CAN INVESTORS PROFIT FROM MEASURING STOCK LIQUIDITY WITH ORDERED FUZZY NUMBERS?

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Abstract: In this paper, we investigate whether measuring stock liquidity based on an ordered fuzzy numbers representation of a limit order book allows investors to develop a profitable investment strategy. To this end, we examine the data from 259 companies listed on the Warsaw Stock Exchange between January 2014 and December 2021. We apply several methods commonly used in asset pricing studies to obtain our baseline results and supplement them with a battery of robustness checks. Our strategy which employs a liquidity measure based on ordered fuzzy numbers generates a weekly return of 0.184% (0.199%) for equal- (value-) weighted portfolios, which translates into an annual return of 9.568% (10.348%). This return remains positive after adjusting for risk and considering trading costs and short-sales restrictions. The strategy outperforms the market buy-and-hold strategy, generating a Sharpe ratio several times higher. These results are useful for investors and portfolio managers as they may help them improve their investment strategies.

Keywords: stock liquidity, liquidity premium, limit order book, ordered fuzzy numbers, transaction costs, investment strategy.

JEL codes: C63, C65, G11, G12.

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Abstract

In this paper, we investigate whether measuring stock liquidity based on an ordered fuzzy numbers representation of a limit order book allows investors to develop a profitable investment strategy. To this end, we examine the data from 259 companies listed on the Warsaw Stock Exchange between January 2014 and December 2021. We apply several methods commonly used in asset pricing studies to obtain our baseline results and supplement them with a battery of robustness checks. Our strategy which employs a liquidity measure based on ordered fuzzy numbers generates a weekly return of 0.184% (0.199%) for equal- (value-) weighted portfolios, which translates into an annual return of 9.568% (10.348%). This return remains positive after adjusting for risk and considering trading costs and short-sales restrictions. The strategy outperforms the market buy-and-hold strategy, generating a Sharpe ratio several times higher. These results are useful for investors and portfolio managers as they may help them improve their investment strategies.

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1. Introduction

How to invest efficiently and beat the market is still an ongoing issue in global finance research. Investors and researchers continue to seek investment strategies that generate an excess return and/or have lower risk exposure. A substantial body of asset pricing literature has identified a significant number of anomalies that could potentially be exploited to generate profits in equity markets (Hou et al., 2020). One of the most important and pervasive anomalies is related to the illiquidity premium, which posits that less liquid stocks generate higher returns (Amihud & Mendelson, 1986). This cross-sectional return pattern has been observed in the US stock market[1], other developed stock markets[2], international markets[3], and some of the emerging markets[4]. However, some studies suggest that although the liquidity premium is a worldwide phenomenon, it exists only among the microcap stocks that are difficult to trade (Cakici & Zaremba, 2021), or appears only during certain periods or market conditions (Eleswarapu & Reinganum, 1993).

Previous research has employed a variety of measures to capture stock liquidity. In recent years a significant number of studies have been conducted to identify the most effective proxy for stock liquidity[5]. Sometimes the use of a specific measure may lead to different outcomes (Brennan et al., 2012; Marshall & Young, 2003), indicating that these measures differ in terms of the liquidity premium they capture. Consequently, the use of a specific liquidity measure may lead to the profitability of an investment strategy that exploits the anomaly related to the stock liquidity premium.

[1] See e.g. (Amihud, 2002; Amihud & Mendelson, 1986; Amihud & Noh, 2021a; Brennan & Subrahmanyam, 1996; Eleswarapu & Reinganum, 1993; Harris & Amato, 2019; Huh, 2014).
[2] See e.g. (Chang et al., 2010; Chen & Sherif, 2016; Marshall & Young, 2003).
[3] See e.g. (Amihud et al., 2015; Amihud & Noh, 2021b; Cakici & Zaremba, 2021; Chiang & Zheng, 2015).
[4] See e.g. (Amihud et al., 2015; Bekaert et al., 2007; Machado & de Medeiros, 2012; Stereńczak, 2021, 2022).
[5] See e.g. (Abdi & Ranaldo, 2017; Będowska-Sójka & Echaust, 2020; Corwin & Schultz, 2012; Fong, Holden, & Trzcinka, 2017; Goyenko et al., 2009; Li et al., 2018).

Some of the liquidity measures require only daily transactional data to be computed. For example, Amihud's (2002) illiquidity ratio, which is one of the most frequently employed measures, requires only data on daily returns and trading volumes to be computed. Other measures, such as those developed by Corwin and Schultz (2012), Abdi and Ranaldo (2017), and Li et al. (2018), require daily open, close, high and low prices as inputs for estimation. Low-frequency measures based on daily data only allow for the proxy of liquidity, with various measures demonstrating varying degrees of effectiveness in this regard. This disadvantage does not pertain to the measures utilising data not on orders placed in a market, as opposed to trades. Such measures are ex-ante liquidity measures and typically require data from the limit order book (LOB).

Beyond the most commonly used order-based measures, such as the bid-ask spread (Amihud & Mendelson, 1986), its fixed and variable components (Brennan & Subrahmanyam, 1996; Huh, 2014) that rely only on the best bid- and ask-quotes, one can model the entire limit order book to measure stock liquidity (Kalay et al., 2004; Marszałek & Burczyński, 2024; Næs & Skjeltorp, 2006). Of the aforementioned three LOB measures, only that of Marszałek and Burczyński (2024) employs the concept of ordered fuzzy numbers (OFN) (Kosiński et al., 2003, 2002). The remaining two measures, those of Kalay et al. (2004) and Næs & Skjeltorp (2006) employ price-level-based data representation schemes that are highly susceptible to subtle perturbations in the LOB data, such as minor fluctuations in order placements and cancellations. This suggests that modelling LOB using OFN may facilitate a more accurate representation of stock liquidity, particularly in less liquid markets where large order flows can exert considerable price movements.

The paper aims to determine whether measuring stock liquidity based on ordered fuzzy numbers representation of a limit order book (Marszałek & Burczyński, 2024) yields benefits to investors by giving them the possibility to develop a profitable investment strategy. In particular, we seek to ascertain whether a liquidity measure based on ordered fuzzy numbers contains distinct or similar information to that contained in other commonly used liquidity measures. This will help us in determining whether the computation of this measure is worth the effort. It would not be worth the effort if our liquidity measure contained the same or highly similar information as other, more readily calculated measures. Additionally, we examine whether the stock liquidity measure based on the limit order book modelled with ordered fuzzy numbers captures a higher liquidity premium, which translates into a higher return to a zero-investment portfolio that goes long with the least liquid and shorts the most liquid stocks.

To this end, we investigate the data from 259 (in total) companies listed in the Warsaw Stock Exchange (WSE) between January 2014 and December 2021. The Warsaw Stock Exchange (WSE) provides an interesting context for examining the potential benefits of OFN representation of the LOB to investors. The WSE is still considered an illiquid market, in which large order flows may exert a significant influence on stock liquidity. Additionally, the majority of trading activity on the WSE is concentrated in a small number of stocks, with the 11 most heavily traded stocks accounting for 2.28% of all companies and representing 80% of the turnover. Smaller and less liquid stocks are subject to infrequent trading and a significant number of zero-trading volume days. These WSE features are likely to bias the commonly used liquidity proxies based on daily data (Chelley-Steeley et al., 2015) thereby rendering the OFN representation of the LOB more efficient in capturing stock liquidity. Although we utilise the data from the Warsaw Stock Exchange, the OFN liquidity measure may also be applicable in other markets with a limit order book.

In addition to the ordered fuzzy numbers limit order book liquidity measure (Marszałek & Burczyński, 2024), our analyses consider four different liquidity measures in our analyses: Amihud's (2002) illiquidity ratio, the bid-ask spread (Amihud & Mendelson, 1986), and two versions of Næs and Skjeltorp's (2006) LOB slope. We employ a range of analytical techniques, including one-way liquidity sorts, cross-sectional regressions and two-way dependent sorts. In addition, we conduct a series of robustness checks to assess the resilience of strategy returns to transaction costs and short-sales constraints, and to evaluate the strategy's efficiency relative to a market in general.

The findings of our study contribute to several streams of research. All contributions arise from the utilisation of a novel measure of stock liquidity, which is based on the ordered fuzzy numbers representation of the limit order book developed by Marszałek and Burczyński (2024). An ordered fuzzy number is defined as a pair of two functions that are used to process imprecise and uncertain data, which certainly includes the concept of stock liquidity, which is considered an elusive concept. Furthermore, ordered fuzzy numbers possess well-defined arithmetic properties. As demonstrated, converting LOB into OFN makes it possible to create a time series to represent input data for deep learning models (Marszałek & Burczyński, 2024). In this study, we go one step beyond and investigate whether investors can benefit from using it in the stock market.

The first contribution of our study is the generation of some intriguing insights regarding the liquidity premium. Therefore, our study contributes to the extensive body of research in asset pricing that has focused on stock liquidity issues. The existing literature on this topic is inconclusive, particularly with regard to emerging markets. Amihud et al. (2015) and Beakert et al. (2007), among others, have reported that the liquidity premium is higher in emerging than in developed stock markets. However, in frontier equity markets, which are even less developed than emerging markets, no liquidity premium is observed (Batten & Vo, 2014; Stereńczak et al., 2020). Cakici and Zaremba (2021) found that the liquidity premium is present across all markets, although it is only observed among microcap stocks. Our study presents further evidence for the liquidity premium puzzle, as it reports a significantly positive return on a long-short liquidity portfolio among the largest companies in the WSE.

Our second contribution emerges from the presentation of a profitable investment strategy that outperforms the market. Hence, our findings contribute to the existing body of knowledge in the field of portfolio management. In our analyses, we consider trading costs and short-sales constraints, thereby ensuring greater realism. Notwithstanding the aforementioned market frictions, our investment strategy continues to generate positive returns and outperforms the market. A comparable approach was put forth by Korajczyk and Sadka (2004), who investigated whether momentum strategies continue to be profitable when trading costs are taken into account. In the context of emerging European markets, Zaremba and Nikorowski (2019) observed that the prevailing majority of anomalies cease to be profitable once trading costs are taken into account, irrespective of the frequency of portfolio rebalancing. Zaremba and Andreu (2018) reached analogous conclusions based on the sample of 42 countries. However, they found that annual rebalancing helps to regain the profitability of the strategies. Our study demonstrates that the strategy that employs the OFN representation of the LOB generates markedly positive returns even after accounting for considerable trading costs associated with weekly portfolio rebalancing.

Our third contribution builds upon the second and relates to the existing literature on market efficiency. The efficient market hypothesis posits that asset prices reflect all available information. Fama (1970) distinguished three forms of informational efficiency: weak, semi-strong and strong. Since it is possible to profit from our strategy, which utilises the information content of stock prices and other information that can be easily made publicly available (i.e. LOB data), our study also proves that the WSE is at most weak efficient. Our strategy is profitable only because stock prices do not reflect the information content of the limit order book. This translates into prices not reflecting all publicly available information, thereby giving rise to the information asymmetry problem (Akerlof, 1970; Stiglitz, 2002).

Finally, we contribute to the ongoing debate on liquidity measurement. Since the publication of the seminal paper by Amihud and Mendelson (1986), academics have sought to identify a proxy for liquidity that is straightforward to calculate and reflects stock liquidity accurately[6]. The results of our analyses indicate that the OFN representation of the LOB may serve as a liquidity measure, but provides information distinct from that of Amihud's (2002) illiquidity ratio and the bid-ask spread. This, in turn, suggests that our OFN liquidity measure captures a different liquidity dimension than Amihud's (2002) illiquidity ratio or the bid-ask spread.

The remainder of the paper is structured as follows. The following section presents a brief overview of the methodology of measuring stock liquidity utilising the ordered fuzzy numbers representation of the limit order book. Section 3 outlines the data sources and analytical techniques employed. Section 4 compares the OFN liquidity measure to alternative proxies for liquidity. Section 5 presents the baseline results, while Section 6 provides a more in-depth analysis of the profitable strategies derived in Section 5. The final section concludes.

2. Measuring stock liquidity with ordered fuzzy numbers

The representation of limit order book data using ordered fuzzy numbers, as proposed by Marszałek and Burczyński (2024), introduces a novel and robust approach to the handling of the dynamic and irregular nature of financial market data. The majority of traditional methods (e.g. Kalay et al., 2004; Næs & Skjeltorp, 2006)often rely on price-level data representations, which can be susceptible to minor perturbations and noise. In contrast, the use

[6] See e.g. (Abdi & Ranaldo, 2017; Corwin & Schultz, 2012; Fong, Holden, & Trzcinka, 2017; Goyenko et al., 2009; Li et al., 2018).

of OFNs provides a more stable and precise mathematical framework for capturing the complexities of LOB data.

Ordered fuzzy numbers, as introduced by Kosinski et al. (2003, 2002), are defined through ordered pairs of continuous real functions on the interval $[0,1]$, i.e.,

$$A = (f, g) \text{ with } f, g: [0,1] \rightarrow R \text{ as continuous functions,}$$

rather than traditional membership functions $\mu_A: R \rightarrow [0,1]$ through which the "classic" fuzzy numbers are defined. This structure allows basic operations like addition, subtraction, multiplication, and division to be performed pairwise, maintaining neutral elements and preventing the support of the fuzzy number from constantly increasing. Thus, OFNs enable the creation of fuzzy models using classical equations. Moreover, essential functions such as logarithm, exponentiation, absolute value, and square root can be similarly defined (Prokopowicz, 2013).

The process of transforming LOB data into OFN is defined as follows (Marszałek & Burczyński, 2024). Let $\{p_a^i(t), v_a^i(t)\}_{i=1}^{L_a}$ and $\{p_b^i(t), v_b^i(t)\}_{i=1}^{L_b}$ be a complete snapshot of LOB at time t (all price levels), where $p_a^i(t)$, $p_b^i(t)$ are the ask and bid prices for price level i at time t and $v_a^i(t)$, $v_b^i(t)$ are the ask and bid volumes, respectively, and L_a, L_b are the number of (nonzero) price levels considered for ask and bid side, respectively. Moreover, let $p_r(t)$ be a reference price at time t (e.g. mid-price). Then the functions f and g of OFN $A_t = (f_t, g_t)$ at time t are defined as follows:

$$f_t(x) = -\begin{cases} 0 & \text{if } (1-x)p_r(t) > \mu_a^1(t), \\ \sum_{i=1}^{L_a} v_a^i(t) & \text{if } (1-x)p_r(t) \leq \mu_a^{L_a}(t), \\ \left(\sum_{i=1}^{l_a} v_a^i(t)\right)\left(1 + \frac{\mu_a^{l_a}(t) - (1-x)p_r(t)}{(1-x)p_r(t) - p_a^{l_a+1}(t)}\right) & \text{otherwise} \end{cases}$$

$$g_t(x) = \begin{cases} 0 & \text{if } (1+x)p_r(t) < \mu_b^1(t), \\ \sum_{i=1}^{L_b} v_b^i(t) & \text{if } (1+x)p_r(t) \geq \mu_b^{L_b}(t), \\ \left(\sum_{i=1}^{l_b} v_b^i(t)\right)\left(1 + \frac{\mu_b^{l_b}(t) - (1+x)p_r(t)}{(1+x)p_r(t) - p_b^{l_b+1}(t)}\right) & \text{otherwise} \end{cases}$$

where $\mu_a^i(t)$ and $\mu_b^i(t)$ are volume-weighted average prices at time t from level 1 to i for ask and bid prices, respectively, l_a is the lowest i that satisfies the relation $\mu_a^i < (1-x)p_r(t)$ and l_b is the lowest i that satisfies the relation $\mu_b^i > (1+x)p_r(t)$.

The conversion of limit order book data into ordered fuzzy numbers is designed to illustrate a given instrument's depth as defined by Sarr and Lybek (2002), indicating how many shares can be sold or bought at any given moment and how it will impact the transaction price. Figure 1 illustrates an example of an OFN generated from the LOB of KGHM on January 3, 2017, at 09:22:52. The values of $|f(x)|$ and $g(x)$ represent the sizes of trades that can be executed at different levels of potential cost $(1-x)$, which is the percentage difference between the actual average transaction price and the reference price for the ask and bid sides. If an investor wishes to purchase 30,000 KGHM shares all at once, the average price to buy per share would be 94.97. Compared to the current mid-price of 94.52, this results in an approximate loss of 0.4%. Similarly, selling 10,000 shares immediately would yield an average selling price of 94.22 per share, leading to an approximate loss of 0.3% compared to the current mid-price.

