Volatility of Price-Earnings Ratio and Return Predictability

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Abstract

We propose that the variance of the log price-earnings ratio is useful in capturing time variation for expected returns, based on our novel second-order dynamic priceearnings ratio model. We demonstrate that the volatility of the log price-earnings ratio significantly predicts positive future stock returns, in various horizons and frequencies, and both in-sample and out-of-sample. We show that the volatility of the log price-earnings ratio significantly predicts negative macroeconomic activities. Further analysis suggests that the volatility of the log price-earnings ratio is correlated with important economic state variables, economic uncertainty, quantity of risk and price of risk.

Keywords: Volatility of Price-Earnings Ratio, Predicting Market Equity Premium, Predicting Macroeconomic Activities, Quantity of Risk, Price of Risk.

JEL: E17, E44, G12, G14

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1 Introduction

Contemporary research on asset pricing asserts that expected aggregate market risk premium is not constant but rather vary over time, suggesting that aggregate market risk premium is predictable. Consistent with this perspective, the asset pricing literature underscores the importance of stochastic volatility in determining asset prices and bears a distinct risk premium (e.g., [Merton,](#page-27-0) [1980,](#page-27-0) [Bloom,](#page-24-0) [2009,](#page-24-0) [Bansal et al.,](#page-24-1) [2014,](#page-24-1) and [Campbell et al.,](#page-24-2) [2018\)](#page-24-2). There is, however, limited empirical evidence supporting time-series return predictability of volatility (measured either based on macro fundamentals or market returns), particularly in long-run.^{[1](#page-0-0)} In this paper, we propose a novel second-order dynamic price-earnings ratio model in the similar spirit of [Gao and Martin](#page-25-0) [\(2021\)](#page-25-0), and argue that both the level and the variance of price-earnings ratio serve as the optimal forecast of future market returns, cash flow growths, and volatility risk, presenting valuable new paths for comprehending the dynamic accounting identity and return predictability.

Empirically, we find that the variance of the log price-earnings ratio (V_{pe}) significantly predicts future positive market returns, in various horizons and frequencies, and both insample and out-of-sample. The predictability is robust across various sample periods, frequencies, and accounting for control variables. In addition, we show that V_{pe} significantly predicts macroeconomic activities, such as GDP growths, consumption growths, net profitability, and net cash flow growths. The predictive power of V_{pe} outperforms that of log price-earnings ratio (*pe*), the variance of market excess returns (V_{re}) , and the variance of market real returns (V_{rr}) . Further analysis indicates that the superior predictability can be attributed to the plausible linkage between V_{pe} and quantity of risk, as well as price of risk. To the best of our knowledge, this is the first study analyzing the predictability of market returns and macroeconomic activities using V_{pe} .

Our paper is motivated by the second-order dynamic price-dividend ratio model presented in [Gao and Martin](#page-25-0) [\(2021\)](#page-25-0) who extend [Campbell and Shiller](#page-24-3) [\(1988\)](#page-24-3) loglinear dynamic dividend growth model and allow the variance of log price-dividend ratio to play an impor-

¹[Guo](#page-26-0) [\(2006\)](#page-26-0) presents empirical evidence showing that aggregate market return volatility joined with consumption-wealth ratio (cay) exhibit significant return predictive power while aggregate market return volatility alone displays negligible predictive power. [Martin](#page-27-1) [\(2017\)](#page-27-1) presents evidence of short-run return predictability using a lower bound on the equity premium, $SVIX^2$, based on option data.

tant role in comprehending the dynamic accounting identity, particularly when the pricedividend ratio is persistent and far deviated with its long-run mean. We further elaborate on the dynamic accounting identity by [Gao and Martin](#page-25-0) [\(2021\)](#page-25-0) in two distinct dimensions. First we relax the homoscedasticity assumption of price-dividend ratio in [Gao and Martin](#page-25-0) [\(2021\)](#page-25-0). Second, we shift our focus on price-earnings ratio instead of price-dividend ratio for several reasons. Empirical evidence shows unstable dividend policy (e.g., [Fama and](#page-25-1) [French,](#page-25-1) [2001;](#page-25-1) and [Vuolteenaho,](#page-28-0) [2002\)](#page-28-0) and stronger connection between earnings and economic activities and fundamentals (e.g., [Penman and Sougiannis,](#page-28-1) [1998](#page-28-1) and [Konchitchki and](#page-26-1) [Patatoukas,](#page-26-1) [2014\)](#page-26-1), as well as better return predictability of price-earnings ratio compared to price-dividend ratio(e.g., [Campbell and Shiller,](#page-24-4) [2005\)](#page-24-4). In light of this observation, we propose a novel second-order dynamic price-earnings ratio model following a similar approach as [Gao and Martin](#page-25-0) [\(2021\)](#page-25-0). This model presents a two-factor structure that both pe and V_{pe} are endogenously associated with the long-run future market returns (risk premium), long-run future growths, and/or long-run volatility.

