the post regime change volatility, also increases the value of the benefit based on the equity value. By contrast, for the case Δq, it reduces the value the value of the benefit, by reducing the value of the borrowing firms' equity. While a firm engaging in more volatile endeavors might result in positive outcomes, increasing the

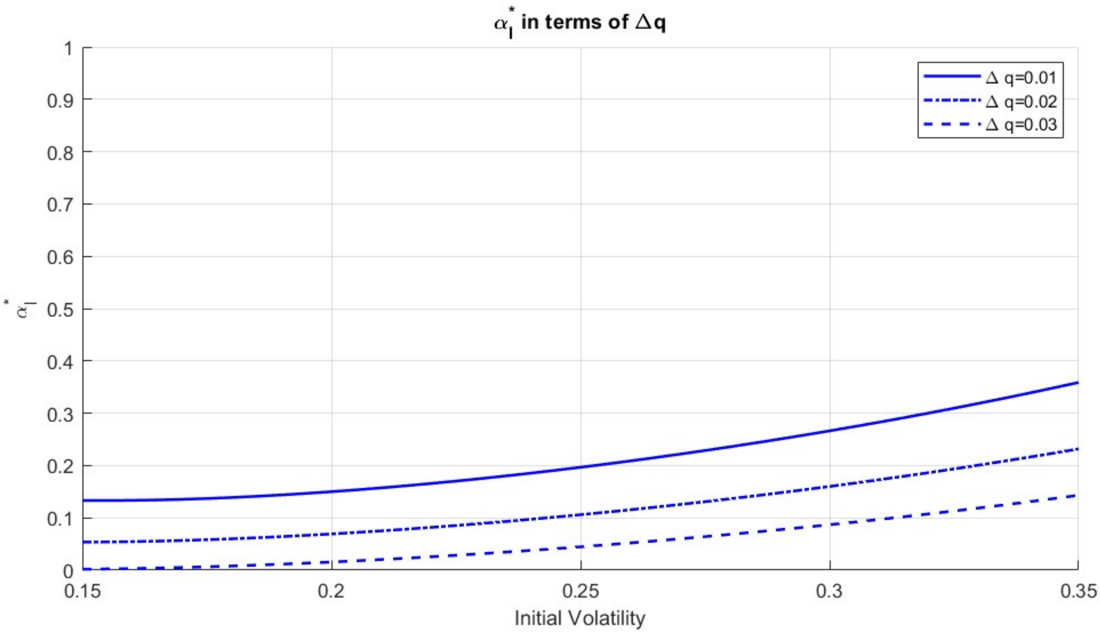

Figure 6: Payoff in terms of α_l with different values for Δq

dividend rate — transferring value from the firm to the equity holders — can only reduce it.

This highlights an implication for the role of covenants. The lead lender has a bigger incentive to prevent risk-shifting in the form of result distribution increases, as it impairs future business endeavors with the borrowing firm. This form of risk-shifting affects the borrower's value, and therefore compromises the future endeavors the lead lender might have with them. This differs from the situation with the volatility increases, as it has a possibility of increasing future business endeavors. This suggests that covenants on results distribution are less impactful on shaping the behavior of the lead lender, as they already have a bigger incentive to prevent risk-shifting than they do in the case of increasing the volatility.

Finally, we can also challenge the assumption of a fixed amount for weighting of the benefit, γ. For instance, one might expect it to be greater for firms with higher volatility, before and after the risk-shifting occurs. We might expect that firms with a higher risk to be more dependent on its financier relations, thus representing a more valuable business relationship with the lead lender. This hypothesis will be studied alongside the possibility of the γ not being fixed for different maturities, as one might also expect longer loans to have a bigger contribution to the relationship with the borrower.