[FIGURE 1 ABOUT HERE]

For practical application, the values of f and g are normalised. Additionally, adjustments such as limiting the number of price levels considered (e.g., to ±10% of the mid-price) and setting maximum cost parameters are applied to refine the representation . In order to facilitate numerical computations, the continuous functions f and g are discretised by sampling them at uniformly or logarithmic-spaced points within the interval $[0,1]$. This discretisation allows the OFNs to be represented as vectors, enabling efficient arithmetic operations. A detailed description is available in the study by Marszałek and Burczyński (2024).

Following the interpretation of the defined ordered fuzzy number, it can be assumed that the ordered fuzzy number constructed based on LOB data is itself a (fuzzy) measure of an asset's liquidity at a given time t. However, since comparing fuzzy numbers is not straightforward, for our research we define the liquidity measure as a crisp value (real number) obtained by applying a defuzzification operator. Let $A_t = \left(f_{A_t}, g_{A_t}\right)$ be ordered fuzzy number generated from the limit order book data at time t. Then, a crisp liquidity measure LIQ^{OFN} of A_t is computed by using the expected value defuzzification operator (Marszałek & Burczyński, 2021) specified by the formula:

$$LIQ^{OFN}(A_t) = E(|A_t|) = E\left(\left|f_{A_t}\right|, \left|g_{A_t}\right|\right) = \frac{1}{2}\int_0^1 \left[\left|f_{A_t}(s)\right| + \left|g_{A_t}(s)\right|\right] ds. \tag{1}$$

Additionally, by applying a similar formula to only one of the functions, we can define separate liquidity measures for the ask and bid sides, respectively:

$$LIQ_{ask}^{OFN}(A_t) = \int_0^1 \left|f_{A_t}(s)\right| ds, \quad LIQ_{bid}^{OFN}(A_t) = \int_0^1 \left|g_{A_t}(s)\right| ds. \tag{2}$$

It is important to note that from equations (1) and (2) the following relationship exists between these defined measures:

$$LIQ^{OFN} = \frac{1}{2}(LIQ_{ask}^{OFN} + LIQ_{bid}^{OFN}). \tag{3}$$

3. Data and methods

To examine whether investors can profit from measuring stock liquidity with ordered fuzzy numbers, we aim to create an investment strategy based on one of the most pervasive asset pricing phenomena, namely the liquidity premium. In order to ascertain whether our LIQOFN is capable of generating a positive liquidity premium, we have conducted a series of examinations. These include one-way (univariate) portfolio sorting, cross-sectional regressions and two-way (bivariate) dependent portfolio sorting. To conduct the aforementioned tests, it is necessary to merge the data gathered from several sources. The data employed in this study, specifically the limit order book (LOB) data comprising all buy and sell orders placed within a specified time interval, were obtained directly from the Warsaw Stock Exchange. The data set encompasses the period from 2014 to 2021 and comprises solely those stocks included in three indices: WIG20, mWIG40 and sWIG80. This means in each period we dispose of the data on 140 stocks (out of approximately 450 listed during the period under scrutiny). Companies' financial data and stock prices adjusted for corporate actions are sourced from the S&P Capital IQ database. The LOB data are matched with other data using the stock issue ISIN.

To calculate our ordered fuzzy numbers liquidity measure, we take LOB snapshots at 10-minute intervals. Taking into account that trading on the WSE occurs between the hours of 9 a.m. and 5 p.m., and given that opening and closing auctions are excluded due to disparate regulations governing their conduct, we are left with 47 LOB snapshots per day, amounting to approximately 235 snapshots per week. For each snapshot, we compute $f_t(x)$ and $g_t(x)$ to get the order fuzzy number A_t at time t as described in Section 2. The resulting values are then averaged across each week. OFNs are generated based on only the price levels that fall within the daily price fluctuation limit of ±10%, which is calculated based on the current best bid and ask prices. The reference price is calculated as the mid-price at time t. The maximum cost parameter to refine the representation is set to 0.035, which is equal to the dynamic price fluctuation limit. Imposing these two limits is aimed at aligning the liquidity measure more closely with the real conditions of trading in the WSE.

For numerical computation, we used discretisation with 11 points from the interval [0, 1], distributed in a regular, evenly spaced manner on a logarithmic scale. Furthermore, in the formulas for f and g, we use the volume multiplied by the price instead of just the volume. Finally, for presentational purposes we scale the obtained values by dividing by a factor of 10^6. Subsequently, for each weekly ordered fuzzy number, we calculate the crisp liquidity measure LIQ^{OFN} as defined in equation (1). Figure 2 illustrates an example of LIQ^{OFN} for 27 weeks of one of the most liquid stocks in the sample, KGHM.

[FIGURE 2 ABOUT HERE]

We compare our measure, LIQ^{OFN}, and the results of the strategy based on it with other commonly used liquidity proxies, namely Amihud's (2002) illiquidity ratio (LIQ^{Amihud}) and bid-ask spread (LIQ^{BAS}). Furthermore, we compare our measure, LIQ^{OFN}, with another measure of stock liquidity based on the limit order book data, namely the LOB slope (Næs & Skjeltorp, 2006). Amihud's (2002) illiquidity ratio is calculated as the absolute value of the mid-price change between two LOB snapshots divided by the trading volume in this 10-minute interval, averaged across the week. Næs and Skjeltorp's (2006) LOB slopes are calculated for each snapshot from 10 ticks from the best quote ($LIQ^{LOBslope10}$) and the full limit order book ($LIQ^{LOBslope}$) and also averaged across the week. Furthermore, the bid-ask spread is calculated from the best buy- and sell orders from each LOB snapshot and averaged across the week.

In the initial analysis, we compare our liquidity measure, LIQ^{OFN}, to other proxies, namely Amihud's (2002) illiquidity ratio (LIQ^{Amihud}), bid-ask spread (LIQ^{BAS}) and Næs and Skjeltorp's (2006) LOB slopes ($LIQ^{LOBslope10}$, $LIQ^{LOBslope}$). The measures are then compared in several areas. The distributional properties of the measures are examined, with particular attention paid to variability and non-normality. Additionally, the correlation among the measures is investigated, as is the extent of prediction errors. Similar comparisons are common in the assessment of the suitability of newly developed liquidity measures (Abdi & Ranaldo, 2017; Corwin & Schultz, 2012; Fong, Holden, & Trzcinka, 2017; Goyenko et al., 2009; Li et al., 2018) and will help in determining whether representation of the LOB by OFNs provides information that is not present in other proxies.

To examine whether investors can leverage the liquidity premium to develop a profitable investment strategy, we have conducted a series of tests, including one-way (univariate) portfolio sorting, cross-sectional regressions and two-way (bivariate) dependent portfolio sorting. To perform the univariate portfolio sorts, at the end of each week *t-1*, we rank all the stocks in the sample according to the value of one of five liquidity measures – LIQ^{OFN}, LIQ^{Amihud}, LIQ^{BAS}, $LIQ^{LOBslope10}$, and $LIQ^{LOBslope}$ and form equal-weighted and value-weighted quintile portfolios. Additionally, a zero-investment portfolio is constructed to serve as an ad hoc test of monotonicity in the cross-section of returns. This portfolio goes long the quintile of the least liquid and short the most liquid shares, according to one of the five measures.

We evaluate the performance of these portfolios with their week *t* raw log-return and risk-adjusted returns (αs) calculated based on CAPM (α_{CAPM}), Fama and French's (1992) three-factor model (α_{FF3}) and Carhart's (1997) four-factor model ($\alpha_{Carhart}$). The Fama and French five- (2015) and six-factor (2018) models are not applied in this study, as the profitability and investment policies of companies are unlikely to be relevant in a weekly horizon. Furthermore, the momentum factor and four-factor Carhart (1997) are the most effective in explaining returns in the Polish stock market (Zaremba et al., 2019). The factor returns are calculated based on all firms listed on the Warsaw Stock Exchange and closely replicating the procedures outlined in original papers by Fama and French (1992) and Carhart (1997). To compute the momentum factor, the cumulative return over the previous 52 weeks (one year) is employed.

The second method employed to ascertain the potential profitability of our LIQ^{OFN} is cross-sectional regression analysis. Unlike Fama and MacBeth (1973), we utilise pooled cross-sectional time-series data to derive the coefficient estimates. In this method, we regress week t stock returns on liquidity measures and other company characteristics in week $t-1$:

$$R_{it} = \alpha_i + \alpha_t + \beta_{LIQ} LIQ_{it-1} + \sum_{j=1}^{J} \beta_j X_{it-1}^{j} + \varepsilon_{it}, \qquad (4)$$

where R_{it} is the weekly excess (over risk-free return as proxied by the WIBOR rate) return on stock i in week t, LIQ_{it-1} is one of the liquidity measures (LIQ^{OFN}, LIQ^{Amihud}, LIQ^{BAS}, $LIQ^{LOBslope10}$ or $LIQ^{LOBslope}$) and X_{it-1}^{j} refers to the j–th control variable. The set of control variables includes the market value (*MV*) represented by the natural logarithm of total stock market capitalisation at the end of the week (Banz, 1981), book-to-market ratio (*B-MV*) for week t calculated as the book value of equity half a year before week t over the most recent market capitalisation (Fama & French, 1992); momentum (*MOM*) is the 52-week average of the weekly log-return (Jegadeesh & Titman, 1993), stock return volatility (*VOL*) calculated as a standard deviation of weekly stock returns in recent 52-week period, and turnover ratio (*TURN*) computed as a trading volume (in units of shares) scaled by the number of outstanding shares. To alleviate the impact of the outliers, all continuous variables were cross-sectionally winsorised at the 1st and 99th percentiles of the distribution.

In the final test, we sought to ascertain whether the magnitude of returns generated by our strategy was independent of the type of equities. To check this, the sample was divided into distinct groups based on various variables, and the performance of the long-short portfolio was evaluated within each group. In particular, we form quartile portfolios from two-way dependent sorts on control variables and a liquidity measure. In the first pass, all the stocks in a given week are ranked according to one of the control variables, i.e. *MV*, *B-MV*, *MOM*, *VOL* or *TURN*. In the second pass, stocks within each quartile are sorted into four portfolios based on one of the liquidity measures. Additionally, long-short quartile portfolios are formed based on *LIQ* within each quartile of the company characteristics. We form both, equal- and value-weighted portfolios and evaluate the performance of these portfolios with their week *t* raw log-return and risk-adjusted returns (αs) calculated based on Carhart's (1997) four-factor model ($\alpha_{Carhart}$).

4. OFN limit order book measure as a liquidity indicator

We begin our analysis of the profitability of measuring stock liquidity using ordered fuzzy numbers with a comparison of our measure, LIQ^{OFN}, to other commonly used liquidity proxies, namely Amihud's (2002) illiquidity ratio (LIQ^{Amihud}) and bid-ask spread (LIQ^{BAS}), and another measure of stock liquidity based on the limit order book data, namely LOB slope (Næs & Skjeltorp, 2006). The full details of the computation of each of these measures are presented in the previous section. The comparison conducted in this section allows us to conclude whether LIQ^{OFN} contains distinct or similar information to that contained in other commonly used liquidity measures, which helps to answer the question of whether the computation of our measure is worth the effort. We compare LIQ^{OFN} to other liquidity proxies in several aspects.

The first area of comparison is the distributional properties of the measures. To this end, we compare the means, standard deviations, coefficients of variation, skewness and kurtosis of the distributions of the measures in question. The results of this comparison are presented in Table 1; Panel A contains time-series averages of the cross-sectional means, standard deviations, skewness and kurtosis. Meanwhile, Panel B demonstrates the cross-sectional averages of the time-series statistics. The information provided by both panels is different and allows us to make inferences about the distributions of the measures for other purposes.

[TABLE 1 ABOUT HERE]

It is of particular importance for the purposes of asset pricing to ensure sufficient variation among companies, and thus cross-sectional distributional properties are of great consequence. As illustrated in Panel A of Table 1, our LIQ^{OFN} is characterised by the second-highest coefficient of variation, with only LIQ^{Amihud} exhibiting a higher degree of cross-sectional variation. The standard deviations of the bid-ask spread and of Næs and Skjeltorp's (2006) LOB slopes' are of a significantly lower order of magnitude. All the measures are right-skewed, indicating that their means are higher than medians. This is an intriguing observation, as LIQ^{Amihud} and LIQ^{BAS} reflect illiquidity, with higher values indicating lower liquidity. In contrast, other proxies reflect stock liquidity, which means liquidity increases with their values. Thus, mean liquidity as gauged by the bid-ask spread or Amihud's (2002) ratio is lower than the median, in contrast to liquidity as measured with measures based on LOB data. All five measures exhibit elevated cross-sectional kurtosis, indicating a greater extremity of outliers than would be expected in a normal distribution. However, our LIQ^{OFN} has the second-lowest kurtosis among all five measures. Therefore, our liquidity measure provides a relatively high degree of cross-sectional variation, is right-skewed and has elevated kurtosis. The two latter statistics are superior (i.e. have lower values) to at least one of the commonly accepted liquidity proxies, namely bid-ask spread and Amihud's (2002) ratio.

When the time-series distributional properties are taken into consideration (Panel B of Table 1), it can be seen that our LIQ^{OFN} exhibits the second-highest volatility. This is unfortunate, as it is likely to make forecasting liquidity more difficult. Similarly, the time-series volatilities of the bid-ask spread and Næs and Skjeltorp's (2006) LOB slopes' are significantly smaller in magnitude. It is also noteworthy that these measures also exhibit lower skewness and kurtosis than LIQ^{OFN}. This information suggests that LIQ^{OFN} may present greater challenges in forecasting liquidity compared to the other measures. Only LIQ^{Amihud}'s time-series distributions are inferior to those of LIQ^{OFN}.

Secondly, the ordered fuzzy numbers measure of liquidity is compared to other proxies through the analysis of the correlations among the measures. To this end, we employ a methodology similar to that used by Goyenko et al. (2009), Corwin and Schultz (2012), Abdi and Ranaldo (2017) or Fong, Holden and Trzcinka (2017). This involves calculating both: 1) the time-series average of the cross-sectional Pearson correlation among liquidity proxies and 2) the cross-sectional average of the time-series Pearson correlation among them. To provide additional insight, we also examine the time-series average of cross-sectional Spearman rank correlation, as Fong, Holden and Tobek (2017) did. The results are presented in Table 2; Panel A reports the time-series averages of the cross-sectional Pearson correlation, while Panel B demonstrates the cross-sectional average of the time-series Pearson correlation among liquidity proxies. Panel C presents time-series averages of cross-sectional Spearman rank correlation and Panel D presents the absolute changes in liquidity rank from one week to another[7]. As LIQ^{BAS} and LIQ^{Amihud} measure illiquidity, only for correlation purposes we multiply their values by -1 to ensure that a positive value of the correlation coefficient denotes a positive correlation of liquidity.

[7] The absolute change in a liquidity rank is calculated as follows: $AbsChange_{it} = |LIQ_{it}^{R} - LIQ_{it-1}^{R}|$, where LIQ_{it}^{R} is the rank of *i*th company in week *t* based on the given *LIQ* measure.

[TABLE 2 ABOUT HERE]

In the absence of an indication of a benchmark, as is the case with the studies by Goyenko et al. (2009), Corwin and Schultz (2012), Abdi and Ranaldo (2017) or Fong, Holden and Trzcinka (2017) among others, we can only claim whether LIQ^{OFN} contains similar or different information about liquidity than other metrics under scrutiny. As illustrated in Panel A of Table 2, LIQ^{OFN} exhibits the highest cross-sectional correlation with Næs and Skjeltorp's (2006) LOB slope calculated from 10 ticks from best quotes. Nevertheless, the correlation is approximately 0.7, indicating that LIQ^{OFN} encompasses information not captured by $LIQ^{LOBslope10}$. The correlation of LIQ^{OFN} with other considered proxies is less than 0.5, with the correlation with LIQ^{Amihud} and $LIQ^{LOBslope}$ equalling only several percent. Thus, the cross-sectional Pearson correlation suggests that our liquidity measure behaves similarly to other measures based on LOB data, but potentially captures a different liquidity dimension than Amihud's (2002) illiquidity ratio and the bid-ask spread.