We argue that V_{pe} better captures the inherent uncertainty and forward-looking in stochastic volatility of market returns and fundamentals. First, the nature of conditional volatility is unobservable or latent, and time-varying. Many empirical estimates of conditional volatility rely on realized returns or realized fundamentals. A growing list of studies explores ex ante measure of expected returns using valuation ratios (e.g., [Claus and Thomas,](#page-25-2) [2001;](#page-25-2) [Easton,](#page-25-3) [2004;](#page-25-3) [Polk et al.,](#page-28-2) [2006](#page-28-2) [Kelly and Pruitt,](#page-26-2) [2013;](#page-26-2) and [Jiang and Kang,](#page-26-3) [2020\)](#page-26-3). V_{pe} reflects the inherent uncertainty and forward-looking nature in expected return volatility if pe ratio is a reasonable proxy for the expected returns.^{[2](#page-0-0)} Second, V_{pe} captures the information on the uncertainty and risk from both market and fundamental factors. Third, the predictability of the V_{pe} is a direct implication of the second-order dynamic price-earnings ratio model. Indeed our empirical results support that V_{pe} is superior to the variance of realized returns (excess returns and real returns) in terms of predictability of market returns and macroeconomic activities.

Our paper contributes to the return predictability literature by demonstrating that V_{pe}

 2 In a recent study, [Ai et al.](#page-24-5) [\(2022\)](#page-24-5) propose and examine the information-driven volatility measured as variance of expected macroeconomic fundamentals, showing that the information-driven volatility induces a negative correlation between past realized volatility and future expected returns.

as a proxy for the volatility of expected returns significantly and robustly predicts positive market returns. Numerous studies have found, though with controversy, that market expected returns are time-varying and predictable with various variables (e.g., [Ang and](#page-24-6) [Bekaert,](#page-24-6) [2007;](#page-24-6) [Cochrane,](#page-25-4) [2008;](#page-25-4) [Goyal and Welch,](#page-26-4) [2008;](#page-26-4) [Campbell and Thompson,](#page-24-7) [2008;](#page-24-7) [Rapach et al.,](#page-28-3) [2010;](#page-28-3) [Koijen and Van Nieuwerburgh,](#page-26-5) [2011;](#page-26-5) [Zhou and Zhu,](#page-28-4) [2015;](#page-28-4) [Yang,](#page-28-5) [2023;](#page-28-5) [Gao and Martin,](#page-25-0) [2021;](#page-25-0) [Goyal et al.,](#page-26-6) [2021;](#page-26-6) [Cederburg et al.,](#page-25-5) [2023;](#page-25-5) and [Bali et al.,](#page-24-8) [2023;](#page-24-8) among others). [Cederburg et al.](#page-25-5) [\(2023\)](#page-25-5) provides an insightful discussion on the economic significance of stock market return predictability. While the existing studies provide comprehensive evaluation on return predictability using various predictors, this study represents the initial exploration of return predictability through the analysis of V_{pe} . We identify a new predictor, V_{pe} , which exhibits substantial and consistent predictability for future market returns and macroeconomic activities.

A growing literature emphasizes the effect of stochastic volatility in macroeconomics and finance. [Bansal et al.](#page-24-1) [\(2014\)](#page-24-1) and [Campbell et al.](#page-24-2) [\(2018\)](#page-24-2) develop an intertemporal asset pricing model with stochastic volatility and demonstrate that volatility risk is indeed an important and separate risk that significantly affects the macroeconomic activities and asset prices. The volatility risk, beyond cash flow risk and discount rate risk, is priced in the cross-section of stock returns. These studies measure stochastic volatility relying on realized market returns or macroeconomic activities (also see [Zhou and Zhu,](#page-28-4) [2015\)](#page-28-4) and concentrate on examining the impact of stochastic volatility on the cross-section of stock returns. We introduce V_{pe} as a novel measure of stochastic volatility. We emphasize the aggregate timeseries dynamic relation between stochastic volatility and expected market risk premium and macroeconomic activities. The time-series predictability using V_{pe} substantially surpasses that achieved through the volatility of realized returns, complementing the cross-sectional study on stochastic volatility.

Our paper also relates to the literature on empirical tests of risk-return trade-off relation. The main challenge in testing the risk-return trade-off relation (e.g., [Merton,](#page-27-0) [1980\)](#page-27-0) is that the expected market return and the conditional variance of the market are not observable. To better understand the inherent stochastic nature of conditional variance, researchers explore various methods involving statistical models, econometric techniques, or other proxies (e.g., [Glosten et al.,](#page-26-7) [1993;](#page-26-7) [Harvey,](#page-26-8) [2001;](#page-26-8) [Ghysels et al.,](#page-25-6) [2005;](#page-25-6) [Jiang and Lee,](#page-26-9) [2014;](#page-26-9) among others). [Scruggs](#page-28-6) [\(1998\)](#page-28-6), [Guo](#page-26-0) [\(2006\)](#page-26-0), and [Guo and Whitelaw](#page-26-10) [\(2006\)](#page-26-10) argue that the misspecification problem caused by omitted variables leads to weak or negative risk-return relation. Using the variance of the log price-earnings ratio as a measure of volatility of expected returns, we provide additional positive risk-return trade-off evidence consistent with [Merton](#page-27-0) [\(1980\)](#page-27-0).