In Figure 8 we can observe the required share to be held by lead lender in order

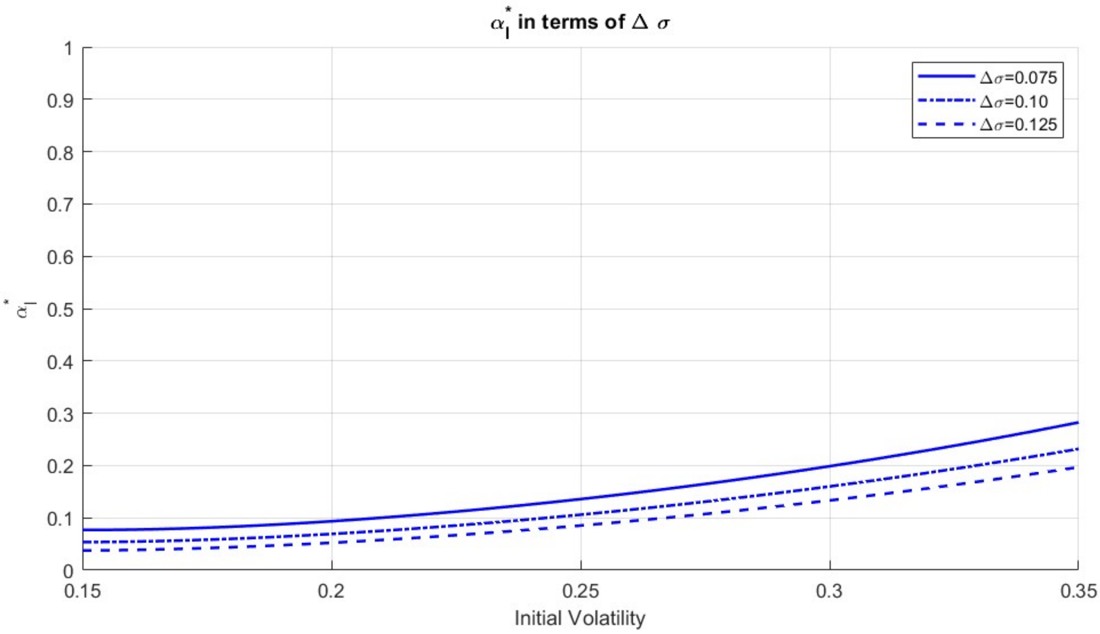

Figure 7: Payoff in terms of α_l with different values for $\Delta\sigma$

to assure the loan is monitored. For the level of volatility of 0.15 and 0.35, the γ is, respectively, 0.05 and 0.10. In the volatility intermediate values, γ increases linearly. As it can be observed, while in the case of the longer maturity the share held by the lead lender increases, this is not the case for the shorter maturity. Given the values of the γ, there is an interval where, as the volatility increases, the share that needs to be held by the lead lender in order to assure monitoring decreases. That is, as the volatility increases, the loss over the value of the bond held by the lead lender is not compensated by the increases in the benefit from not monitoring.

In Figure 9 we study α_L^* for the case where the maturity is 5 years, while the value of γ ranges from 0.02 to 0.08, increasing linearly with volatility. As it can be observed, now, with increases in volatility the share held by the lead lender in order to prevent monitoring increases in volatility. With the stylized values used for the syndicated loan setup, this suggests that, as the maturity and volatility increase, the value the lead lender attributes to relationship with the borrower increases.

In Figure 10 we are able to note the behavior of the required loan share to be held as the capacity to monitor becomes weaker. So, as the monitoring becomes more difficult to enforce, the value of L_M rises, as the risk-shifting can occur when the firm value crosses a higher values than it would otherwise. The values assumed for L_M are 0.20, 0.30 and 0.40. As the value of L_M increases, for the different volatility values, the required share of