A weaker correlation is observed between LIQ^{OFN} and $LIQ^{LOBslope10}$ when the time-series Pearson correlation is considered. Conversely, a stronger (than cross-sectional) time-series correlation of LIQ^{OFN} with LIQ^{BAS}, LIQ^{Amihud} and $LIQ^{LOBslope}$ is observed. Nevertheless, it can be argued that our liquidity measure based on ordered fuzzy numbers contains some distinct information and reflects a different dimension of stock liquidity than other proxies.

It is agreed in the literature that stock liquidity is a multifaceted concept, which encompasses several transactional properties of the market. Sarr and Lybek (2002) distinguished five distinct dimensions of stock liquidity: tightness, immediacy, depth, breadth and resiliency. The bid-ask spread is typically regarded as a gauge of tightness (transaction costs), while Amihud's (2002) illiquidity ratio is thought to reflect market breadth or resiliency (price impact). Our LIQ^{OFN}, similar to LOB slope measures by Næs and Skjeltorp (2006), refers to market depth, indicating the existence of numerous and large orders above and below the current trading price. Our findings corroborate the notion of stock liquidity as a multidimensional concept, which cannot be encapsulated by a single measure (Sarr & Lybek, 2002).

Although the cross-sectional Pearson correlation among the liquidity proxies under consideration is relatively weak, all except $LIQ^{LOBslope}$ appear to rank stocks according to their liquidity in a manner that is broadly similar. This is evidenced by Panel C of Table 2. The Spearman rank correlation coefficients are considerably higher than the Pearson correlation coefficients, indicating the potential for non-linearity in the relationship between the considered measures. It is noteworthy that, in the context of asset pricing and portfolio formation based on percentiles of certain companies' characteristics, the Pearson correlation is of lesser importance as these issues are focused on a rank, not the specific value. Consequently, the use of LIQ^{OFN} in asset pricing and portfolio formation purposes can yield similar results to those obtained using other considered proxies.

Nevertheless, which is also very important in creating a profitable investment strategy, one should notice that LIQ^{OFN} provides more stable stock rankings, which is evidenced by mean absolute change in liquidity rank (Panel D of Table 2). It is noteworthy that despite the relatively elevated time-series volatility and a notable degree of correlation with other liquidity proxies exhibited by LIQ^{OFN}, each week it consistently ranks the analysed stocks in an order that closely resembles the previous week's ranking. This suggests that a long-short portfolio based on this measure will be subject to relatively lower turnover than long-short portfolios based on other measures. Lower portfolio turnover translates into lower transaction costs thereby making LIQ^{OFN} a superior proxy in this context.

Finally, we compare our liquidity measure to other proxies in terms of their predictability. Given that investors are interested not only in liquidity-related costs incurred at the time of purchase but also at the time of sale (Amihud et al., 2005), it is reasonable to expect stock liquidity to be predictable. In case of considerable discrepancy between predicted and actual outcomes, the usefulness of a liquidity measure can be called into question. In order to verify the predictive capacity of a given liquidity proxy, we compare them in terms of the average relative errors[8] (ARE), mean absolute errors[9] (MAE) and root mean squared errors[10] (RMSE) of the forecasts generated by an AR(1) model. Such an approach is similar to that employed by Li et al. (2018), but we do not consider the discrepancies between estimated and true spread. Instead, we focus on the differences between the predicted and observed values of a liquidity measure.

In order to make forecasts of liquidity measures, we employ a simple AR(1) model. In particular, to predict the value of a given liquidity measure for week *t*, an AR(1) model is estimated using the values of this liquidity measure from weeks *t-27* to *t-1* (26 weeks provides approximately half a year of data). Subsequently, the week *t* liquidity measure value is predicted based on the week *t-1* value, and the prediction error is calculated as a difference between the estimated liquidity for week *t* ($E_{t-1}[LIQ_t]$) and the observed value (LIQ_t), scaled by the observed value of the liquidity measure (LIQ_t). Table 3 presents the values of forecasting errors for all five considered liquidity measures.

[TABLE 3 ABOUT HERE]

[8] Average relative error is computed as $ARE = E[(E_{t-1}[LIQ_t] - LIQ_t)/LIQ_t]$.
[9] Mean absolute error is calculated as $MAE = E[|E_{t-1}[LIQ_t] - LIQ_t|/LIQ_t = |RE|]$.
[10] Root mean squared error is calculated as $RMSE = \sqrt{E[(E_{t-1}[LIQ_t] - LIQ_t)/LIQ_t]^2}$.

The results from Table 3 are in accordance with the expected outcomes. Given that LIQ^{OFN} exhibits the second-highest time-series volatility, it is to be expected that this measure will be hardly predictable. The results from Table 3 somehow resemble those presented in Panel B of Table 1, i.e. Amihud's (2002) illiquidity ratio has the highest time-series volatility and so do the prediction errors. In contrast, other liquidity measures, such as the bid-ask spread and Næs and Skjeltorp's (2006) LOB slopes, display less volatility and are thus more predictable.

Overall, it is our contention that our ordered fuzzy numbers limit order book measure is likely to reflect stock liquidity and, potentially, yields profits for investors who utilise it. Firstly, it provides a sufficient cross-sectional variation to differentiate between stocks of low and high liquidity. Secondly, it correlates with another LOB measure in the cross-section, with other liquidity measures in the time series, and ranks stocks according to their liquidity in a manner that is consistent with other proxies. However, it should be noted that the measure still contains information not captured by the other liquidity measures. Thirdly, since LIQ^{OFN} provides relatively stable cross-sectional stock rankings according to their liquidity, it provides a premise for a lower turnover of long-short portfolios. This latter issue is likely to make our liquidity measure superior to other proxies when creating an efficient investment strategy, even taking its relatively high prediction errors into account.

5. Basic results

5.1. Univariate portfolio sorting

We start our mainstream analyses with the examination of the performance of quintile portfolios sorted by liquidity. This is measured with our LIQ^{OFN} proxy and other liquidity measures for comparison. This will facilitate the determination of whether the sorting of stocks into portfolios based on their liquidity results in a cross-sectional return pattern, thereby enabling investors to develop a profitable investment strategy. The summary of the results of one-way sorted portfolios is presented in Table 4, and Figure 3 illustrates the cumulative return on zero-investment long-short portfolios that short the most liquid stocks and go long the least liquid ones, utilising five different liquidity proxies.

[TABLE 4 ABOUT HERE]

[FIGURE 3 ABOUT HERE]

In contrasto to recent evidence from developed markets (Amihud & Noh, 2021a; Chiang & Zheng, 2015; Huh, 2014), other emerging markets (Bekaert et al., 2007; Stereńczak, 2017), and also from the Warsaw Stock Exchange (Stereńczak, 2021, 2022), the zero-investment long-short portfolios based on stock liquidity yield negative returns. Irrespective of the liquidity measure employed, the average weekly return on an equal-weighted long-short portfolio is observed to range from -0.183% to -0.288%. This translates into an annual return of approximately -9.52% to -14.98%. These negative returns are statistically significant and are not attributable to the higher or lower risk exposure as evidenced by the significantly negative risk-adjusted returns (αs) on these long-short portfolios. Both the returns and alphas of value-weighted zero-investment portfolios also yield negative results, though these are not statistically significant.

The results of univariate portfolio sorting are generally consistent with those reported by Marshall and Young (2003) and Brennan and Subrahmanyam (1996). Both studies found a negative and statistically significant relationship between the bid-ask spread and stock returns. However, they attributed this relationship to the fact that the bid-ask spread is likely acting as a proxy for a risk variable related to the reciprocal of the stock's price due to an inaccurate beta estimation. This, in turn, suggests that the performances of our one-way sorted portfolios may be driven by other companies' characteristics somehow captured by or correlated with our liquidity measures. We address this concern in the following section by running cross-sectional regressions.

5.2. Cross-sectional regressions

Table 5 presents the results of the cross-sectional regressions. To test the robustness of the inferences, several models' specifications are employed. In each specification, however, we use robust standard errors clustered by a firm and by a week to take potential heteroskedasticity and autocorrelation of residuals into account.

[TABLE 5 ABOUT HERE]

Panel A of Table 5 presents the slope coefficients for univariate models, which include a single explanatory variable, namely stock liquidity. Therefore, it is comparable to the analyses presented in Table 4, although on a single stock rather than a portfolio level. It is noteworthy that only the coefficients on LIQ^{Amihud} and LIQ^{BAS} are significantly negative, thereby corroborating the findings presented in Table 4. The coefficients on liquidity measures based on the LOB data, namely LIQ^{OFN}, $LIQ^{LOBslope10}$ and $LIQ^{LOBslope}$, are positive but not significantly different from zero. All aforementioned coefficients' values indicate that less liquid stocks yield lower returns than more liquid shares as higher values of LIQ^{Amihud} and LIQ^{BAS} denote lower liquidity. Given that the negative returns on long-short liquidity portfolios are driven mostly by the negative return on the "long leg" of the portfolio, such results likely suggest that the relationship between stock liquidity and returns is non-linear or depends on some stock features.

The coefficients presented in Panel B of Table 5 are derived from multivariate models that, in addition to stock liquidity, incorporate a range of other companies' characteristics related to future stock returns. However, these models do not account for time-invariant, elusive features (companies' fixed effects), nor do they control for macroeconomic conditions (time dummies). The results remain qualitatively unchanged. That is to say, the slope coefficients on LIQ^{Amihud} and LIQ^{BAS} are significantly negative, while the slope coefficients on LIQ^{OFN}, $LIQ^{LOBslope10}$ and $LIQ^{LOBslope}$ remain statistically insignificant.

The inclusion of companies' fixed effects (Panel C) and both companies' and time-fixed effects (Panel D) does not alter the conclusions derived from univariate and multivariate regressions. The sole alteration is the statistical insignificance of the coefficient on LIQ^{Amihud}. The signs of the coefficients on control variables are largely consistent with previous literature and with expectations. A negative coefficient on *MV* indicates that larger firms tend to yield lower future returns, whereas a positive coefficient on *B-MV* suggests higher returns on value stocks. A positive slope on *MOM* indicates that the momentum is likely to continue, whereas a positive coefficient on *VOL* signifies that investors demand higher returns on more risky stocks. Given that *TURN* may serve as a proxy for both liquidity and investors' holding period, it is expected to exert a negative effect on stock returns (Stereńczak, 2022), a hypothesis that is corroborated by our findings.

In conclusion, the results of the cross-sectional regressions indicate the potential existence of non-linearities in the relationship between stock liquidity and returns. Such non-linearities may influence the profitability of the long-short liquidity portfolios, which is thus likely to vary according to the type of equities. To investigate this further, we proceed to the subsample analysis by forming portfolios from two-way dependent sorts.

5.3. Subsample analysis – bivariate portfolio sorting

For the subsample analysis, both equal- and value-weighted zero investment liquidity portfolios were formed. The returns on these portfolios, which were sorted according to various companies' characteristics and alternative liquidity measures are presented in Table 6 (equal-weighted portfolios) and Table 7 (value-weighted portfolios). The results reported therein provide some intriguing insights.

[TABLE 6 ABOUT HERE]

[TABLE 7 ABOUT HERE]

The most intriguing insight is that long-short liquidity portfolios sorted by LIQ^{OFN} yield significantly positive returns in the subsample of the largest companies listed in the WSE. The weekly return of 0.184%, which translates into an annual return of 9.568%, is statistically significant at the 0.1 level. The risk-adjusted return on the aforementioned portfolio ($\alpha_{Carhart}$) is even higher, indicating that the positive return on the portfolio is not attributable to higher risk exposure. Returns on value-weighted long-short liquidity portfolios for the subsample of the largest companies are positive when one employs LIQ^{OFN}, LIQ^{BAS} and $LIQ^{LOBslope10}$. All these returns are statistically significant, even after adjusting for risk exposure. However, the LIQ^{OFN} portfolio yields the highest raw return.

The aforementioned inferences contradict the findings of Cakici and Zaremba (2021), who found that the liquidity premium is exclusive to microcap stocks. In their study, the average market value of a microcap stock was 0.2 USD billion. The average capitalisation of the subsample of the largest companies in our study was approximately 3 USD billion, with a mean median value of approximately 1.8 USD billion. This demonstrates that the average size of companies among which we detected a liquidity premium is approximately 3.8 times higher than those of Cakici and Zaremba (2021).

The aforementioned discrepancies in the presence of a liquidity premium across companies of different sizes may be attributed to disparate time horizons. In contrast to the approach taken by Cakici and Zaremba (2021), who have examined the effect of stock liquidity on monthly returns, our study employs weekly returns and liquidity as the unit of analysis. The divergence in the results across studies conducted over different time horizons may indicate that the importance of liquidity varies depending on investors' portfolio rebalancing horizons. However, it is important to note that the study by Cakici and Zaremba (2021) covered companies listed in 45 markets globally, whereas our study focused on a subset of companies listed in a single market. This difference in scope could also contribute to the observed discrepancies in findings.

On the other hand, in their study of frontier markets, Stereńczak et al. (2020) found that long-short liquidity portfolios yielded significantly positive returns among companies, whose stock prices exhibited the most pronounced co-movement with international equities. For these companies, the diversification benefits are the smallest due to their high integration with the global economy. As a result, our results may simply capture that effect, indirectly supporting the hypothesis by Batten and Vo (2014) that the lack of a liquidity premium in less developed markets may be linked to their low integration with the global economy, resulting in some diversification benefits that offset low stock liquidity. This is particularly the case for the largest companies listed on the WSE, which are likely to be the most integrated with the global economy.

It is noteworthy that zero-investment liquidity portfolios do not yield significantly positive returns in other subsamples, particularly those based on *B-MV*, *MOM*, *VOL* and *TURN*. The sole exception is the capitalisation-weighted portfolio of companies with a moderately high turnover ratio, which exhibits a significant return of 0.298% per week, translating into an annual return of 15.496%. Furthermore, the return on this portfolio is also significantly positive after adjusting for risk.

6. Further analyses

The previous section demonstrates that an investment strategy that exploits the illiquidity premium generates positive returns only among the subset of the largest companies listed on the Warsaw Stock Exchange. To further substantiate this conclusion, a series of tests will be conducted in this section in order to assess the resilience of this strategy's returns. In particular, the following tests were conducted: 1) we investigate whether the utilisation of the bid or ask side of liquidity results in the generation of higher liquidity premia, 2) we assess whether the returns observed on the long-short portfolios remain positive when short-selling constraints are taken into account, 3) we evaluate the resilience of the profitability of our investment strategy to trading costs, and 4) we conduct a comparative analysis of the performance of our investment strategy with that of alternative strategies, with a particular focus on the buy-and-hold strategy. For the sake of brevity, we focus exclusively on the performance of long-short liquidity portfolios within the quartile of the largest companies. Figure 4 illustrates the cumulative return on these portfolios.

[FIGURE 4 ABOUT HERE]

6.1. Buy- and sell-side liquidity

In Section 5, we employ stock liquidity measures that are averaged across the bid and ask sides of the limit order book. Given that the buy and sell sides of the limit order book may differ in terms of the liquidity provision and the generation of liquidity premium (Brennan et al., 2012), we examine whether the use of a liquidity measure computed based on one of the LOB sides is associated with higher returns on strategy. To this end, we calculate the LIQ^{OFN}, $LIQ^{LOBslope10}$ and $LIQ^{LOBslope}$ measures separately for the buy and sell sides of the limit order book. Subsequently, the analyses conducted in Section 5.3 are repeated separately for the buy- and sell-side liquidity measures. This approach also enables the detection of whether the liquidity premium captured is due to the bid- or ask-side of stock liquidity. It should be noted that, at this stage of the analyses, LIQ^{BAS} and LIQ^{Amihud} are discarded, as they are not suitable for such analyses by construction.