The rest of the paper is organized as follows. Section [2](#page-4-0) outlines our framework. We discuss data and the construction of volatility of log price-earnings ratio in Section [3.](#page-7-0) Section [4](#page-9-0) presents our main empirical results. Section [5](#page-18-0) explores the potential sources of predictability and we conclude in Section [6.](#page-23-0)

2 Framework

We start with the loglinear present value identity of [Campbell and Shiller](#page-24-3) [\(1988\)](#page-24-3). Let P_{t+1}, D_{t+1} , and R_{t+1} be the price, dividend, and gross return of the market, respectively, we define gross return as:

$$
R_{t+1} = \frac{P_{t+1} + D_{t+1}}{P_t} = \frac{P_{t+1} + D_{t+1}}{D_{t+1}} \frac{D_{t+1}}{D_t} \frac{D_t}{P_t}
$$
(1)

Taking logarithm of both sides of equation [\(1\)](#page-4-1) yields:

$$
r_{t+1} = \Delta d_{t+1} - p d_t + \log(1 + e^{p d_{t+1}})
$$
\n(2)

where $pd_t = p_t - d_t = log(P_t) - log(D_t)$ and $\Delta d_{t+1} = d_{t+1} - d_t$.

Applying the first-order Taylor approximation to linearize the last term in equation [\(2\)](#page-4-2), [Campbell and Shiller](#page-24-3) [\(1988\)](#page-24-3) derive that the log of the price-dividend ratio can be expressed as a linear function of expected future returns and expected dividend growth rates; that is

$$
pd_t = k + \sum_{j=0}^{\infty} \rho^j E_t(\Delta d_{t+1+j} - r_{t+1+j}),
$$
\n(3)

where lowercase letters denote logs of the corresponding uppercase letters, $pd_t = p_t - d_t$,

 $\Delta d_{t+j+1} = d_{t+j+1} - d_{t+j}$, and $\rho = \frac{\mu}{1+\mu}$ $\frac{\mu}{1+\mu}$, where $\mu = e^{\bar{p}\bar{d}}$, and $k = \frac{\log(1+\mu)-\rho\log(\mu)}{1-\rho}$ $\frac{\mu(-\rho log(\mu))}{1-\rho}$. Equation [\(3\)](#page-4-3) demonstrates that the log price-dividend ratio provides the optimal forecast of the long-run growth rates and long-run returns.

[Gao and Martin](#page-25-0) [\(2021\)](#page-25-0) expand upon the present value identity of [Campbell and Shiller](#page-24-3) [\(1988\)](#page-24-3) using the second-order Taylor expansion and allow the second movement of log pricedividend ratio to enter the present value identity. Assuming that the log price-dividend ratio follows an AR(1) process: $pd_{t+1} - \bar{pd} = \phi (pd_t - \bar{pd}) + \epsilon_{t+1}$, where $var_t \epsilon_{t+1} = \sigma^2$, [Gao and](#page-25-0) [Martin](#page-25-0) [\(2021\)](#page-25-0) derive the following novel present value identity:

$$
pd_t = \alpha + \sum_{j=0}^{\infty} \rho^j E_t(g_{d,t+1+j} - r_{t+1+j}) + \frac{\rho(1-\rho)\phi^2}{2(1-\rho\phi^2)}var(pd_t)
$$
(4)

where $var(pd_t)$ denotes the variance of log price-dividend ratio and $\alpha = k + \frac{\rho \sigma^2}{2(1-\rho \sigma^2)}$ $\frac{\rho \sigma^2}{2(1-\rho\phi^2)}$. [Gao](#page-25-0) [and Martin](#page-25-0) [\(2021\)](#page-25-0) view the variance of log price-dividend ratio as convexity correction, and show that the variance of log price-dividend ratio can be quantitatively important when price-dividend ratio is persistent and deviates far away from its mean.

In this study, we propose a novel second-order dynamic price-earnings ratio model, expanding upon the work of [Gao and Martin](#page-25-0) [\(2021\)](#page-25-0) in two key aspects. First, we emphasize on price-earnings ratio rather than price-dividend ratio. Corporate dividend policies are widely recognized for their instability and modeling challenge with many firms refraining from paying dividends in their early stages (e.g., [Fama and French,](#page-25-1) [2001;](#page-25-1) and [Vuolteenaho,](#page-28-0) [2002\)](#page-28-0). In addition, earnings are more directly related to economic activities and fundamentals, and better predict future stock returns than price-dividend ratio (e.g., [Penman and Sougiannis,](#page-28-1) [1998;](#page-28-1) and [Konchitchki and Patatoukas,](#page-26-1) [2014\)](#page-26-1). Price-earnings ratio also shows better return predictability [\(Campbell and Shiller,](#page-24-4) [2005\)](#page-24-4). Second, to explore the information content of V_{pe} , we relax the homoscedasticity assumption in [Gao and Martin](#page-25-0) [\(2021\)](#page-25-0) and allow stochastic volatility reflecting time-varying economic uncertainty to enter the dynamic identity.