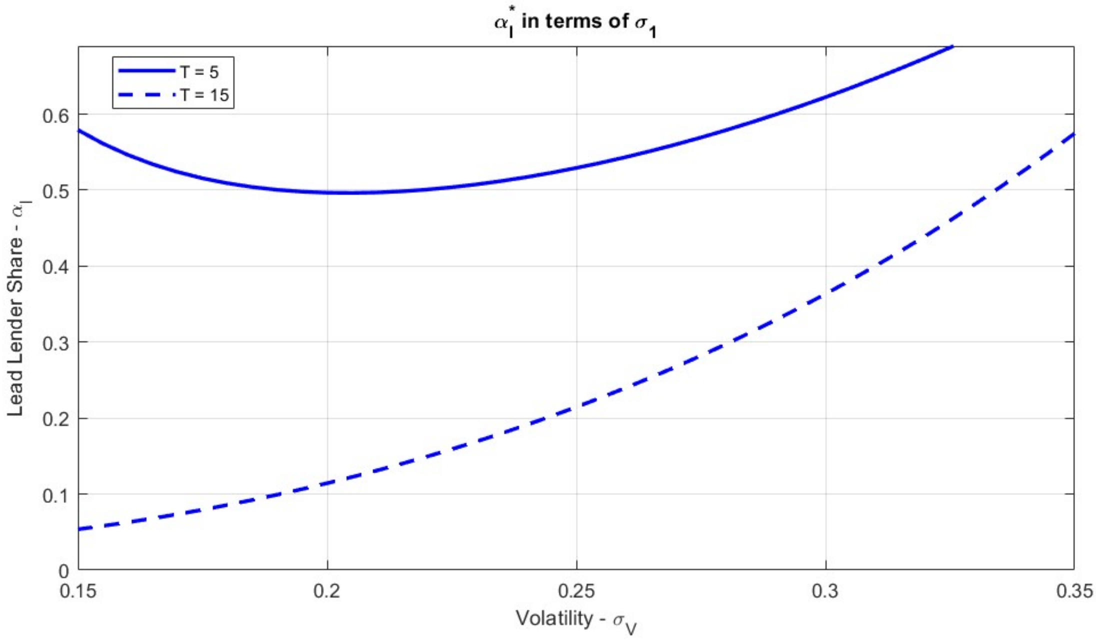

Figure 8: Lead loan shares for different volatilities for a γ ranging from 0.05 to 0.10. Maturity of 5 years and 15 years.

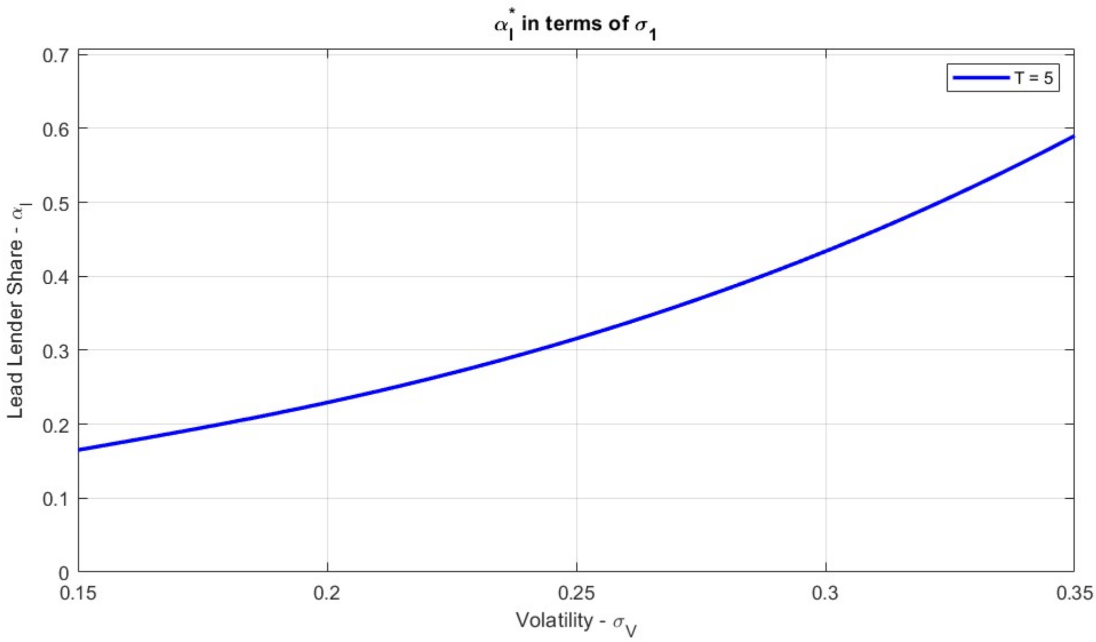

Figure 9: Lead loan shares for different volatilities for a γ ranging from 0.05 to 0.10. Maturity of 5 years and 15 years.