[TABLE 8 ABOUT HERE]

Table 8 presents the returns on bi-variate portfolios, which have been sorted based on stock capitalisation and sell-side (buy-side) liquidity. In considering equal-weighted portfolios, it is evident that both bid-side and ask-side liquidity portfolios exhibit no superior performance compared to portfolios based on the averaged liquidity, irrespective of the measure employed. However, when value-weighted portfolios are considered, portfolios sorted on bid-side liquidity perform better than those based on the ask-side and average liquidity. The above findings pertain to both raw and risk-adjusted returns. This suggests that bid-side stock liquidity may contribute more to the performance of our investment strategy that exploits the liquidity premium.

6.2. Long- or short-side of the portfolio

The results thus far indicate that the investment strategy which exploits the liquidity premium has yielded significantly positive returns within the quartile of the companies with the highest market capitalisation. This strategy is, in fact, a composite of two portfolios: a long portfolio of the least liquid stocks and a portfolio of the most liquid stocks that are short-sold. The efficacy of this strategy is contingent upon the ability to freely trade all stocks assigned to the two portfolios, without the constraints imposed on short selling. As Umar et al. (2024) observed, abnormal returns are predominantly observed in stocks that are difficult to trade. Consequently, it is essential to assess whether the presented investment strategy can provide significantly positive returns when confronted with trading difficulties and constraints.

It seems probable that the main source of trading difficulties and constraints for our strategy is its “short leg”. Despite the absence of explicit constraints on short selling in European Union regulations (Regulation (EU) No 236/2012 of the European Parliament and of the Council of 14 March 2012 on Short Selling and Certain Aspects of Credit Default Swaps, 2012), it is reasonable to expect greater challenges in short selling rather than in going long. Consequently, an investigation is conducted to ascertain whether the positive return on the long-short strategy among the largest companies is predominantly attributable to the long portfolio comprising the least liquid stocks or the short portfolio comprising the most liquid stocks.

The Regulation (EU) No 236/2012 of the European Parliament and of the Council of 14 March 2012 on Short Selling and Certain Aspects of Credit Default Swaps has been in force in Poland since June 2015. In an earlier period, the Warsaw Stock Exchange imposed constraints on short sales, requiring that the stock subject to short sales should be of sufficient liquidity. Prior to June 2015, the WSE published a list of stocks that were permitted for sale through the short-selling mechanism. Our time scope of the study encompasses the period from 2014 to 2021, which means that short sales constraints were in force for part of the study period. Consequently, the analysis of the returns on long and short portfolios excludes the period from January 2014 to May 2015. This ensures that the same rules for short selling were in force throughout the entire period under scrutiny.

[TABLE 9 ABOUT HERE]

Table 9 presents the average weekly returns on portfolios of the most and least liquid stocks within the quartile of the largest companies. Both raw and risk-adjusted returns are presented for both equal- and value-weighted portfolios. Although the study period is somewhat shorter than that in Section 5.3, the results presented in Table 9 are qualitatively similar to those observed in Tables 6 and 7. When examining equal-weighted portfolios, only the long-short liquidity portfolio constructed using LIQ^{OFN} exhibits statistically significant positive return. In contrast, when analysing value-weighted portfolios, also LIQ^{BAS} and $LIQ^{LOBslope10}$ long-short portfolios demonstrate positive raw and risk-adjusted returns.

It is noteworthy that the raw returns on portfolios comprising the most and the least liquid stocks are statistically indistinguishable from zero. This indicates that the profitability of our strategy stems from the combination of both portfolios. However, it is important to note that portfolios comprising the most liquid stocks have generated significantly negative alphas, which suggests that portfolios comprising the most and least liquid stocks differ in terms of their risk exposure.

It is also noteworthy that the "long leg" of our strategy generates approximately two-thirds of the entire strategy's raw return, while a short portfolio comprising the most liquid stocks accounts for only approximately one-third of the zero-investment portfolio's return. After adjusting for risk, the contributions of the long and short portfolios are approximately equal, irrespective of the liquidity measure employed. This further suggests that the portfolios comprising the most and the least liquid stocks differ in their risk exposure.

The most significant conclusion that can be drawn from Table 9 is that our strategy, which involves taking a short position on the most liquid stocks and long position on the least liquid ones (among the largest companies), generates significantly positive returns only when the two portfolios are combined. Of the three liquidity measures that generate statistically significant profits, the returns on portfolios formed based on our LIQ^{OFN} measure are the highest. However, for value-weighted portfolios, this highest return can be attributed to slightly higher risk exposure compared to other portfolios. This further corroborates our findings that investors can benefit from measuring stock liquidity with our measure that utilises ordered fuzzy number representation of a limit order book.

6.3. Transaction costs

Thus far, we have not considered trading costs for our investment strategy that exploits liquidity premium among the largest companies. Given that the strategy entails weekly portfolio rebalancing it is probable that high transaction costs will be incurred, thereby reducing profits. We now turn to an examination of whether the long-short liquidity portfolios within the quartile of the largest companies continue to be profitable after accounting for trading costs. To this end, we calculate the portfolio turnover and compare the resulting values among zero-investment portfolios formed on various liquidity measures. This would not only allow us to test the resilience of our strategy's profitability to transaction costs, but also to compare our LIQ^{OFN} measure to other liquidity proxies.

The portfolio turnover is calculated as follows. At each instance of portfolio rebalancing, that is to say at the end of each week within the study period, two weights of a given stock in a portfolio are calculated: the weight at the end of the week and the weight at the beginning of the next week. The weight at the end of the given week is a function of the weight at the beginning of that week, the given’s stock return and the return on all stocks in a portfolio:

$$w_{it}^{end} = \frac{w_{it}^{begin} exp(r_{it})}{\sum_{j=1}^{N} w_{jt}^{begin} exp(r_{jt})}, \tag{5}$$

where w_{it}^{end} is the stock's *i* weight at the end of the week *t*, w_{it}^{begin} is the stock's *i* weight at the beginning of the week *t* and r_{it} is its log return in that week. w_{it}^{begin} reflects either equal weight or capitalisation weight.

Subsequently, for each stock in a portfolio, we calculate the value of stock *i* trading volume as a share of a portfolio value: $\left|w_{it}^{end} - w_{it+1}^{begin}\right|$. Total portfolio turnover is a sum of $\left|w_{it}^{end} - w_{it+1}^{begin}\right|$ for each stock in a portfolio, which is represented by the following equation:

$$turn_t = \sum_{i=1}^{N} \left|w_{it}^{end} - w_{it+1}^{begin}\right|. \tag{6}$$

Figure 5 illustrates the dynamics of the turnover of long-short liquidity portfolios formed based on various liquidity measures within the quartile of the largest companies. Table 10 presents the mean values of these portfolios' turnover. For the sake of clarity, the initial formation of the portfolios (i.e. the beginning of the study period) and their subsequent liquidation (i.e. the end of the study period) have been omitted. Table 10 presents the average turnover of long-short liquidity portfolios for two distinct time periods: the entire study period (January 2014 to December 2021) and the period without short sales constraints (June 2015 to December 2021).

[FIGURE 5 ABOUT HERE]

[TABLE 10 ABOUT HERE]

As demonstrated in Table 10, long portfolios comprising the least liquid shares are subject to more significant rebalancing than short portfolios comprising the most liquid stocks.

This is observed to be the case regardless of whether the portfolio is equal- or value-weighted. Nevertheless, portfolios formed based on LIQ^{OFN} values demonstrate markedly lower turnover rates than portfolios sorted according to alternative measures. To illustrate, the equal-weighted portfolio of illiquid stocks formed on LIQ^{OFN} has a mean turnover of 0.2426, which equates to approximately a quarter of the portfolio value being traded on average each week. This figure may appear considerable, but the same portfolio formed on LIQ^{BAS} has an average turnover of 0.3751, indicating that over a third of the portfolio structure changes every week. The lowest portfolio turnover for LIQ^{OFN} is a consequence of the lowest absolute changes in stock ranks, which are presented in Panel D of Table 2.

A low portfolio turnover rate is indicative of relatively lower trading costs incurred during the rebalancing of a portfolio. Table 10 additionally presents the maximum per cent trading cost that can be incurred for the strategy to continue to be profitable, that is to say, to yield a non-negative return. Each transaction generates a trading cost expressed as a percentage of the value traded (x). Thus, when one trades (buys or sells) stocks with a value of P, the trading costs are $x*P$. The maximum per cent trading costs (x^{max}) are calculated using the following logic:

1. Let us assume that the initial portfolio value is P.
2. After a week a portfolio generates a log return of r, which signifies that the portfolio value is $P*exp(r)$.
3. A portfolio is rebalanced, which entails an investor selling a portion (*turn*) of the portfolio's value and purchasing stocks of an equivalent value. Consequently, each each trade is of value $turn*P*exp(r)$.
4. Each trade incurs a trading cost of x, resulting in a loss of a portfolio value equal to $x* turn*P*exp(r)$. Two trades (one sale and one purchase) generate a cost of $2* x* turn*P*exp(r)$.

5. The portfolio value after rebalancing is *P*exp(r)- 2* x* turn*P*exp(r).*

Consequently, we aim to identify an x value that ensures the portfolio value following rebalancing is no less than the initial portfolio value. This can be expressed as follows:

$$P * exp(r) - 2 * x * turn * P * exp(r) > P, \tag{7}$$

where P represents the initial portfolio value, r denotes the average weekly log return, *turn* is the average weekly turnover, and x is a per cent trading cost. In order to solve the aforementioned inequality with respect to x, a series of transformations can be employed to ascertain the maximum value of x (x^{max}), which is given by:

$$x^{max} = \frac{exp(r) - 1}{2 * turn * exp(r)}. \tag{8}$$

The maximum per cent trading costs presented in Table 10 illustrate that our strategy of purchasing the least and selling the most liquid stocks (based on LIQ^{OFN} indications) among the largest companies is the most resilient to transaction costs among the considered liquidity measures. Upon examination of value-weighted portfolios, the majority of which demonstrate statistically significant positive returns, the maximum trading costs for portfolios sorted on LIQ^{OFN} is 0.622% (0.794%) for the entire period (period without short sales constraints). In the case of other liquidity proxies, x^{max} does not exceed 0.5% and is highest for LIQ^{BAS} throughout the period without short-selling constraints (0.466%). This evidence corroborates the assertion that investors can benefit from measuring stock liquidity using ordered fuzzy numbers.

It is important to note that the figures presented above represent the average trading cost, which has been calculated using the average portfolio turnover (of both long and short positions) and the composite long-short portfolio return. It is noteworthy that the long portfolios of the least liquid shares are subject to higher turnover, which consequently results in higher trading costs. Nevertheless, as illustrated in Table 9, these portfolios also generate higher raw returns, which (in accordance with equation (8)) may offset the effect of elevated turnover. It is important to note that illiquid stocks are subject to higher transaction costs, particularly bid-ask spreads, which everyone should be aware of. Our analysis indicates that the average maximum trading cost of 0.622% (or even 0.794%) is significant, suggesting that our strategy that utilises LIQ^{OFN} is likely to continue to be profitable after accounting for transaction costs.

6.4. Liquidity portfolios vs. buy-and-hold

The results thus far demonstrate unequivocally that LIQ^{OFN} can be employed to construct a profitable investment strategy. The strategy employs bivariate dependent sorts on market value and LIQ^{OFN}, with a long position in the portfolio comprising the least liquid stocks among the largest companies and a short position in the portfolio comprising the most liquid shares among the largest companies. The average weekly return on the zero-investment portfolio is 0.184% (0.199%) for equal-weighted (value-weighted) liquidity portfolios, generates positive Carhart's (1997) α of 0.226% (0.187%), and is resilient to transaction costs. The maximum percentage of trading costs for the strategy to continue to be profitable is 0.462% (0.622%). If we examine a period devoid of short-selling constraints (i.e. June 2015 to December 2021), these figures are markedly higher. However, do these figures indicate that our strategy outperforms the market? To address this question, we compare our strategy to a straightforward market buy-and-hold strategy that does not generate trading costs.

[FIGURE 6 ABOUT HERE]

[TABLE 11 ABOUT HERE]

Figure 6 illustrates the cumulative returns on our LIQ^{OFN} strategy in comparison to the cumulative returns on the market buy-and-hold strategy. I order to proxy the return of the strategy in question, two indices are used: the WIG index and the WIG20 index. Both are considered to be the two main indices on the Warsaw Stock Exchange. Table 11 provides a summary of the weekly returns on the aforementioned strategies. Panel A demonstrates the average excess returns (over a risk-free return, which is proxied by a WIBOR rate), while Panel B reports the standard deviations of weekly returns. Finally, Panel C presents the Sharpe ratios for the strategies.

As illustrated in Figure 6, our strategy, which exploits a liquidity measure based on the OFN representation of a limit order book, outperforms a market in terms of return. The average excess return on the LIQ^{OFN} strategy is approximately 3.63 – 4 times higher than the excess market return. The long-short liquidity portfolio sorted by LIQ^{OFN} also exhibits a lower risk exposure, resulting in a Sharpe ratio that is over four times higher. The Sharpe ratio for our strategy is 0.068 (0.076) for equal-weighted (value-weighted) liquidity portfolios, while the market buy-and-hold strategy generates a Sharpe ratio of 0.017.

It is important to note that the aforementioned figures do not account for trading costs, which are considerably higher for our strategy due to the significantly more frequent trading. Assuming a trading cost of 0.2% and real turnovers of our liquidity portfolios, our strategy still generates a Sharpe ratio of 0.033 (0.047) for equal-weighted (value-weighted) portfolios. In order to depress the Sharpe ratio of our equal-weighted (value-weighted) LIQ^{OFN} strategy below the Sharpe ratio of the market buy-and-hold return, the average trading costs would have to increase to 0.295% (0.414%). When we limit our analyses to a period without short-selling constraints (i.e. June 2015 to December 2021), these figures are correspondingly higher.

7. Discussion and conclusions

The stock liquidity premium is arguably one of the most pervasive cross-sectional asset pricing patterns – or is it? If this is indeed the case, why not take advantage of this phenomenon and develop an efficient investment strategy? How to measure stock liquidity to make sure this strategy will prove profitable over the long term? This study addresses these and other questions. The objective of this paper is to determine whether measuring stock liquidity based on ordered fuzzy numbers representation of a limit order book (Marszałek & Burczyński, 2024) yields benefits to investors by giving them the possibility to develop a profitable investment strategy. To this end, we analysed whether our LIQ^{OFN} contains distinct or similar information to other commonly used liquidity measures and examined whether the LIQ^{OFN} captures a higher liquidity premium, and thus helps to develop a profitable investment strategy.

Among the liquidity measures that were subject to analysis, our LIQ^{OFN} exhibited the second-highest degree of variation, both in the cross-section and in the time-series. Although high cross-sectional variation is a desirable feature, high time-series volatility is rather not, as it renders our LIQ^{OFN} more difficult to predict in comparison to other liquidity proxies. The correlation of LIQ^{OFN} with other considered liquidity measures indicates that our measure based on the OFN representation of the LOB reflects stock liquidity. However, it contains a different piece of information than the bid-ask spread and Amihud's (2002) illiquidity measure. This is presumably due to the fact that our measure reflects a distinct liquidity dimension in comparison to the latter two metrics. Our findings thus lend support to the view of the multidimensionality of stock liquidity and the inability of a single measure to fully capture it (Sarr & Lybek, 2002). Although our LIQ^{OFN} measure exhibits considerable time-series volatility, it provides relatively stable cross-sectional stock rankings according to their liquidity. Consequently, the turnover of long-short portfolios based on this proxy is lower, which makes our liquidity measure superior to other proxies in developing an efficient investment strategy. This is because lower portfolio turnover translates into significantly lower trading costs induced by portfolio rebalancing.

Our mainstream analysis reveals that a one-way sorted zero-investment portfolio comprising long positions in the least liquid shares and short positions in the most liquid stocks exhibits statistically significant negative weekly returns, irrespective of the liquidity measure employed. This pertains to both raw and risk-adjusted returns (alphas). These results are corroborated by the pooled cross-sectional time-series regressions, which, in addition to liquidity, encompass a number of firm characteristics that have been identified as drivers of stock return. However, when two-way sorted portfolios are considered, a zero-investment long-short liquidity portfolio yields significantly positive returns within the quartile of the largest companies.