Following [Nelson](#page-27-2) [\(1999\)](#page-27-2) and [Sharpe](#page-28-7) [\(2002\)](#page-28-7), we express log price-dividend ratio (pd_t) as log price-earnings ratio(pe_t) and log dividend payout ratio(λ_t). Dividend smoothing is one of the most well-documented phenomena in corporate financial policy. [Lintner](#page-27-3) [\(1956,](#page-27-3) [1963\)](#page-27-4) observes that firms are primarily concerned with the stability of dividends and attempt to make adjustments of dividends toward some desirable (target) payout ratio. Here, since our interest is the variance of price-earnings ratio, we model the dividend payout ratio as the long-term target ratio($\bar{\Lambda}$) times a random variable (Γ_t) for simplicity: $\Lambda_t = \bar{\Lambda} \Gamma_t$, and $\gamma_t \equiv log(\Gamma_t) \sim N(0, \kappa^2)$. Formally,

$$
pd_t = pe_t - \lambda_t \tag{5}
$$

where $\lambda_t = log(\Lambda_t) = log(\bar{\Lambda}) + \gamma_t$ ^{[3](#page-0-0)}

Denoting the aggregate earning of the market by E , equation (2) can be rewritten as:

$$
r_{t+1} = \Delta e_{t+1} + \lambda_{t+1} - pe_t + \log(1 + e^{pe_{t+1} - \lambda_{t+1}})
$$
\n(6)

where $\Delta e_{t+1} = e_{t+1} - e_t = log(E_{t+1}) - log(E_t)$.

Following [Gao and Martin](#page-25-0) [\(2021\)](#page-25-0) and taking a second-order Taylor expansion, we derive the following novel present value identify:

$$
pe_t = \delta^* + E_t[\sum_{j=0}^{\infty} \rho^j(\Delta e_{t+1+j} - r_{t+1+j})] + \frac{1}{2}\rho(1-\rho)E_t[\sum_{j=0}^{\infty} \rho^j(pe_{t+1+j} - \bar{pe})^2]
$$
(7)

where $\rho = \frac{e^{\bar{p}e-\bar{\lambda}}}{1+\bar{p}e^{-\bar{\lambda}}}$ $\frac{e^{\bar{p}e-\bar{\lambda}}}{1+e^{\bar{p}e-\bar{\lambda}}}=\frac{e^{\bar{p}\bar{d}}}{1+e^{\bar{p}}}$ $\frac{e^{pd}}{1+e^{pd}} > 0$, and $\delta^* = \frac{1}{2}$ $\frac{1}{2}\rho\kappa^2+\frac{k-\rho\bar{p}e+\bar{\lambda}}{1-\rho}$ $\frac{\rho \bar{p} \bar{e} + \bar{\lambda}}{1-\rho}$, all are constant. $\bar{p} \bar{e}$ and $\bar{\lambda}$ are defined as the aggregate mean of log price-earnings ratio and log long-term target dividend payout ratio.

The habit model in [Campbell and Cochrane](#page-24-9) [\(1999\)](#page-24-9) reveals that time-varying risk aversion directly affects time-varying price-dividend ratio (price-earnings ratio). The higher volatility of risk aversion is associated with higher volatility of price-earnings ratio, and then higher expected returns. On the other hand, [Bansal and Yaron](#page-24-10) [\(2004\)](#page-24-10)'s long-run risk model indicates that the volatility of valuation ratios can be attributed to variation in expected growth rates and fluctuating economic uncertainty (conditional volatility of consumption). Accordingly we model the dynamics of price-earnings ratio with the time-varying stochastic volatility. We assume log price-earnings ratio follows $AR(1)$ process. However, we relax the homoscedasticity assumption in [Gao and Martin](#page-25-0) [\(2021\)](#page-25-0) and allow time-varying conditional

³Empirically, the aggregate long-term target dividend payout ratio $(\bar{\Lambda})$ is about 0.55 over the sample period of 1927 - 2021, and the regression of log dividend payout ratio (λ_t) on the constant shows a high R^2 of 0.81.

volatility of a state variable in the economy.

$$
pe_{t+1} - \bar{pe} = \phi(pe_t - \bar{pe}) + \psi \sigma_t u_{t+1},
$$
\n(8)

where $u_{t+1} \sim N i.i.d(0,1)$ and σ_t denotes the stochastic volatility reflecting time-varying economic uncertainty. Substituting equation (8) into equation (7) , we obtain the following second-order dynamic price-earnings ratio model ^{[4](#page-0-0)}:

$$
pe_t - \pi E_t[(pe_t - \bar{pe})^2] = \alpha^* + E_t[\sum_{j=0}^{\infty} \rho^j(\Delta e_{t+1+j} - r_{t+1+j} + \nu \sigma_{t+1+j}^2)]
$$
(9)

where $\pi = \frac{\rho(1-\rho)\phi^2}{2(1-\rho\phi^2)}$ $\frac{\rho(1-\rho)\phi^2}{2(1-\rho\phi^2)}, \alpha^* = \frac{\psi^2\sigma^2}{1-\rho\phi^2} + \delta^*, \nu = \frac{\psi^2\rho(1-\rho)}{2(1-\rho\phi^2)}$ $\frac{\psi^2 \rho(1-\rho)}{2(1-\rho\phi^2)}$, and $E_t[(pe_t-\bar{pe})^2]$ is the conditional volatility of log price-earnings ratio. It is important to note that even though the unconditional variance of pe ratio is constant, the conditional variance of pe ratio is time-varying. In the empirical work below, we use the variance of ten-year rolling log S&P price-earnings ratio as a proxy for the conditional variance of pe ratio. Equation (9) closely aligns with three risk factors, cash flow risk, discount rate risk, and volatility risk, emphasized in [Bansal et al.](#page-24-1) [\(2014\)](#page-24-1) and [Campbell et al.](#page-24-2) [\(2018\)](#page-24-2). It states that both level and variance of log price-earnings ratio exhibit the optimal forecast of the long-run earnings growths, long-run expected returns and long-run volatility. It highlights the importance of the second movement of price-earnings ratio and provides valuable new paths for comprehending the dynamic accounting identity and return predictability.