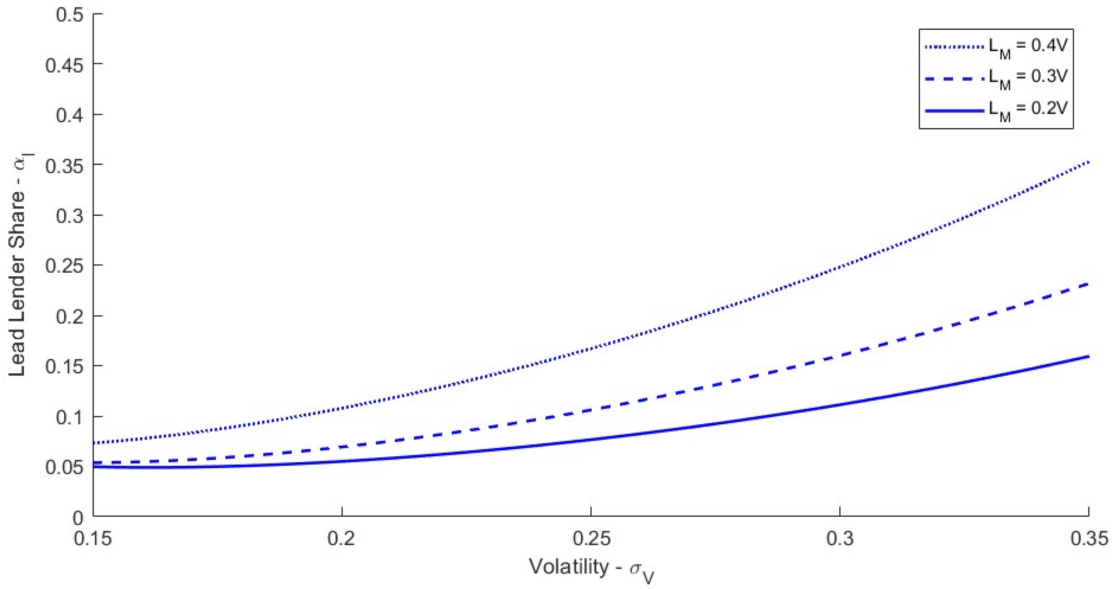

Figure 10: Lead loan shares for different volatilities for a L_M assuming the values of 0.20, 0.30, 0.40.

the loan to be held by lead lender increases. In other words, as it becomes more difficulty in impose discipline to the borrower, the value of α_L^* increases.

5. Conclusions

Syndicated lending facilitates risk-sharing among multiple lenders and provides borrowers access to substantial capital that may not be attainable from a single source. Typically, the lead bank assumes a more active role in overseeing the financial and operating activities of the borrower. Syndicated lending also presents avenues for risk-shifting.

In this study, we develop a structural model for analyzing risk-shifting behaviors, departing from the seminal Merton model and employing a a framework which departs from barrier options formulae. Utilizing the Stopping Time approach developed by Dias et al. (2015), our paper is, to our knowledge, the first to delve into the dynamics of syndicated loans using a regime change framework. This approach allows for a comprehensive understanding of various factors, particularly the monitoring role of the lead lender.

Our analysis of monitoring and non-monitoring payoffs suggests that the lead bank is expected to retain higher shares of the loan, as there is a greater incentive to prevent risk-shifting, especially for longer maturities and as volatility increases. Interestingly, we point that different kinds covenants may have a differentiated impact on shaping the lead bank's behavior. Furthermore, our study reveals a non-linear relationship in the monitoring of risk-shifting behavior, with volatility exerting a stronger (weaker) influence on debt value for shorter (longer) maturities, compared to increases in dividend payouts. Overall, we also indicate that longer loan maturities should be associated with greater value future for future collaborations. As an intermediate step, we also present regime-change frameworks to represent a bond, for which there is the possibility of risk-shifting.

By introducing this novel model, we contribute to the understanding of lending dynamics in the literature on syndicated lending, paving the way for future research in this area.

Appendix A. Risk-shifting in credit risk

Appendix A.1. The baseline Merton model

We begin by introducing the Merton (1974) model from which this study follows. For the studied firm, the face value of debt is represented by D and the maturity of the zero-coupon bond is T. The parameter r stands for the risk-less rate, and the firm's total payout to debt and equity holders is given by q — both parameters are constant in the baseline model.

The firm value — the sum of the equity and the market value of debt — is V. V follows the usual (risk-neutral) geometric Brownian motion, thus, its dynamics are given by

$$\frac{dV_t}{V_t} = (r - q)dt + \sigma dW_V(t), \tag{A.1}$$

which is solved by

$$V_t = V_0 \exp\left[\left(r - q - \frac{1}{2}\sigma^2\right) t + \sigma W_V(t)\right], \tag{A.2}$$

where σ represents the (constant) standard deviation of the firm value and $W_V(t)$ being the standard Brownian motion defined under the measure Q which generates the filtration $\mathbb{F} := \{\mathcal{F}_t, t \geq t_0\}$.