In further analyses, we conducted a series of tests to ascertain the resilience of the return on the zero-investment long-short liquidity portfolio among the largest companies. The results indicate that not only does LIQ^{OFN} generate significantly positive returns on a long-short liquidity portfolio, but so do other liquidity measures. Nevertheless, the returns generated by the LIQ^{OFN} strategy are the most substantial and demonstrate the greatest resilience to transaction costs. The long-short strategy that utilises LIQ^{OFN} generates a weekly return of 0.184% (0.199%) for equal- (value-) weighted portfolios, which translates into an annual return of 9.568% (10.348%), significant at 0.1 level. The average turnover of portfolios formed based on LIQ^{OFN} is significantly (by approximately one-third) lower than that of portfolios sorted on other measures. This translates into lower rebalancing costs under the assumption that all other factors remain constant. For this strategy to continue to be profitable, the maximum average trading cost is 0.462% (0.622%) for equal- (value-) weighted portfolios. Furthermore, our strategy generates higher Sharpe ratios than the market buy-and-hold strategy, even after accounting for an average trading cost of 0.2%.

The findings of our study offer some intriguing insights, which, when considered alongside the existing evidence on liquidity premiums, appear to challenge the recent evidence.

As the WSE is still regarded as an emerging market (in accordance with MSCI classification), our findings are at odds with those of (Amihud et al., 2015; Bekaert et al., 2007). As these studies demonstrate, given the lower liquidity of emerging markets, it is expected that the liquidity premium in these markets will be higher. The findings of our study indicate that the liquidity premium is negative among all companies listed on the WSE. Furthermore, our findings challenge the conclusions of Batten and Vo (2014) and Stereńczak et al. (2020), who identified a negative correlation between stock illiquidity and returns but found it to be statistically insignificant. This discrepancy may be attributed to the presence of diversification opportunities that offset poor liquidity. The results of our study indeed indicate a negative relationship between illiquidity and returns, but this relationship is significant at 0.05 level or below. This is unlikely to be explained by low market integration with the global economy, which results in some diversification opportunities that offset stock illiquidity. Nevertheless, our study results do, to some extent, concur with those of Stereńczak et al. (2020), who discovered a statistically significant positive liquidity premium among stocks that are the most integrated with the global market. These stocks are typically those with potentially higher participation of international institutional investors. The same can be said of the largest companies in the WSE, which also exhibited a significantly positive liquidity premium. However, this inference contradicts the findings of Cakici and Zaremba (2021), who found that the liquidity premium exists only among microcap stocks. In this context, our findings contribute a new piece of evidence on the liquidity premium across the globe.

It can be reasonably asserted that the possibility of developing an investment strategy that yields significantly positive returns even after accounting for trading costs provides evidence of the informational inefficiency of the Warsaw Stock Exchange. The profitability of our strategy is contingent upon the existence of information that is not reflected in stock prices. In this case, the information in question is LOB data, which is not incorporated into stock prices. This can be identified as a sign of market inefficiency. It seems probable that this is because LOB data (including orders that have been placed and executed in the past) is not freely and immediately available to all interested parties. This is closely related to the information asymmetry problem (Akerlof, 1970; Stiglitz, 2002). If all interested parties were to have instant access to the full limit order book, an information asymmetry between investors would be reduced. This would improve market efficiency by making our strategy no longer profitable.

The results presented in the paper are of great interest, particularly for practitioners – investors in equity markets. The findings presented herein may prove useful in the development or improvement of an efficient investment strategy that generates significant returns that are resilient to short-selling constraints and substantial trading costs induced by frequent portfolio rebalancing. The tested strategy, however, requires continuous access to an up-to-date limit order book, which may entail significant costs. Nevertheless, these costs may be offset by robust profits on a portfolio formed with the use of that data. In this context, our findings may also be of interest to policymakers and capital market organisers. As market efficiency should be the exchange organisers' strategic objective in and of itself, they should be interested in providing continuous and instant access to the LOB data for all market participants. Such a measure would likely mitigate information asymmetry between investors and somewhat reduce the advantage of the big traders, thereby improving efficiency and making the market more attractive for retail investors.

It should be noted that our analyses are limited to companies listed on the Warsaw Stock Exchange, which may limit the generalisability of the conclusions drawn. It would be beneficial to conduct further tests to assess the efficacy of the LIQ^{OFN} on other stock markets, particularly those developed. It would also be valuable to examine the efficacy of LIQ^{OFN} in capturing liquidity and the liquidity premium in a variety of economic conditions and time intervals. The limited scope of our study, encompassing only eight years of data, limits the possibility of rigorous testing of our strategy at monthly intervals. The insufficient number of observed returns within this timeframe hinders the reliability of inferences drawn from such tests. Furthermore, the eight-year period is inadequate for discerning the effectiveness of our LIQ^{OFN} measure in diverse economic conditions, as it lacks a sufficient number of periods of market turmoil or stress.

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Table 1. Distributions of liquidity measures

Panel A: Time-series averages of cross-sectional statistics					
Measure	**Mean**	**Standard deviation**	**Coefficient of variation**	**Skewness**	**Kurtosis**
LIQ^{OFN}	0.2291	0.5666	2.4599	4.1091	19.853
LIQ^{Amihud}	21.267	96.777	3.6981	6.9421	58.907
LIQ^{BAS}	0.0093	0.0086	0.9030	2.6862	13.441
$LIQ^{LOBslope10}$	503.62	622.84	1.2258	3.9704	23.387
$LIQ^{LOBslope}$	162.34	141.69	0.8306	5.6670	50.062
Panel B: Cross-sectional averages of time-series statistics					
Measure	**Mean**	**Standard deviation**	**Coefficient of variation**	**Skewness**	**Kurtosis**
LIQ^{OFN}	0.1556	0.1023	0.8750	2.1968	11.276
LIQ^{Amihud}	54.551	89.097	1.6328	3.7388	23.564
LIQ^{BAS}	0.0117	0.0054	0.4300	1.2979	4.2768
$LIQ^{LOBslope10}$	400.73	170.57	0.4267	1.3209	3.9555
$LIQ^{LOBslope}$	157.31	74.579	0.4077	1.6333	7.3048

Note: The table presents the descriptive statistics of compared liquidity proxies. Panel A demonstrates time-series averages of cross-sectional statistics. Panel B reports cross-sectional averages of time-series statistics.

Table 2. Correlations among liquidity measures

Panel A: Time-series averages of cross-sectional Pearson correlations					
Measure	LIQ^{OFN}	LIQ^{Amihud}	LIQ^{BAS}	$LIQ^{LOBslope10}$	$LIQ^{LOBslope}$
LIQ^{OFN}	1	0.1124	0.3613	0.7285	0.1502
LIQ^{Amihud}	0.1124	1	0.2519	0.1490	0.0361
LIQ^{BAS}	0.3613	0.2519	1	0.4431	0.1629
$LIQ^{LOBslope10}$	0.7285	0.1490	0.4431	1	0.5619
$LIQ^{LOBslope}$	0.1502	0.0361	0.1629	0.5619	1
Panel B: Cross-sectional averages of time-series Pearson correlations					
Measure	LIQ^{OFN}	LIQ^{Amihud}	LIQ^{BAS}	$LIQ^{LOBslope10}$	$LIQ^{LOBslope}$
LIQ^{OFN}	1	0.2619	0.5593	0.5497	0.2748
LIQ^{Amihud}	0.2619	1	0.3617	0.2358	0.1107
LIQ^{BAS}	0.5593	0.3617	1	0.6269	0.3167
$LIQ^{LOBslope10}$	0.5497	0.2358	0.6269	1	0.6992
$LIQ^{LOBslope}$	0.2748	0.1107	0.3167	0.6992	1
Panel C: Time-series averages of cross-sectional Spearman rank correlations					
Measure	LIQ^{OFN}	LIQ^{Amihud}	LIQ^{BAS}	$LIQ^{LOBslope10}$	$LIQ^{LOBslope}$
LIQ^{OFN}	1	0.7108	0.9056	0.8620	0.3696
LIQ^{Amihud}	0.7108	1	0.7018	0.6833	0.2927
LIQ^{BAS}	0.9056	0.7018	1	0.8941	0.3635
$LIQ^{LOBslope10}$	0.8620	0.6833	0.8941	1	0.5830
$LIQ^{LOBslope}$	0.3696	0.2927	0.3635	0.5830	1
Panel D: Absolute changes in liquidity ranks					
Measure	LIQ^{OFN}	LIQ^{Amihud}	LIQ^{BAS}	$LIQ^{LOBslope10}$	$LIQ^{LOBslope}$
The time-series avg of cross-sectional means	7.7113	17.383	10.674	12.408	22.030
Cross-sectional avg of time-series means	8.4543	18.273	11.078	13.226	22.463
Pooled mean	7.7108	17.384	10.673	12.407	22.031

Note: The table presents the correlation coefficients among analysed liquidity measures. Panel A demonstrates time-series averages of cross-sectional Pearson correlations. Panel B reports cross-sectional averages of time-series Pearson correlations. Panel C presents time-series averages of cross-sectional Spearman rank correlations, and Panel D reports absolute changes in a weekly change of a liquidity rank.

Table 3. Prediction errors of liquidity measures

Measure	LIQ^{OFN}	LIQ^{Amihud}	LIQ^{BAS}	$LIQ^{LOBslope10}$	$LIQ^{LOBslope}$
ARE	-0.2102	-1.6726	-0.0727	-0.0888	-0.0914
MAE	0.3913	1.9798	0.2127	0.2287	0.2301
RMSE	0.7867	3.4403	0.4615	0.4161	0.4158

Note: The table presents the values of prediction errors of compared liquidity proxies. Forecasts are done using a simple AR(1) model using the data on the previous 26 weeks (half a year).

Table 4. Returns on univariate portfolio sorts

Panel A: Equal-weighted portfolios

Measure	*Illiq*	*2*	*3*	*4*	*Liq*	*Illiq-Liq*	α_{CAPM}	α_{FF3}	$\alpha_{Carhart}$
LIQ^{OFN}	-0.183*	-0.086	-0.067	0.062	0.024	-0.207**	-0.185**	-0.244***	-0.234**
	(2.087)	(2.708)	(2.789)	(2.615)	(2.846)	(2.090)	(2.12)	(2.67)	(2.57)
LIQ^{Amihud}	-0.229*	-0.080	0.001	0.047	0.001	-0.235**	-0.221**	-0.267**	-0.247**
	(2.644)	(2.614)	(2.514)	(2.360)	(2.878)	(2.081)	(2.24)	(2.49)	(2.39)
LIQ^{BAS}	-0.174*	-0.127	-0.075	0.121	0.001	-0.183*	-0.162*	-0.216**	-0.208**
	(2.069)	(2.813)	(2.705)	(2.671)	(2.724)	(2.000)	(1.93)	(2.43)	(2.35)
$LIQ^{LOBslope10}$	-0.182*	-0.089	-0.037	0.059	0.001	-0.186*	-0.168*	-0.189*	-0.182*
	(2.240)	(2.670)	(2.781)	(2.674)	(2.690)	(2.078)	(1.81)	(1.91)	(1.84)
$LIQ^{LOBslope}$	-0.211*	-0.001	-0.070	-0.039	0.078	-0.288***	-0.283***	-0.264***	-0.263***
	(2.487)	(2.660)	(2.666)	(2.511)	(2.521)	(1.681)	(3.46)	(2.79)	(2.79)

Panel B: Value-weighted portfolios

Measure	*Illiq*	*2*	*3*	*4*	*Liq*	*Illiq-Liq*	α_{CAPM}	α_{FF3}	$\alpha_{Carhart}$
LIQ^{OFN}	-0.080	0.040	-0.026	0.059	-0.047	-0.033	-0.008	-0.021	-0.033
	(1.914)	(2.307)	(2.578)	(2.540)	(2.824)	(2.221)	(0.10)	(0.27)	(0.42)
LIQ^{Amihud}	-0.175	-0.001	0.077	0.017	-0.045	-0.129	-0.110	-0.094	-0.102
	(2.293)	(2.345)	(2.517)	(2.455)	(2.838)	(2.182)	(1.12)	(0.97)	(1.06)
LIQ^{BAS}	-0.062	-0.072	0.014	0.140	-0.062	-0.000	0.024	0.011	-0.004
	(2.013)	(2.499)	(2.537)	(2.651)	(2.794)	(2.307)	(0.27)	(0.12)	(0.04)
$LIQ^{LOBslope10}$	-0.092	0.000	0.058	0.110	-0.072	-0.020	0.002	0.002	-0.008
	(2.065)	(2.493)	(2.614)	(2.711)	(2.763)	(2.167)	(0.02)	(0.02)	(0.09)
$LIQ^{LOBslope}$	0.064	0.045	-0.074	-0.107	0.074	-0.010	-0.001	0.003	0.016
	(2.672)	(2.947)	(2.926)	(2.698)	(2.869)	(2.178)	(0.00)	(0.03)	(0.15)

Note: The table presents the returns on quintile portfolios sorted by stock liquidity alongside the return on a zero investment portfolio that goes short on most liquid stocks and long on least liquid ones. The table also reports risk-adjusted returns on these portfolios computed based on CAPM (α_{CAPM}), Fama and French's (1992) three-factor model (α_{FF3}) and Carhart's (1997) four-factor model ($\alpha_{Carhart}$). Both returns and alphas are expressed in percentage terms. Panel A demonstrates returns on equal-weighted portfolios and Panel B reports returns on value-weighted portfolios. The values in brackets are the standard deviations of returns (for portfolio returns) and *t*-statistics based on Newey and West's (1987) adjusted standard errors (for alphas). The asterisks ***, ** and * denote statistical significance at 0.01, 0.05 and 0.1 levels respectively.