3 Data

In our empirical analysis, we use an annual sample from 1937 to 2021.^{[5](#page-0-0)} Market returns (R_m) are from CRSP value-weighted market returns, while stock market prices (P) , dividends $(D,$ four-quarter moving sum of dividends), and earnings $(E,$ four-quarter moving sum of earnings) of the Standard and Poor's (S&P) 500 index are from Robert J. Shiller's website.

⁴Detailed derivation can be found in the Appendix.

⁵Our original sample is from 1927 to 2021. Due to the construction of volatility, we lose ten years of observations. The testing data sample period is from 1937 to 2021.

The risk-free rates (R_f) are measured as one-month T-bill rates from CRSP. Inflation rates are based on the Consumer Price Index from the Bureau of Labor Statistics.

The log price-earnings (pe) ratio is measured as the difference between the log of price and log of earnings. The stock market excess returns (R_e) and real returns (R_r) are measured as the difference between the stock market returns and risk-free rates as well as the difference between market returns and inflation rates. We construct the volatility of log price-earnings ratio (V_{pe}) as the variance of log price-earnings ratios using a ten-year window. So are the volatility of excess returns (V_{re}) and the volatility of real returns (V_{rr}) . [Asness](#page-24-11) [\(2000\)](#page-24-11) argues that investors' perception of the relative risk of equity is shaped by the volatility it has experienced. To capture the long-run component of the volatility, we select 10-year window for volatility estimation.^{[6](#page-0-0)} The relatively long window, smoothing out short-run transitory component, tends to reveal information on long-run risk. Summary statistics for these variables are presented in Table 1.

In Panel A, we report mean, standard deviation, first-order autocorrelation, and unit-root tests based on the augmented Dickey-Fuller test[\(Dickey and Fuller,](#page-25-7) [1979\)](#page-25-7) and the Philips-Perron test[\(Phillips and Perron,](#page-28-8) [1988\)](#page-28-8) of these variables. All variables considered appear stationary, and the first-order autocorrelation coefficients are less than or equal to 0.9. We are particularly interested in the property of V_{pe} . The null hypothesis of a unit root has been statistically rejected at 5% significance level based on both augmented Dickey-Fuller test and Philips-Perron test. The V_{pe} shows reasonable persistence with the AR(1) coefficient of 0.834, lower than the coefficients of the variance of excess returns (V_{re}) and variance of real returns (V_{rr}) . The V_{pe} has a higher mean and a slightly higher standard deviation compared to V_{re} and V_{rr} . In Panel B, we report a correlation matrix between the considered variables. The pe as expected is positively correlated with excess return and real return and negatively correlated with various volatility measures. V_{re} and V_{rr} are highly correlated with the coefficient of 0.969. V_{re} (V_{rr}) is negatively correlated to excess returns (real returns) with notable smaller values. The correlation between V_{re} and excess return is -0.075 and the correlation between V_{rr} and real return is -0.084. It is interesting to note that V_{pe} is positively correlated with both market returns $(R_e$ and $R_r)$ and realized market volatility

⁶The results are robust when the volatility of pe ratio is estimated using 5-year and 15-year windows.

 $(V_{re}$ and V_{rr}). The correlations between V_{pe} and R_e and between V_{pe} and R_r are 0.247 and 0.170, respectively, while the correlations between V_{pe} and V_{re} and between V_{pe} and V_{rr} are 0.165 and 0.320, respectively.

 V_{pe} is quite volatile and persistent in our sample period. We plot V_{pe} with one-year, five-year, seven-year, and ten-year ahead stock excess return in Figure 1. We find that V_{pe} shows similar long up and down swings as long-run returns. The property of slow mean reversion in V_{pe} hints at long-run return predictability.

4 Predicting equity market returns and macroeconomic activities

4.1 In-sample return prediction

We primarily employ the OLS predictive regressions in our in-sample analysis. However, as well-documented in the return predictability literature, predictive regressions pose certain serious econometric issues (e.g., [Granger and Newbold,](#page-26-11) [1974;](#page-26-11) [Mark,](#page-27-5) [1995;](#page-27-5) [Nelson and Kim,](#page-27-6) [1993;](#page-27-6) [Stambaugh,](#page-28-9) [1999;](#page-28-9) [Lewellen,](#page-27-7) [2004,](#page-27-7) and [Kostakis et al.,](#page-27-8) [2015\)](#page-27-8). To mitigate potential heteroskedasticity and autocorrelation concerns, we calculate and report [Newey and West](#page-28-10) [\(1987\)](#page-28-10) heteroskedasticity and autocorrelation consistent (HAC) standard error (computed with lags set to the number of horizons plus one) for parameter estimates. To address the statistical issue with overlapping data in long-horizon predictive regressions, we calculate and report [Hodrick](#page-26-12) [\(1992\)](#page-26-12) standard error. [Stambaugh](#page-28-9) [\(1999\)](#page-28-9) shows that there is a small sample bias in the estimated predictive coefficient in forecast regressions with price-scaled variables, particularly when they are highly persistent. Although our key predictor, V_{pe} , is not a pricescaled variable per se, and the null hypothesis of a unit root has been statistically rejected, it is quite persistent. To remedy this issue, we apply the bootstrap procedure, impose the null of no predictability in calculating the critical values, and report the bootstrap p -values for each parameter estimate.