The firm issues a zero coupon bond. The default can only occur at maturity and does so when the firm value is below the face value of debt, $V_T < D$. When the default occurs, the debt-holder obtains a fraction of the firm's value $V_T\phi_{dwl}$. The inclusion of $\phi_{dwl} \leq 1$

regards the possibility of a dead-weight loss, that is, upon default, only a fraction of the value ϕ_{dwl} is recovered.

Following the known solution, the bond value at time-0 will be given by

$$\begin{aligned} B(V,D,T) &= e^{-rT}\mathbb{E}\left[D\mathbb{1}_{\{V_T>D\}} + \phi_{dwl}V_T\mathbb{1}_{\{V_T\leq D\}}|\mathcal{F}_{t_0}\right] \\ &= De^{-rT}N\left(d_2\right) + \phi_{dwl}V_{t_0}e^{-qT}N\left(-d_1\right), \end{aligned} \tag{A.3}$$

where

$$d_1 = \frac{\ln(V_0/D) + \left(r - q + \sigma^2/2\right)T}{\sigma\sqrt{T}},\ d_2 = d_1 - \sigma\sqrt{T}. \tag{A.4}$$

The bond value can also be written in the terms of a put option as

$$B(V,D,T) = De^{-rT} - \underbrace{\left(De^{-rT}N\left(-d_2\right) - \phi_{dwl}V_{t_0}e^{-qT}N\left(-d_1\right)\right)}_{G(V,D,T)=P(V,D,T;1)} \tag{A.5}$$

where $P(V,D,T;1)$ represents a put option. In relation to the usual option formulation, we use the debt value in the place of the strike price and the firm value as the underlying asset. The term represented by the put corresponds to the bond discount, $G(V,D,T)$.

In this modeling setting, the value of the firm equity at time-0 is given by the expression

$$\begin{aligned} E_0(V_0) &= e^{-rT}\mathbb{E}\left[\phi_{dwl}V_T\mathbb{1}_{\{V_T>D\}} - \phi_{dwl}D\mathbb{1}_{\{V_T>D\}}|\mathcal{F}_{t_0}\right] \\ &= \phi_{dwl}V_0e^{-qT}N\left(d_1\left(V_0,D,T\right)\right) - De^{-rT}N\left(d_2\left(V_0,D,T\right)\right), \end{aligned} \tag{A.6}$$

which is equivalent to the price of a call option, $P(V,D,T;-1)$.

Appendix A.2. Options with regime switching

Now, we present options which, upon crossing a barrier, the process of the underlying asset, V, is changed[2]. This change is instantaneous, can occur in one or more of the parameters and each of the process' parameters is piecewise continuous.

In the case of one regime switch, we have as follows

$$\frac{dV_t}{V_t} = \begin{cases} (r_1 - q_1)dt + \sigma_1 dW_V(t) & \text{if } \tau_i > t \\ (r_2 - q_2)dt + \sigma_2 dW_V(t) & \text{if } \tau_i \leq t \end{cases} \tag{A.7}$$

where (r_1, q_1, σ_{V_1}) is the set of the asset's process parameters in the first regime and (r_2, q_2, σ_{V_2}) is the set in the second regime. The regime change occurs after the underlying asset crosses a barrier, which occurs at time τ_i. This corresponds to the first passage time, which is as follows.

Definition 1. *For single barrier option contracts, we define*

$$\tau_L := \inf\{u > t_0 : V_u = L\}, \tag{A.8}$$

as the first hitting time of the lower barrier L, by the asset price V_u, for $u \in [0, T]$. While we denote

$$\tau_U := \inf\{u > t_0 : V_u = U\}, \tag{A.9}$$

as the first hitting time of the upper barrier U, by the asset price V_u, also for $u \in [0, T]$.

As before, with $\phi = 1$ corresponding to a put option and $\phi = -1$ to a call option, the price of a European option with the possibility of a regime change upon crossing a barrier, B, will be given by P_{rc}, such that

[2]While S tends to designate the asset "spot" price, here we use V, given our focus on the firm value. In addition, we will always use D, the debt, in the place of the strike value, K.