Table 5. Results of cross-sectional regressions

Panel A: Univariate tests					
Measure	LIQ^{OFN}	LIQ^{Amihud}	LIQ^{BAS}	$LIQ^{LOBslope10}$	$LIQ^{LOBslope}$
const	-0.045	-0.025	0.093	-0.054	-0.055
	(0.37)	(0.21)	(0.75)	(0.41)	(0.40)
LIQ	0.034	-0.001**	-14.12***	0.000	0.000
	(0.78)	(2.24)	(2.95)	(0.57)	(0.50)
R^2	0.0000	0.0003	0.0005	0.0000	0.0000
Number of obs.	57,935	57,935	57,935	57,935	57,935
Panel B: Multivariate tests					
Measure	LIQ^{OFN}	LIQ^{Amihud}	LIQ^{BAS}	$LIQ^{LOBslope10}$	$LIQ^{LOBslope}$
const	-1.22	-0.745	0.074	-1.05	-0.839
	(1.45)	(1.08)	(0.10)	(1.23)	(1.21)
LIQ	-0.083	-0.001*	-14.35***	-0.000	0.000
	(1.48)	(1.94)	(2.66)	(0.47)	(0.23)
MV	0.051	0.028	-0.004	0.044	0.032
	(1.41)	(0.96)	(0.12)	(1.15)	(1.07)
B-MV	0.068*	0.068*	0.064	0.066	0.066
	(1.66)	(1.67)	(1.60)	(1.63)	(1.64)
MOM	18.29**	17.79**	16.92**	18.33**	18.52**
	(2.08)	(2.02)	(1.93)	(2.07)	(2.11)
VOL	0.845	0.842	0.302	0.713	0.650
	(0.29)	(0.30)	(0.11)	(0.25)	(0.23)
TURN	0.021***	0.017***	0.012***	0.021***	0.017***
	(4.32)	(4.70)	(2.58)	(2.79)	(2.75)
Fixed effects	No	No	No	No	No
Time effects	No	No	No	No	No
R^2	0.0008	0.0010	0.0012	0.0008	0.0008
Number of obs.	55,361	55,361	55,361	55,361	55,361
Panel C: Multivariate tests with fixed effects					
Measure	LIQ^{OFN}	LIQ^{Amihud}	LIQ^{BAS}	$LIQ^{LOBslope10}$	$LIQ^{LOBslope}$
const	13.64***	13.93***	15.89***	13.94***	13.78***
	(4.38)	(4.49)	(4.94)	(4.48)	(4.39)
LIQ	-0.062	-0.000	-29.37***	0.000	0.000
	(0.51)	(0.30)	(5.20)	(0.78)	(1.16)
MV	-0.679***	-0.693***	-0.775***	-0.695***	-0.688***
	(4.58)	(4.69)	(5.07)	(4.69)	(4.61)
B-MV	0.137**	0.138**	0.149**	0.139**	0.139**
	(2.12)	(2.12)	(2.28)	(2.15)	(2.16)
MOM	20.46***	20.57***	18.39***	20.56***	20.54***
	(3.69)	(3.78)	(3.35)	(3.76)	(3.75)
VOL	5.05***	4.91***	4.69**	4.94***	4.99***
	(2.73)	(2.69)	(2.52)	(2.69)	(2.72)
TURN	-0.083***	-0.084***	-0.094***	-0.085***	-0.085***
	(4.84)	(4.94)	(5.32)	(4.95)	(4.95)
Fixed effects	Yes	Yes	Yes	Yes	Yes
Time effects	No	No	No	No	No
R^2	0.0119	0.0119	0.0126	0.0119	0.0119
Number of obs.	55,361	55,361	55,361	55,361	55,361
Panel D: Multivariate tests with fixed effects and time dummies					
Measure	LIQ^{OFN}	LIQ^{Amihud}	LIQ^{BAS}	$LIQ^{LOBslope10}$	$LIQ^{LOBslope}$
const	16.68***	16.98***	18.23***	16.92***	16.92***
	(5.90)	(6.01)	(6.28)	(6.00)	(5.98)
LIQ	-0.125	-0.000	-21.86***	-0.000	0.000
	(1.11)	(0.37)	(3.76)	(0.38)	(0.49)
MV	-0.721***	-0.736***	-0.790***	-0.733***	-0.734***
	(5.44)	(5.55)	(5.80)	(5.54)	(5.52)
B-MV	0.141***	0.143***	0.153***	0.142***	0.142***
	(2.62)	(2.65)	(2.82)	(2.63)	(2.65)

MOM	17.49***	17.45***	16.66**	17.59***	17.52***
	(2.71)	(2.73)	(2.59)	(2.77)	(2.75)
VOL	-2.74	-2.84	-2.82	-2.85	-2.88
	(1.54)	(1.61)	(1.52)	(1.60)	(1.62)
TURN	-0.059***	-0.060***	-0.066***	-0.059***	-0.060***
	(3.50)	(3.56)	(3.81)	(3.54)	(3.55)
Fixed effects	Yes	Yes	Yes	Yes	Yes
Time effects	Yes	Yes	Yes	Yes	Yes
R^2	0.1907	0.1907	0.1911	0.1907	0.1907
Number of obs.	55,361	55,361	55,361	55,361	55,361

Note: The table reports the slope coefficients (βs, multiplied by 100) of the pooled cross-sectional time-series regressions. The raw returns are regressed on liquidity measures (Panel A) and additional control variables (Panels B, C and D). Panel B reports slope coefficients for models without any effects; Panel C demonstrates the slopes for the models with fixed effects and Panel D – for the models with both fixed effects and time dummies. The control variables are: market value (*MV*), book-to-market ratio (*B-MV*), momentum (*MOM*), return volatility (*VOL*), and stock turnover (*TURN*). The numbers in brackets are *t*-statistics with robust standard errors clustered by a firm and by week (Petersen, 2009). The asterisks ***, **, and * denote statistical significance at 0.01, 0.05 and 0.1 levels respectively.

Table 6. Returns on equal-weighted bivariate portfolio sorts

Panel A: Portfolios sorted on MV and LIQ

	Raw returns				$\alpha_{Carhart}$			
Measure	***Low MV***	***2***	***3***	***High MV***	***Low MV***	***2***	***3***	***High MV***
LIQ^{OFN}	-0.343**	-0.232*	-0.139	0.184*	-0.514***	-0.330**	-0.175	0.226**
	(2.07)	(1.66)	(1.18)	(1.64)	(2.72)	(2.42)	(1.50)	(1.87)
LIQ^{Amihud}	-0.306	-0.273**	-0.160	0.091	-0.370*	-0.350***	-0.237**	0.099
	(1.52)	(2.16)	(1.41)	(0.84)	(1.72)	(2.62)	(2.09)	(0.83)
LIQ^{BAS}	-0.271	-0.310**	-0.110	0.079	-0.481***	-0.487***	-0.096	0.055
	(1.58)	(2.25)	(0.99)	(0.79)	(2.58)	(3.56)	(0.85)	(0.51)
$LIQ^{LOBslope10}$	-0.255	-0.177	-0.034	0.085	-0.417**	-0.268*	-0.059	0.155
	(1.55)	(1.28)	(0.31)	(0.84)	(2.09)	(1.83)	(0.52)	(1.51)
$LIQ^{LOBslope}$	-0.376*	-0.304***	-0.078	-0.087	-0.258	-0.267**	-0.087	-0.033
	(1.96)	(2.81)	(0.68)	(0.81)	(1.13)	(2.18)	(0.75)	(0.29)

Panel B: Portfolios sorted on B-MV and LIQ

	Raw returns				$\alpha_{Carhart}$			
Measure	***Low B-MV***	***2***	***3***	***High B-MV***	***Low B-MV***	***2***	***3***	***High B-MV***
LIQ^{OFN}	-0.257*	0.033	-0.120	-0.168	-0.394***	0.049	-0.100	-0.081
	(1.84)	(0.27)	(0.92)	(1.05)	(2.65)	(0.39)	(0.82)	(0.46)
LIQ^{Amihud}	-0.322**	0.010	-0.202	-0.117	-0.457***	0.161	-0.208*	0.008
	(2.29)	(0.09)	(1.48)	(0.68)	(2.95)	(1.34)	(1.61)	(0.04)
LIQ^{BAS}	-0.209*	0.070	-0.172	-0.317*	-0.277**	0.050	-0.171	-0.306*
	(1.63)	(0.57)	(1.38)	(1.88)	(2.02)	(0.38)	(1.46)	(1.64)
$LIQ^{LOBslope10}$	-0.220*	0.022	-0.099	-0.329**	-0.316**	-0.005	-0.083	-0.301*
	(1.73)	(0.19)	(0.81)	(2.09)	(2.44)	(0.04)	(0.69)	(1.70)
$LIQ^{LOBslope}$	-0.184	-0.017	-0.143	-0.113	-0.265**	0.057	-0.123	-0.060
	(1.49)	(0.15)	(1.24)	(0.85)	(1.96)	(0.47)	(1.01)	(0.38)

Panel C: Portfolios sorted on MOM and LIQ

	Raw returns				$\alpha_{Carhart}$			
Measure	***Low MOM***	***2***	***3***	***High MOM***	***Low MOM***	***2***	***3***	***High MOM***
LIQ^{OFN}	-0.068	-0.044	-0.019	-0.242*	-0.045	-0.011	-0.044	-0.301**
	(0.40)	(0.35)	(0.18)	(1.76)	(0.24)	(0.09)	(0.41)	(2.14)
LIQ^{Amihud}	-0.161	-0.070	-0.035	-0.281**	-0.139	-0.087	-0.059	-0.299**
	(0.87)	(0.54)	(0.34)	(2.14)	(0.64)	(0.64)	(0.52)	(2.22)
LIQ^{BAS}	-0.224	0.025	-0.003	-0.447***	-0.272	0.007	-0.034	-0.514***
	(1.25)	(0.20)	(0.03)	(3.52)	(1.34)	(0.06)	(0.32)	(3.93)
$LIQ^{LOBslope10}$	-0.082	-0.009	-0.058	-0.323**	-0.097	-0.013	-0.100	-0.381***
	(0.48)	(0.08)	(0.56)	(2.49)	(0.49)	(0.10)	(0.89)	(2.97)
$LIQ^{LOBslope}$	-0.137	-0.032	-0.150	-0.317**	-0.138	-0.046	-0.111	-0.325**
	(0.84)	(0.30)	(1.53)	(2.34)	(0.70)	(0.40)	(1.02)	(2.09)

Panel D: Portfolios sorted on VOL and LIQ

	Raw returns				$\alpha_{Carhart}$			
Measure	***Low VOL***	***2***	***3***	***High VOL***	***Low VOL***	***2***	***3***	***High VOL***
LIQ^{OFN}	0.023	0.081	-0.144	-0.507***	0.057	0.069	-0.112	-0.539***
	(0.24)	(0.67)	(1.00)	(3.16)	(0.57)	(0.57)	(0.75)	(3.21)
LIQ^{Amihud}	0.044	-0.006	-0.132	-0.432**	0.046	0.037	-0.067	-0.480**
	(0.48)	(0.05)	(0.95)	(2.49)	(0.50)	(0.32)	(0.46)	(2.51)
LIQ^{BAS}	-0.003	0.112	-0.203	-0.395**	-0.008	0.107	-0.110	-0.437**
	(0.03)	(0.92)	(1.46)	(2.44)	(0.08)	(0.89)	(0.77)	(2.45)
$LIQ^{LOBslope10}$	0.045	0.000	-0.138	-0.537***	0.104	0.012	-0.064	-0.627***
	(0.47)	(0.01)	(0.96)	(3.48)	(1.04)	(0.10)	(0.41)	(3.70)
$LIQ^{LOBslope}$	-0.164*	-0.096	-0.110	-0.263	-0.158	-0.090	-0.062	-0.214
	(1.87)	(0.98)	(0.85)	(1.55)	(1.59)	(0.87)	(0.41)	(1.09)

Panel E: Portfolios sorted on TURN and LIQ

	Raw returns				$\alpha_{Carhart}$			
Measure	***Low TURN***	***2***	***3***	***High TURN***	***Low TURN***	***2***	***3***	***High TURN***
LIQ^{OFN}	-0.163*	-0.219*	0.096	-0.278*	-0.208**	-0.200	0.175	-0.162
	(1.61)	(1.78)	(0.70)	(1.73)	(1.97)	(1.59)	(1.20)	(0.96)

LIQ^{Amihud}	-0.292***	-0.099	0.030	-0.197	-0.274**	-0.086	0.076	-0.013
	(3.02)	(0.90)	(0.22)	(1.18)	(2.54)	(0.72)	(0.50)	(0.07)
LIQ^{BAS}	-0.089	-0.126	-0.048	-0.359**	-0.143	-0.129	0.030	-0.232
	(0.84)	(1.17)	(0.36)	(2.12)	(1.29)	(1.27)	(0.20)	(1.23)
$LIQ^{LOBslope10}$	-0.127	-0.148	0.017	-0.445***	-0.177	-0.089	0.092	-0.284
	(1.28)	(1.27)	(0.13)	(2.59)	(1.60)	(0.75)	(0.61)	(1.46)
$LIQ^{LOBslope}$	-0.214**	-0.053	-0.257**	-0.378**	-0.223**	-0.014	-0.201*	-0.259
	(2.24)	(0.49)	(2.29)	(2.41)	(1.99)	(0.13)	(1.65)	(1.43)

Note: The table presents the returns on equal-weighted two-way sorted quartile zero investment portfolios that go short on most liquid stocks and long on least liquid ones. In the first pass, stocks are ranked according to the value of one of the variables, and then, within each quartile, a long-short quartile portfolio based on LIQ is formed. The left side of the table reports raw returns on long-short portfolios and the right side demonstrates risk-adjusted returns on these portfolios computed from Carhart's (1997) four-factor model ($\alpha_{Carhart}$). Both returns and alphas are expressed in percentage terms. Panel A demonstrates the results for portfolios sorted on *MV* and *LIQ*; Panel B reports the returns on portfolios sorted by *B-MV* and *LIQ*; Panel C is devoted to portfolios sorted on *MOM* and *LIQ*; Panel D reports returns on portfolios sorted on *VOL* and *LIQ*; and Panel E demonstrates the results for portfolios sorted on *TURN* and *LIQ*. The values in brackets are the *t*-statistics based on Newey and West's (1987) adjusted standard errors. The asterisks ***, ** and * denote statistical significance at 0.01, 0.05 and 0.1 levels respectively.

Table 7. Returns on value-weighted bivariate portfolio sorts

Panel A: Portfolios sorted on MV and LIQ

	Raw returns				$\alpha_{Carhart}$			
Measure	***Low MV***	***2***	***3***	***High MV***	***Low MV***	***2***	***3***	***High MV***
LIQ^{OFN}	-0.212	-0.212	-0.172	0.199*	-0.277*	-0.248*	-0.173	0.187*
	(1.35)	(1.53)	(1.42)	(1.80)	(1.77)	(1.96)	(1.59)	(1.74)
LIQ^{Amihud}	-0.316*	-0.249**	-0.197*	0.164	-0.311*	-0.269**	-0.178	0.145
	(1.92)	(2.03)	(1.66)	(1.51)	(1.87)	(2.20)	(1.60)	(1.32)
LIQ^{BAS}	-0.227	-0.278**	-0.121	0.192*	-0.291*	-0.329***	-0.108	0.202*
	(1.47)	(2.08)	(1.05)	(1.83)	(1.88)	(2.69)	(0.99)	(1.90)
$LIQ^{LOBslope10}$	-0.214	-0.150	-0.045	0.175*	-0.260*	-0.157	-0.015	0.200*
	(1.40)	(1.13)	(0.37)	(1.69)	(1.71)	(1.25)	(0.14)	(1.93)
$LIQ^{LOBslope}$	-0.346**	-0.278***	-0.004	-0.036	-0.313*	-0.247**	0.029	-0.003
	(2.21)	(2.63)	(0.03)	(0.29)	(1.91)	(2.25)	(0.26)	(0.02)

Panel B: Portfolios sorted on B-MV and LIQ

	Raw returns				$\alpha_{Carhart}$			
Measure	***Low B-MV***	***2***	***3***	***High B-MV***	***Low B-MV***	***2***	***3***	***High B-MV***
LIQ^{OFN}	-0.016	0.037	-0.003	-0.029	-0.119	0.116	0.008	0.019
	(0.11)	(0.27)	(0.02)	(0.17)	(0.91)	(0.91)	(0.06)	(0.12)
LIQ^{Amihud}	-0.106	0.055	-0.163	-0.051	-0.170	0.155	-0.120	0.037
	(0.73)	(0.37)	(1.15)	(0.26)	(1.28)	(1.08)	(0.88)	(0.20)
LIQ^{BAS}	-0.027	0.130	-0.096	-0.126	-0.118	0.197	-0.089	-0.078
	(0.18)	(0.92)	(0.70)	(0.71)	(0.90)	(1.54)	(0.70)	(0.45)
$LIQ^{LOBslope10}$	-0.068	0.142	-0.032	-0.072	-0.157	0.215	-0.036	-0.035
	(0.47)	(0.98)	(0.24)	(0.43)	(1.18)	(1.57)	(0.29)	(0.21)
$LIQ^{LOBslope}$	0.147	0.223	-0.117	-0.129	0.101	0.290*	-0.099	-0.133
	(0.96)	(1.50)	(0.72)	(0.74)	(0.66)	(1.96)	(0.65)	(0.74)

Panel C: Portfolios sorted on MOM and LIQ

	Raw returns				$\alpha_{Carhart}$			
Measure	***Low MOM***	***2***	***3***	***High MOM***	***Low MOM***	***2***	***3***	***High MOM***
LIQ^{OFN}	-0.117	-0.005	0.093	-0.150	-0.139	0.019	0.076	-0.134
	(0.68)	(0.03)	(0.80)	(1.00)	(0.81)	(0.14)	(0.69)	(0.89)
LIQ^{Amihud}	-0.315*	0.010	0.133	-0.217	-0.300	0.031	0.142	-0.189
	(1.63)	(0.07)	(1.04)	(1.54)	(1.53)	(0.23)	(1.11)	(1.30)
LIQ^{BAS}	-0.310*	0.027	0.071	-0.349**	-0.313*	0.036	0.045	-0.316**
	(1.70)	(0.18)	(0.59)	(2.28)	(1.72)	(0.28)	(0.39)	(2.06)
$LIQ^{LOBslope10}$	-0.166	0.036	-0.000	-0.164	-0.189	0.071	-0.017	-0.131
	(0.92)	(0.25)	(0.00)	(1.05)	(1.02)	(0.55)	(0.14)	(0.83)
$LIQ^{LOBslope}$	-0.285*	0.046	-0.128	-0.163	-0.338*	0.121	-0.104	-0.074
	(1.72)	(0.30)	(0.99)	(0.89)	(1.95)	(0.78)	(0.80)	(0.42)