The second-order dynamic price-earnings ratio model in Equations [\(9\)](#page-7-2) implies that both

the level and the variance of pe are associated with future market returns, cash flow growths and volatility. However, few studies have investigated the return predictability using V_{pe} . [Guo](#page-26-0) [\(2006\)](#page-26-0) and [Guo and Savickas](#page-26-13) [\(2006\)](#page-26-13) document that predicting market returns based solely on aggregate stock market volatility yields little predictive ability. However, market volatility jointly with either idiosyncratic volatility or consumption wealth ratio (cay) by [Lettau and Ludvigson,](#page-27-9) [2001](#page-27-9) exhibits strong predictive power for market excess return, supporting positive risk-return relation. For comparison, we also examine the return predictability of variance of excess returns and variance of real returns. We report forecast regression coefficients, Newey-West corrected standard errors [\(Newey and West,](#page-28-10) [1987\)](#page-28-10), Ho-drick standard errors [\(Hodrick,](#page-26-12) [1992\)](#page-26-12), bootstrapping p-value and adjusted R^2 statistics.

4.1.1 Univariate return prediction

We start with the in-sample prediction by conducting the following univariate longhorizon predictive regression:

$$
R_{t,t+k} = \alpha_k + \beta_k x_t + \epsilon_{t+k}.\tag{10}
$$

where $R_{t,t+k}$ is the compounded excess return (R_e) or real return (R_r) in k years in the future, and x_t represents a predictive variable known at time t, which includes pe, V_{re} , V_{rr} , and V_{pe} , respectively. The predictive horizons are 1, 3, 5, 7, and 10 years ahead, respectively.

The results for the univariate long-horizon predictive regressions for excess return and real return are presented in Table 2. Whether using R_e or R_r as the dependent variable, we find that pe predicts future market returns negatively while V_{pe} predicts market returns positively, consistent with our second-order dynamic price-earnings ratio model. V_{pe} significantly predicts R_e and R_r at each horizon, from one year to ten years ahead. The positive relation between V_{pe} and future market returns is consistent with the positive risk-return relation (e.g., [Merton,](#page-27-0) [1980\)](#page-27-0). The adjusted R^2 monotonically increases as the number of prediction horizons increases. For example, V_{pe} predicts 5.2% of the variation of market excess returns one-year ahead; the predictive coefficient is 1.362 with a Newey-West standard error of 0.547, Hodrick standard error of 0.685, and bootstrapping p -value of 0.027. In three-year

horizon, V_{pe} predicts 18.9% of the variation of market excess returns; the predictive coefficient is 4.452 with a Newey-West standard error of 1.196, Hodrick standard error of 1.922, and bootstrapping p-value of 0.000. For five-year ahead forecast, 23.2% of the variation of market excess returns is predicted by V_{pe} ; the predictive coefficient is 7.241 with a Newey-West standard error of 2.152, Hodrick standard error of 2.904, and bootstrapping p -value of 0.000. In seven-year horizon, V_{pe} predicts 27.0% of the variation of market excess returns; the predictive coefficient is 10.340 with a Newey-West standard error of 3.220, Hodrick standard error of 3.662, and bootstrapping p-value of 0.000. Finally, in ten-year horizon, V_{pe} predicts 34.0% of the variation of market excess returns; the predictive coefficient is 18.408 with a Newey-West standard error of 5.293, Hodrick standard error of 4.352, and bootstrapping pvalue of 0.000. Similar results are found when the dependent variable is market real return. V_{pe} significantly predicts positive real returns in each of the horizons, from one-year ahead to ten-year ahead. The adjusted R^2 monotonically increases as the number of prediction horizons increases, spanning from 3.2% in one-year horizon to 33.7% in ten-year horizon.

 V_{re} , in contrast, shows no return predictability in one-year, three-year, or five-year horizons. In seven-year and ten-year horizons, it exhibits moderate return predictability with adjusted R^2 of 3.0% and 7.8%, respectively. V_{rr} , on the other hand, exhibits no return predictability in any prediction horizon. Our result of no return predictability of V_{re} and V_{rr} is consistent with the empirical work of the univariate predictive regression in [Guo](#page-26-0) [\(2006\)](#page-26-0) and [Guo and Savickas](#page-26-13) [\(2006\)](#page-26-13). The superior performance of return predictability using V_{pe} compared to V_{re} and V_{rr} suggests that V_{pe} better captures time-varying market risk premium than V_{re} and V_{rr} . Plausibly, V_{pe} contains information on both the uncertainty of market and the uncertainty of fundamentals.[7](#page-0-0)

pe ratio alone also exhibits reasonable return predictability. However, in each prediction horizon, the adjusted R^2 s for pe exhibit notably lower values compared to those of V_{pe} . While existing studies focus on the level of pe ratio in return prediction, we provide novel evidence that the second moment of pe ratio, V_{pe} , significantly predicts positive future excess returns