$$
\begin{aligned}
P_{rc}(V_0, K, B, T; \phi) &= \mathbb{E}_{\mathrm{Q}}\Big[e^{-\int_0^T r(l)dl}\,(\phi K - \phi V_T)^+ \left(\mathbb{1}_{\{\tau_i > T\}} + \mathbb{1}_{\{\tau_i \le T\}}\right) \mid \mathcal{F}_0\Big]. \\
&= \mathbb{E}_{\mathrm{Q}}\Big[e^{-\int_0^T r(l)dl}\,(\phi K - \phi V_T)^+\, \mathbb{1}_{\{\tau_i > T\}} + e^{-\int_0^T r(l)dl}\,(\phi K - \phi V_T)^+\, \mathbb{1}_{\{\tau_i \le T\}} \mid \mathcal{F}_0\Big] \\
&= \underbrace{\mathbb{E}_{\mathrm{Q}}\Big[e^{-\int_0^T r(l)dl}\,(\phi K - \phi V_T)^+\, \mathbb{1}_{\{\tau_i > T\}} \mid \mathcal{F}_0\Big]}_{P_{ko}} \\
&\quad + \underbrace{\mathbb{E}_{\mathrm{Q}}\Big[e^{-\int_0^T r(l)dl}\,(\phi K - \phi V_T)^+\, \mathbb{1}_{\{\tau_i \le T\}} \mid \mathcal{F}_0\Big]}_{P_{ki}}, \qquad \text{(A.10)}
\end{aligned}
$$

As it can be observed, the first component is a down-and-out option, P_{ko}, is always ruled by the initial regime, as it is deactivated when the barrier is crossed.

As for the second component,

$$
P_{ki}(V_0, K, B, T; \phi) = \mathbb{E}_{\mathrm{Q}}\Big[e^{-\int_0^T r(l)dl}\,(\phi K - \phi V_T)^+\, \mathbb{1}_{\{\tau_i \le T\}} \mid \mathcal{F}_0\Big], \qquad \text{(A.11)}
$$

it is also a barrier option, albeit ruled by a different regime setting in its process. It is also ruled by the first regime, but only until the barrier is crossed. Upon crossing the barrier, a vanilla option is activated, with the barrier value coinciding with the knocked-in value for the initial spot price of the underlying asset and the second regime ruling the underlying asset process until the maturity date.

Thus, by using a Stopping Time (ST) approach, we can separate the two regimes, the first is used until the hitting of the barrier, the second is used afterwards with the approach from Dias et al. (2015). Therefore, in order to obtain the values for equation (A.11), we use Dias et al. (2015, Proposition 3.1), with $\Psi(V_T) = (\phi K - \phi V_T)^+$. Since we are under a GBM diffusion, we are only considering the pure diffusion process without the possibility of hitting the absorbing barrier at zero. Thus we write

$$
\begin{aligned}
P_{ki} =& \mathbb{E}_{\mathrm{Q}}\Big[e^{-\int_0^T r(l)dl}\,(K - V_T)^+\, \mathbb{1}_{\{\tau_i \le T\}} \mid \mathcal{F}_0\Big] \\
=& \int_0^T e^{-\int_0^u r(l)dl} P^{(2)}(B(u), K, T, \phi)\mathbb{Q}^{(1)}\left(\tau_i \in du \mid \mathcal{F}_0\right), \qquad \text{(A.12)}
\end{aligned}
$$

where $\mathbb{Q}^{(1)}\left(\tau_L \in du \mid \mathcal{F}_0\right)$ represents the density function of the first passage time τ_L,

$P^{(2)}(B(u), K, T, \phi)$ represents a vanilla option and $B(u)$ is the value of the barrier crossed at τ_i. Using the i subscript in $P^{(i)}$ and $Q^{(i)}$, allows us to distinguish which regime is being used in equation (A.7). In this case, since the change occurs at τ_i, just as the barrier is crossed, it allows for a separation of the regime change — the first regime occurs until the barrier is crossed, the second regime afterwards, as the vanilla option with the second regime is activated.

The solution to the setting above can be found using the Stopping Time approach as developed by Dias et al. (2015), which also originally also encompasses the Constant Elasticity of Variance (CEV) model and the Jump do Default CEV (JDCEV) model of Carr and Linetsky (2006). By calculating the probabilities of crossing a given barrier separately, the ST approach allows a separation between the V_t regimes as presented in equation (A.7).

Therefore, an option which changes regimes when a barrier is breached can be decomposed into two barrier options. The first is a regular barrier option which is knocked out once the barrier is crossed, and the second is barrier option which is knocked in once that barrier is crossed - but while until the barrier is crossed, the first regime is used for the underlying asset process, and the second process is used is used afterwards, until maturity.

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