Panel D: Portfolios sorted on VOL and LIQ

	Raw returns				$\alpha_{Carhart}$			
Measure	***Low VOL***	***2***	***3***	***High VOL***	***Low VOL***	***2***	***3***	***High VOL***
LIQ^{OFN}	0.080	0.121	-0.049	-0.233	0.084	0.159	-0.016	-0.214
	(0.74)	(0.84)	(0.29)	(1.13)	(0.85)	(1.32)	(0.10)	(1.08)
LIQ^{Amihud}	0.021	0.044	-0.165	-0.388**	0.026	0.092	-0.103	-0.345*
	(0.18)	(0.33)	(0.96)	(2.00)	(0.24)	(0.76)	(0.58)	(1.78)
LIQ^{BAS}	0.073	0.126	-0.125	-0.265	0.064	0.177	-0.080	-0.226
	(0.65)	(0.84)	(0.71)	(1.27)	(0.63)	(1.37)	(0.46)	(1.11)
$LIQ^{LOBslope10}$	0.066	-0.001	-0.134	-0.182	0.062	0.076	-0.068	-0.136
	(0.58)	(0.01)	(0.77)	(0.88)	(0.58)	(0.61)	(0.38)	(0.66)
$LIQ^{LOBslope}$	-0.142	0.050	-0.188	-0.008	-0.143	0.141	-0.142	0.007
	(1.15)	(0.32)	(1.14)	(0.03)	(1.10)	(0.96)	(0.85)	(0.03)

Panel E: Portfolios sorted on TURN and LIQ

	Raw returns				$\alpha_{Carhart}$			
Measure	***Low TURN***	***2***	***3***	***High TURN***	***Low TURN***	***2***	***3***	***High TURN***
LIQ^{OFN}	-0.184	-0.088	0.298**	0.130	-0.157	-0.072	0.361***	0.120
	(1.46)	(0.57)	(2.15)	(0.77)	(1.34)	(0.53)	(2.73)	(0.75)

LIQ^{Amihud}	-0.266**	0.097	0.174	0.038	-0.235*	0.142	0.219*	0.060
	(2.08)	(0.67)	(1.26)	(0.23)	(1.82)	(1.06)	(1.63)	(0.38)
LIQ^{BAS}	-0.185	0.101	-0.021	-0.001	-0.189	0.147	0.046	-0.021
	(1.40)	(0.72)	(0.14)	(0.01)	(1.51)	(1.20)	(0.32)	(0.13)
$LIQ^{LOBslope10}$	-0.211*	-0.008	0.144	0.021	-0.180	0.039	0.219	0.036
	(1.68)	(0.05)	(1.01)	(0.12)	(1.50)	(0.28)	(1.56)	(0.21)
$LIQ^{LOBslope}$	-0.270**	0.232	-0.225	-0.019	-0.267**	0.265*	-0.199	0.024
	(2.23)	(1.53)	(1.41)	(0.08)	(2.10)	(1.85)	(1.26)	(0.11)

Note: The table presents the returns on capitalisation-weighted two-way sorted quartile zero investment portfolios that go short on most liquid stocks and long on least liquid ones. In the first pass, stocks are ranked according to the value of one of the variables, and then, within each quartile, a long-short quartile portfolio based on LIQ is formed. The left side of the table reports raw returns on long-short portfolios and the right side demonstrates risk-adjusted returns on these portfolios computed from Carhart's (1997) four-factor model ($\alpha_{Carhart}$). Both returns and alphas are expressed in percentage terms. Panel A demonstrates the results for portfolios sorted on *MV* and *LIQ*; Panel B reports the returns on portfolios sorted by *B-MV* and *LIQ*; Panel C is devoted to portfolios sorted on *MOM* and *LIQ*; Panel D reports returns on portfolios sorted on *VOL* and *LIQ*; and Panel E demonstrates the results for portfolios sorted on *TURN* and *LIQ*. The values in brackets are the *t*-statistics based on Newey and West's (1987) adjusted standard errors. The asterisks ***, ** and * denote statistical significance at 0.01, 0.05 and 0.1 levels respectively.

Table 8. Returns on equal- and value-weighted bivariate portfolio sorts – bid- and ask-side of liquidity among big companies

Panel A: Equal-weighted portfolios sorted on MV and LIQ

	Raw returns		***$\alpha_{Carhart}$***	
Measure	***Bid-side liquidity***	***Ask-side liquidity***	***Bid-side liquidity***	***Ask-side liquidity***
LIQ^{OFN}	0.149	0.125	0.203*	0.158
	(1.30)	(1.14)	(1.65)	(1.35)
$LIQ^{LOBslope10}$	0.104	0.080	0.173*	0.147
	(1.01)	(0.79)	(1.64)	(1.46)
$LIQ^{LOBslope}$	-0.084	-0.160	-0.061	-0.122
	(0.78)	(1.49)	(0.55)	(1.09)

Panel B: Value-weighted portfolios sorted on MV and LIQ

	Raw returns		***$\alpha_{Carhart}$***	
Measure	***Bid-side liquidity***	***Ask-side liquidity***	***Bid-side liquidity***	***Ask-side liquidity***
LIQ^{OFN}	0.219**	0.184*	0.214**	0.174
	(1.99)	(1.69)	(2.01)	(1.62)
$LIQ^{LOBslope10}$	0.198*	0.188*	0.229**	0.215**
	(1.89)	(1.87)	(2.20)	(2.12)
$LIQ^{LOBslope}$	-0.018	-0.087	0.024	-0.075
	(0.14)	(0.72)	(0.19)	(0.61)

Note: The table presents the returns on equal-weighted (Panel A) and capitalisation-weighted (Panel B) two-way sorted quartile zero investment portfolios that go short on most liquid stocks and long on least liquid ones. In the first pass, stocks are ranked according to their capitalisation, and then, within each quartile, a long-short quartile portfolio based on LIQ is formed. Only the returns on portfolios within the quartile of the largest companies are presented. The left side of the table reports raw returns on long-short portfolios and the right side demonstrates risk-adjusted returns on these portfolios computed from Carhart's (1997) four-factor model ($\alpha_{Carhart}$). Both returns and alphas are expressed in percentage terms. The values in brackets are the *t*-statistics based on Newey and West's (1987) adjusted standard errors. The asterisks ***, ** and * denote statistical significance at 0.01, 0.05 and 0.1 levels respectively.

Table 9. Returns on quartile portfolios sorted by stock liquidity among the largest companies

Panel A: Equal-weighted portfolios

	Raw returns			$\alpha_{Carhart}$		
Measure	***Illiq***	***Liq***	***Illiq-Liq***	***Illiq***	***Liq***	***Illiq-Liq***
LIQ^{OFN}	0.167	-0.069	0.236*	0.109	-0.161***	0.270*
	(1.15)	(0.43)	(1.82)	(1.00)	(2.76)	(1.89)
LIQ^{Amihud}	0.132	0.012	0.120	0.034	-0.078	0.113
	(0.92)	(0.07)	(0.98)	(0.33)	(1.29)	(0.81)
LIQ^{BAS}	0.155	0.039	0.116	0.053	-0.036	0.090
	(1.03)	(0.25)	(1.03)	(0.50)	(0.65)	(0.72)
$LIQ^{LOBslope10}$	0.138	0.020	0.118	0.124	-0.079	0.203*
	(0.94)	(0.12)	(1.02)	(1.33)	(1.24)	(1.72)
$LIQ^{LOBslope}$	0.068	0.099	-0.031	0.048	0.021	0.027
	(0.41)	(0.61)	(0.25)	(0.56)	(0.25)	(0.21)

Panel B: Value-weighted portfolios

	Raw returns			$\alpha_{Carhart}$		
Measure	***Illiq***	***Liq***	***Illiq-Liq***	***Illiq***	***Liq***	***Illiq-Liq***
LIQ^{OFN}	0.156	-0.093	0.250**	0.107	-0.126***	0.234**
	(1.05)	(0.60)	(1.96)	(1.12)	(2.61)	(1.96)
LIQ^{Amihud}	0.107	-0.086	0.193	0.048	-0.120**	0.168
	(0.69)	(0.55)	(1.56)	(0.50)	(2.45)	(1.38)
LIQ^{BAS}	0.182	-0.068	0.249**	0.150	-0.102**	0.253**
	(1.15)	(0.43)	(2.08)	(1.59)	(2.12)	(2.10)
$LIQ^{LOBslope10}$	0.107	-0.093	0.200*	0.093	-0.138***	0.232**
	(0.70)	(0.58)	(1.70)	(1.07)	(2.71)	(1.98)
$LIQ^{LOBslope}$	0.052	0.022	0.030	0.045	-0.023	0.067
	(0.31)	(0.13)	(0.21)	(0.50)	(0.27)	(0.48)

Note: The table presents the returns on quartile portfolios sorted by stock liquidity within the quartile of the largest companies, alongside the return on a zero investment portfolio that goes short on most liquid stocks and long on least liquid ones. The table also reports risk-adjusted returns on these portfolios computed based on Carhart's (1997) four-factor model ($\alpha_{Carhart}$). Both returns and alphas are expressed in percentage terms. Panel A demonstrates returns on equal-weighted portfolios and Panel B reports returns on value-weighted portfolios. The values in brackets are the *t*-statistics based on Newey and West's (1987) adjusted standard errors. The asterisks ***, ** and * denote statistical significance at 0.01, 0.05 and 0.1 levels respectively.

Table 10. Average long-short liquidity portfolios' turnovers

Panel A: Equal-weighted portfolios

Measure	***Illiq***	***Liq***	***Average (entire period)***	***Max. % trading costs (entire period)***	***Average (from June 2015)***	***Max. % trading costs (from June 2015)***
LIQ^{OFN}	0.2426	0.1554	0.1990	0.462	0.1984	0.594
LIQ^{Amihud}	0.5219	0.3007	0.4113	0.111	0.3976	0.151
LIQ^{BAS}	0.3751	0.3102	0.3426	0.115	0.3276	0.177
$LIQ^{LOBslope10}$	0.3659	0.2970	0.3315	0.128	0.3226	0.183
$LIQ^{LOBslope}$	0.7155	0.6113	0.6634	N/A	0.6441	N/A

Panel B: Value-weighted portfolios

Measure	***Illiq***	***Liq***	***Average (entire period)***	***Max. % trading costs (entire period)***	***Average (from June 2015)***	***Max. % trading costs (from June 2015)***
LIQ^{OFN}	0.2316	0.0882	0.1600	0.622	0.1572	0.794
LIQ^{Amihud}	0.5446	0.1556	0.3501	0.234	0.3371	0.286
LIQ^{BAS}	0.3963	0.1477	0.2720	0.353	0.2671	0.466
$LIQ^{LOBslope10}$	0.3782	0.1550	0.2666	0.328	0.2566	0.389
$LIQ^{LOBslope}$	0.7051	0.6266	0.6659	N/A	0.6587	0.023

Note: The table presents the turnovers of quartile portfolios sorted by stock liquidity within the quartile of the largest companies. The table also reports the maximum average per cent trading costs that can be incurred for the strategy to continue to be profitable. Panel A demonstrates the turnovers of equal-weighted portfolios and Panel B reports turnovers of value-weighted portfolios. Portfolio turnover in a given week is computed as a sum of absolute net change in the positions in all considered stocks divided by the value of a portfolio.

Table 11. Returns on OFN liquidity long-short and market buy-and-hold strategies

Panel A: Average excess return				
Period	***EW LIQ^{OFN} strategy***	***VW LIQ^{OFN} strategy***	***Buy-and-hold market (WIG index)***	***Buy-and-hold market (WIG20 index)***
January 2014 – December 2021	0.156	0.171	0.043	-0.043
June 2015 – December 2021	0.211	0.225	0.039	-0.046
Panel B: Standard deviations				
Period	***EW LIQ^{OFN} strategy***	***VW LIQ^{OFN} strategy***	***Buy-and-hold market (WIG index)***	***Buy-and-hold market (WIG20 index)***
January 2014 – December 2021	2.290	2.259	2.531	2.833
June 2015 – December 2021	2.387	2.345	2.654	2.988
Panel C: Sharpe ratios				
Period	***EW LIQ^{OFN} strategy***	***VW LIQ^{OFN} strategy***	***Buy-and-hold market (WIG index)***	***Buy-and-hold market (WIG20 index)***
January 2014 – December 2021	0.068	0.076	0.017	-0.015
June 2015 – December 2021	0.088	0.096	0.015	-0.015

Note: The table presents the comparison of the efficiency of investment strategies. Panel A demonstrates the average excess (over risk-free return) return on a strategy, Panel B reports standard deviations of weekly returns on strategies, and Panel C demonstrates their Sharpe ratios. All values are expressed in percentage terms.

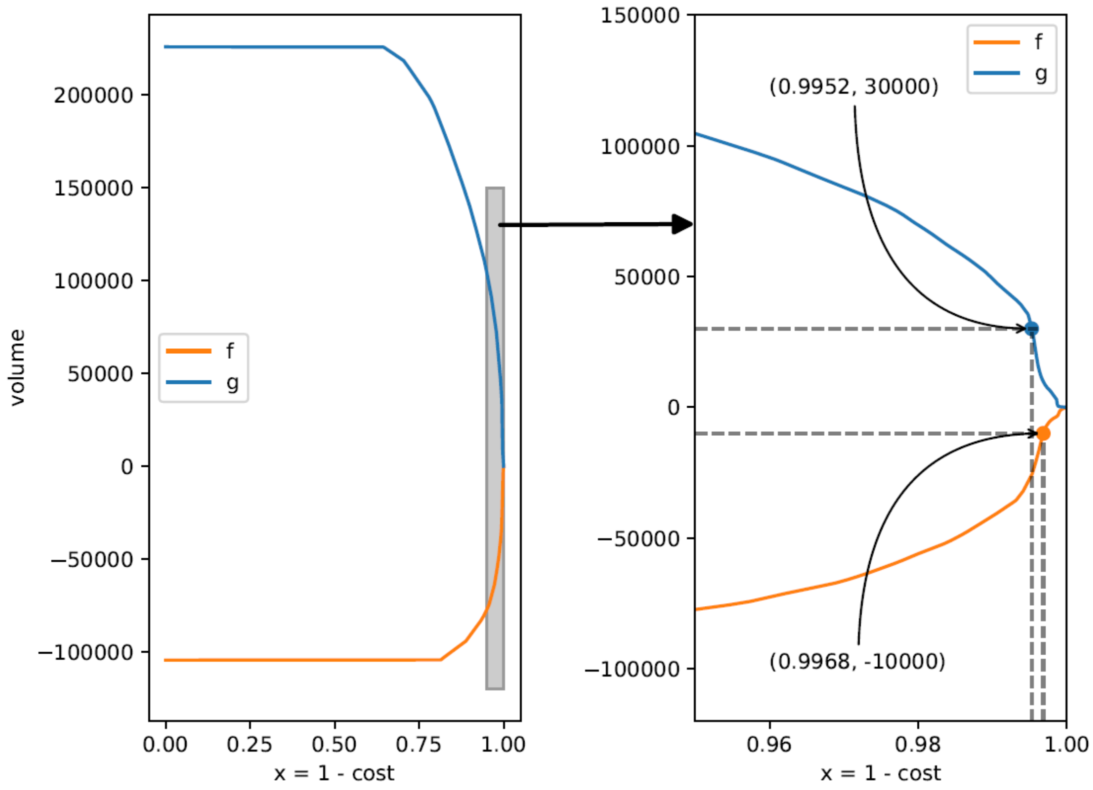

Figure 1. The ordered fuzzy number generated from the limit order book of KGHM
Note: The figure illustrates an OFN generated from the LOB of KGHM on January 3, 2017, at 09:22:52.299826.
Source: (Marszałek & Burczyński, 2024).