⁷We also examine the return predictability of variance of log price-dividend ratio(V_{pd}), variance risk premium (vrp) in [Bollerslev et al.](#page-24-12) [\(2009\)](#page-24-12), and stock variance(svar) in [Guo and Whitelaw](#page-26-10) [\(2006\)](#page-26-10) in Table B.1. V_{pd} and vrp exhibit no predictability at all horizons. svar shows some predictability in three-year and five-year horizons.

and real returns, even better than the level of pe ratio.^{[8](#page-0-0)}

The return predictability using valuation ratios appears to be unstable (e.g. [Ang and](#page-24-6) [Bekaert,](#page-24-6) [2007,](#page-24-6) [Lettau et al.,](#page-27-10) [2008,](#page-27-10) [Lettau and Van Nieuwerburgh,](#page-27-11) [2008,](#page-27-11) [Goyal and Welch,](#page-26-4) [2008\)](#page-26-4). To examine whether the return predictability using V_{pe} exhibits instability, also for the purpose of conducting robustness check, we explore different sample periods. In Table 3, using four different subsamples, 1937-1999 (excluding Tech Bubble, 2008 Financial Crisis and Covid-19), 1937-2007 (excluding 2008 Financial Crisis and Covid-19), 1937-2019 (excluding Covid-19) and 1950-2021 (excluding World War II), we examine the stability of return predictability using V_{pe} .

Panel A of Table 3 reports the results when applying market excess return as the dependent variable. pe ratio significantly predicts future returns with a negative coefficient in different subsamples, 1937-1999, 1937-2007 and 1937-2019, respectively. However, in the 1950-2021 subsample, pe ratio shows insignificant predictability in one-year, three-year, and five-year horizons. The adjusted R^2 s of univariate regression using pe are 3.6%, 6.9%, 3.6% and 0.2% in the one-year horizon, and $28.0\%, 28.2\%, 23.6\%$ and 10.4% in the ten-year horizon, for the subsample period 1937-1999, 1937-2007, 1937-2019 and 1950-2021, respectively. This evidence is consistent with the unstable return predictability using valuation ratios in the literature. V_{re} consistently shows insignificant return predictability at the 5% level for any prediction horizon and subsample except for the ten-year horizon in the subsamples from 1937 to 1999 and from 1937 to 2007, and seven-year and ten-year horizons in the subsample from 1937 to 2019. In contrast, V_{pe} predicts future excess returns with a significantly positive coefficient in each horizon and each subsample. The adjusted R^2 s of one-year ahead prediction regression using V_{pe} are relatively stable, 6.0%, 7.2%, 6.4% and 3.6%, for the subsample period 1937-1999, 1937-2007, 1937-2019 and 1950-2021, respectively, and 35.5%, 34.9%, 32.7% and 30.7%, for the subsample period 1937-1999, 1937-2007, 1937-2019 and 1950-2021, respectively, in the ten-year horizon. The results reported in Panel B using real return as a dependent variable are qualitatively similar.

[Lundblad](#page-27-12) [\(2007\)](#page-27-12) argues that the primary challenge in estimating the risk-return rela-

⁸We report robust univariate regression results in quarterly frequency with predictive horizons of 1, 2, 3, 4, 12, 20, 28 and 40 quarters in Table B.2 and show that V_{pe} predicts market excess returns in all horizons considered and predicts market real returns from the fourth quarter onward.

tionship is due to small samples. Using information from a longer historical record of the U.S. and U.K. equity market experience, [Lundblad](#page-27-12) [\(2007\)](#page-27-12) presents a significantly positive risk-return relationship. Following [Lundblad](#page-27-12) [\(2007\)](#page-27-12), we examine the return predictability of V_{pe} using the longer sample of S&P index data from Robert J. Shiller's website to extend our sample period back to 1881. Due to the lack of available risk-free rate data, we use real returns as the dependent variable and report the results in Table 4. pe ratio continuously predicts future returns with a negative coefficient but loses the statistical significance in oneyear horizon. V_{rr} displays no predictability for any horizon. In contrast, our V_{pe} consistently predicts the positive S&P real returns in every horizon with a significant level of at least 5%. This robustness check confirms that the return predictive power of V_{pe} is both superior and consistently stable.

4.1.2 Multivariate return prediction

Next we examine whether the return predictability of V_{pe} can be subsumed by existing predictors. For this purpose, we execute the following multivariate predictive regression:

$$
R_{t,t+k} = \alpha_k + \beta_k V_{pe,t} + \theta_k x_t + \epsilon_{t+k}.
$$
\n⁽¹¹⁾

where $R_{t,t+k}$ is the k-period ahead cumulative compounded excess return, V_{pe} denotes the variance of pe ratio. x denotes a set of control variables. The forecast horizons are 1, 3, 5, 7, and 10 years ahead, respectively. We consider two sets of control variables. First, Equation (9) states that both level and variance of pe ratio provide the optimal forecast of the longrun expected returns. In addition, we want to compare the return predictability between variance of pe ratio and variance of market returns. In the first set of controls, we examine three pairs of predictors: pe and $V_{re}(V_{rr})$, pe and V_{pe} , and $V_{re}(V_{rr})$ and V_{pe} , separately. The second set of return predictive regression is the kitchen sink regression includes pe , V_{pe} and popular predictive variables used in [Goyal and Welch](#page-26-4) [\(2008\)](#page-26-4) with data available from 1937 to 2021.