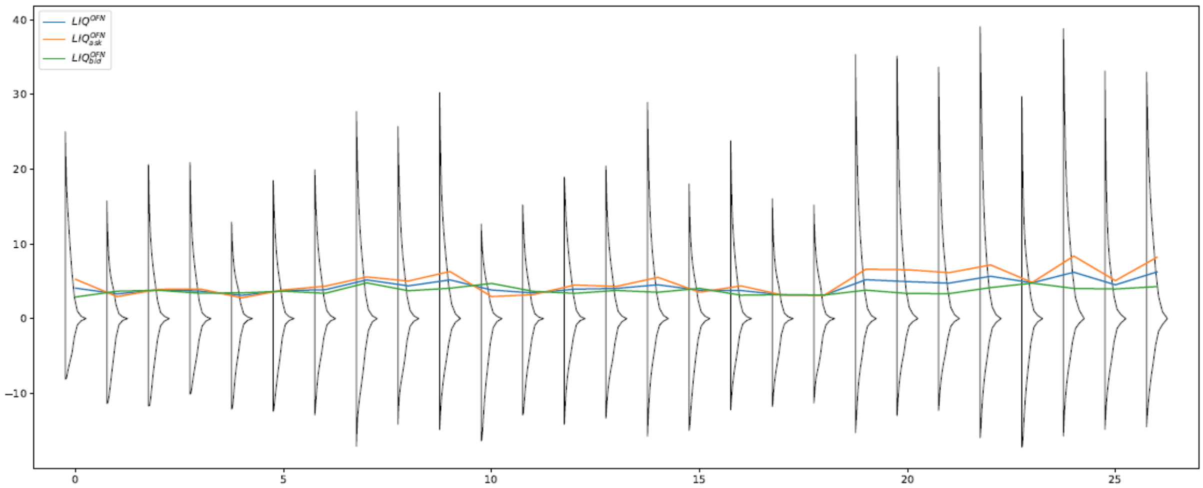

Figure 2. The ordered fuzzy numbers and LIQ^{OFN} generated from the LOB of KGHM

Note: The figure illustrates OFNs and respective LIQ^{OFN} generated from the LOB of KGHM in the period from January 3rd, 2014 to July 4th, 2014, which constitutes the first 27 weeks of a study period. LIQ^{OFN}s, LIQ^{OFN}_{ask}s and LIQ^{OFN}_{bid}s are computed according to formulae (1) and (2).

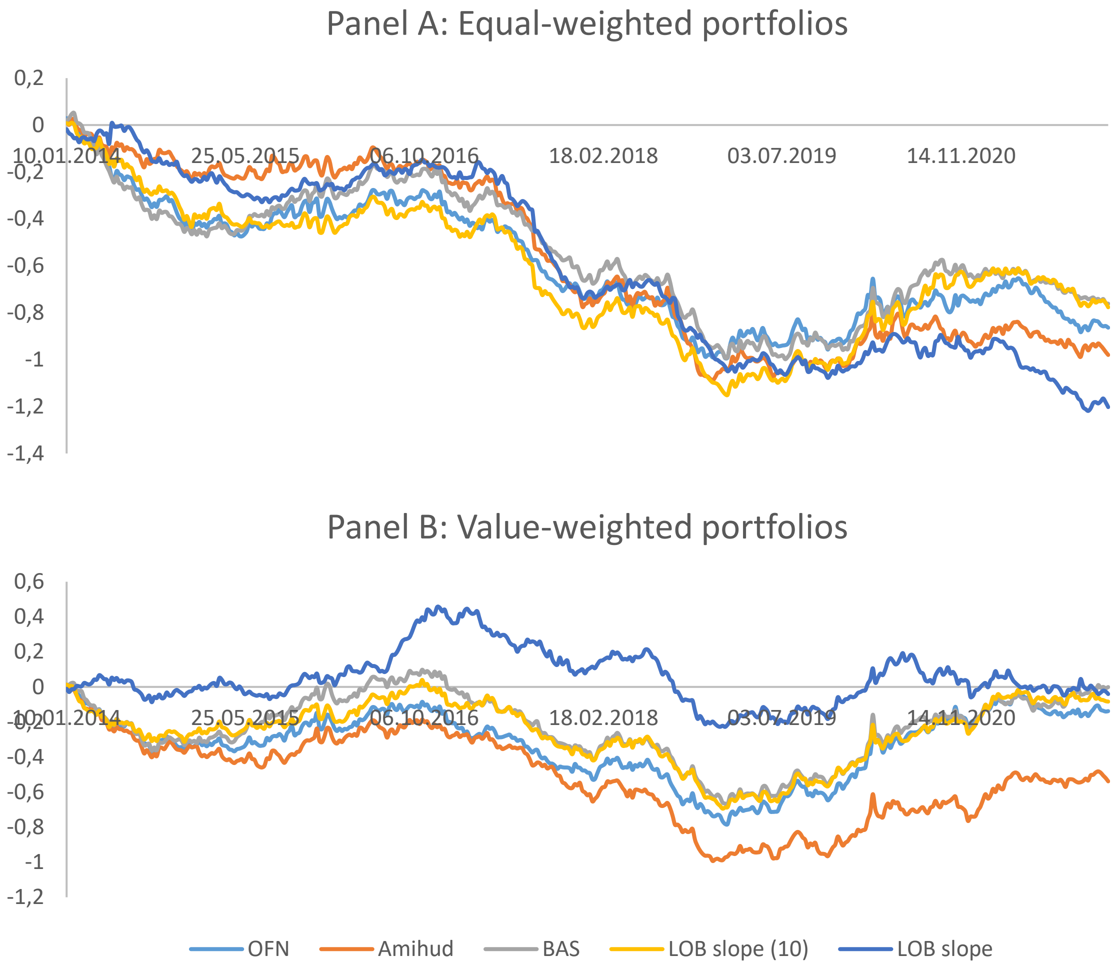

Figure 3. Cumulative long-short returns on long-short portfolios formed on stock liquidity
Note: The figure presents the cumulative returns on zero-investment equal-weighted (Panel A) and value-weighted (Panel B) portfolios formed on various liquidity measures. The portfolios go long the quintile of the least liquid stocks and short the quintile of the most liquid ones. The returns are expressed in percentage terms.

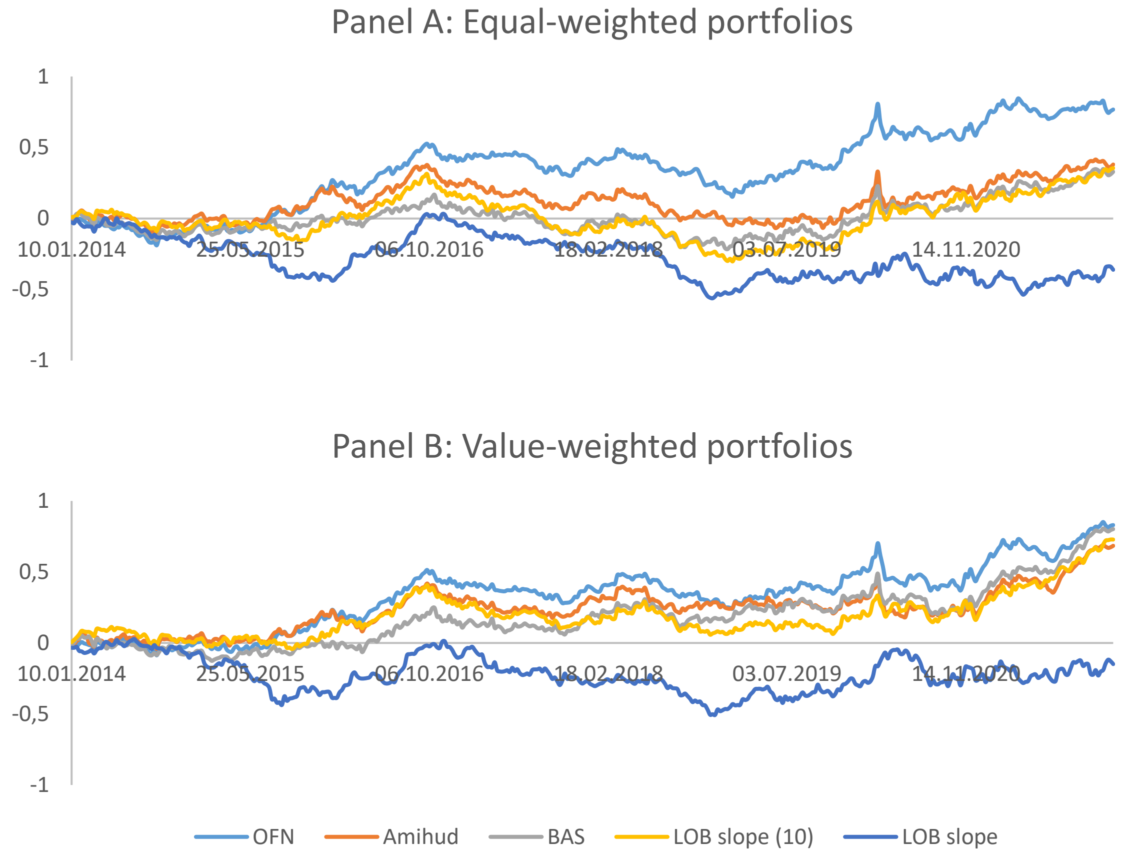

Figure 4. Cumulative long-short returns on long-short portfolios formed on stock liquidity within the quartile of the largest companies

Note: The figure presents the cumulative returns on zero-investment equal-weighted (Panel A) and value-weighted (Panel B) portfolios formed on various liquidity measures among the largest 25% companies. The portfolios go long in the quartile of the least liquid stocks and short in the quartile of the most liquid ones. The returns are expressed in percentage terms.

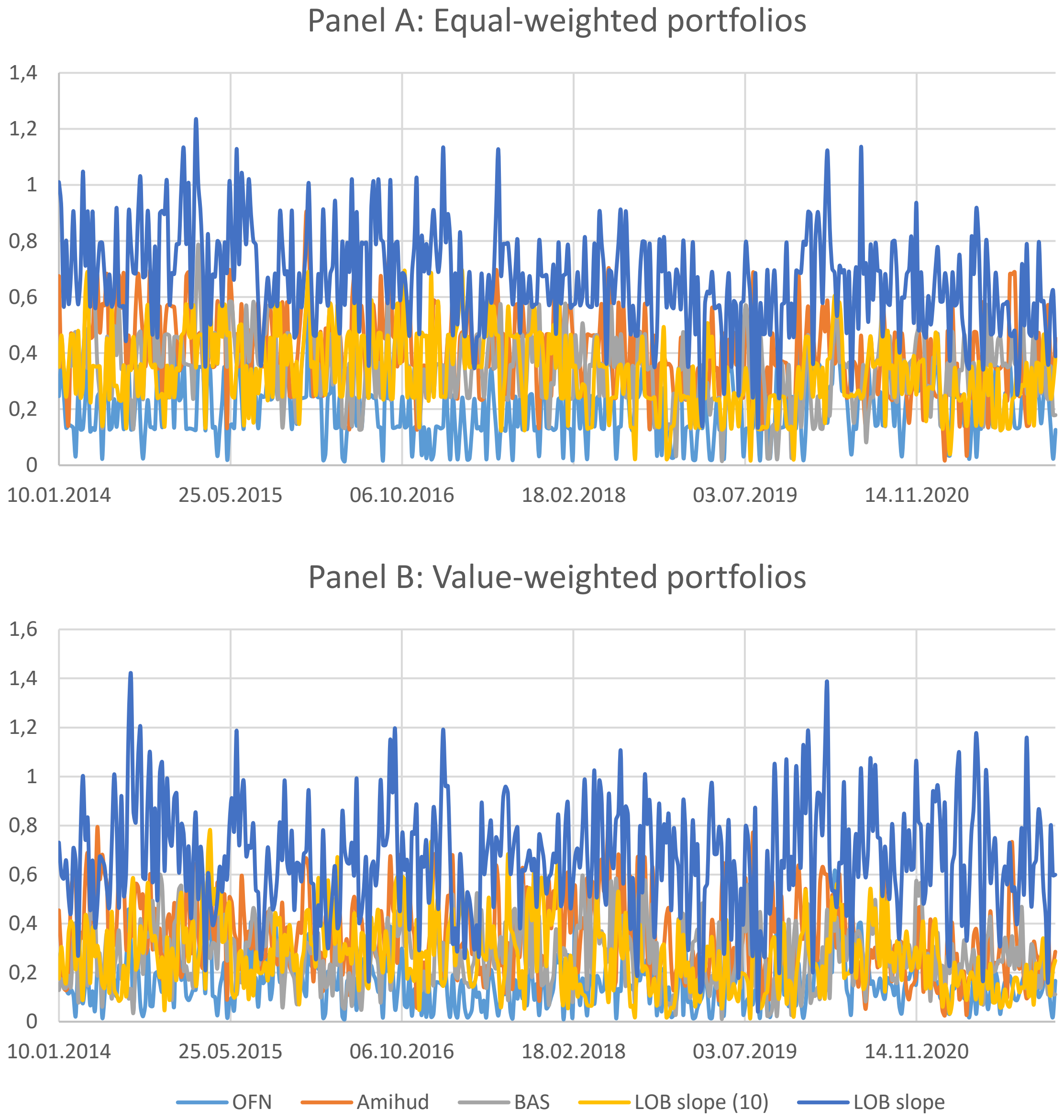

Figure 5. Average turnover of long-short portfolios formed on stock liquidity among the largest companies
Note: The figure presents the average turnover of zero-investment equal-weighted (Panel A) and value-weighted (Panel B) portfolios formed on various liquidity measures among the largest 25% companies. The portfolios go long in the quartile of the least liquid stocks and short in the quartile of the most liquid ones.

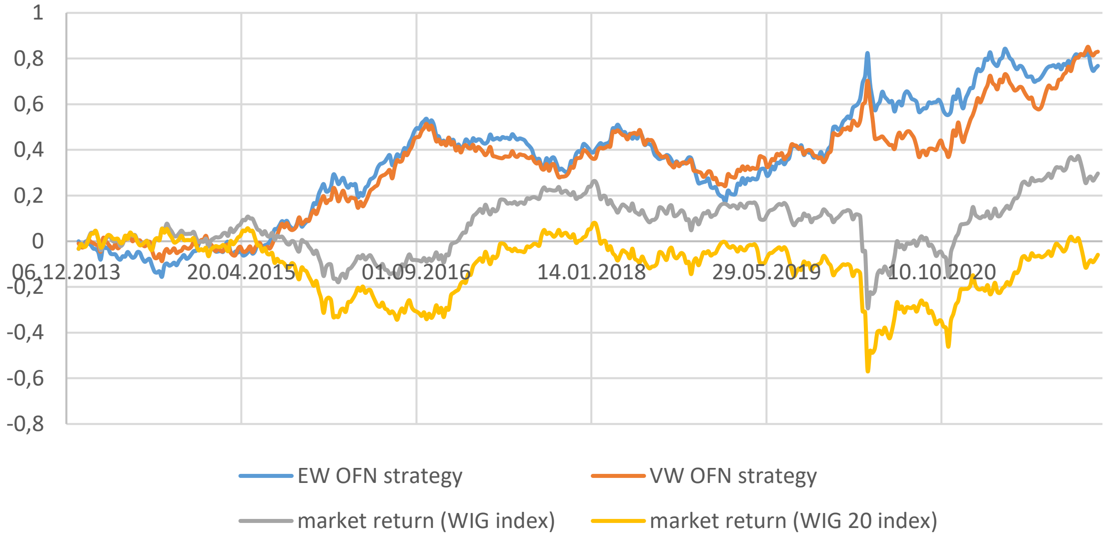

Figure 6. Cumulative returns on OFN liquidity long-short and market buy-and-hold strategies
Note: The figure presents the cumulative returns on zero-investment equal- and value-weighted portfolios formed on LIQ^{OFN} among the largest 25% companies alongside the cumulative returns on the market buy-and-hold strategy. The market return is proxied by two main indices in the Warsaw Stock Exchange: the WIG index and the WIG20 index. The returns are expressed in percentage terms.