In Panel A of Table 5, we report the results of the first set of multivariate predictive regressions of market excess return. When pe and V_{re} are predictors, pe continues to sig-

nificantly predict negative market excess return in each prediction horizon considered while V_{re} exhibits only weakly predictability in long-run, consistent with the results in univariate regressions. When employing pe and V_{pe} as predictors, regardless of the prediction horizon, V_{pe} continues to significantly predict positive market excess return while pe significantly predicts market excess return with a negative sign except for one-year horizon. The evidence of the positive predictive coefficient of V_{pe} and the negative predictive coefficient of pe is consistent with our second-order dynamic price-earning ratio model. Considering V_{re} and V_{pe} as predictors, V_{re} exhibits no predictability except for the ten-year horizon. V_{pe} continues to significantly predict positive market excess return in each horizon, exhibiting superior predictive power than V_{re} . The result in Panel B of Table 5 is qualitatively similar when real return is employed as the dependent variable. Comparing three pairs of multivariate predictive regressions, we show that V_{pe} has superior predictive power compared to pe or $V_{re}.^{\boldsymbol{9}}$ $V_{re}.^{\boldsymbol{9}}$ $V_{re}.^{\boldsymbol{9}}$

We present the results of the kitchen sink regression in Table 6. Controlling priceearning ratio, dividend-price ratio, dividend yield, dividend payout ratio, relative T-bill rate, Book-to-Market ratio, default yield spread, long-term rate of returns, net equity expansion, inflation rate, percent equity issuing, stock variance, default return spread, term spread from [Goyal and Welch](#page-26-4) [\(2008\)](#page-26-4), V_{pe} continuously predicts positive future excess returns with statistic significance in all horizons. Overall we find return predictability of V_{pe} can not be subsumed by existing predictors from the literature.

4.2 Out-of-sample return prediction

[Goyal and Welch](#page-26-4) [\(2008\)](#page-26-4) argue that the evidence of in-sample predictability should be considered cautiously and that it is important to test the out-of-sample (OOS) performance of return predictors. Taking into consideration the concerns regarding in-sample predictions, we adhere to their recommendation and proceed with the OOS test. For the robustness check, we choose two initial estimation periods, from 1937 to 1946 (ten years) and from 1937 to 1966 (thirty years), and then recursively conduct the OOS forecasting test until 2021.

⁹We report the multivariate predictive regression results in quarterly frequency in Table B.3 and find similar results.

We employ the [Clark and McCracken](#page-25-8) [\(2001\)](#page-25-8) test to carry out the nested-model OOS forecasting analysis. Two restricted(benchmark) models commonly used in the literature (e.g., [Lettau and Ludvigson,](#page-27-9) [2001\)](#page-27-9)are the constant mean model and the first-order autoregressive model, AR(1), respectively. The constant mean model has only one regressor, that is, the constant, and the AR(1) model includes two regressors, the constant, and the one-period lagged market excess returns or real returns. Given each restricted model, the corresponding unrestricted model includes one additional return predictor. [Clark and McCracken](#page-25-8) [\(2001\)](#page-25-8) provide two types of OOS tests: the equal forecast accuracy test and the forecast encompassing test. For the equal forecast accuracy test, the null hypothesis is that the restricted and unrestricted models have equal mean-squared forecasting errors (MSE), and the alternative is that the restricted model has a higher MSE. MSE-F provides the results of the equal forecast accuracy F test. For the encompassing test, the null hypothesis is that the restricted model forecast encompasses the unrestricted model, and the alternative is that the unrestricted model contains information that can significantly improve the restricted model's prediction. ENC-NEW provides the modified test statistics on forecast encompassing tests (e.g., [Harvey et al.,](#page-26-14) [1997\)](#page-26-14).

The OOS- R^2 is measured as

$$
OOS R^{2} = 1 - (1 - \bar{R}^{2})(\frac{T - 1}{T - k - 1})
$$
\n(12)

where $\bar{R}^2 = 1 - \frac{\sum_t (r_{t+n} - \hat{r}_{t+n|t})^2}{\sum_{t} (r_{t+n} - \bar{r}_t)^2}$ $\frac{d^{(t)}(t+n-r_{t+n|t})}{\sum_{t}(r_{t+n}-\bar{r}_{t})^2}$, $\hat{r}_{t+n|t}$ is the return forecast based on an unrestricted model, and \bar{r}_t is the historical average return for the constant mean model or return forecast based on AR(1) model.

In Table 7, we present the ratio of mean-squared forecasting errors $(\frac{MSE_u}{MSE_r})$, MSE-F, ENC-NEW and OOS- R^2 . We expect $\frac{MSE_u}{MSE_r}$ is less than one, MSE-F and ENC-NEW are statistically significant, and OOS- R^2 is positive if a predictor exhibits OOS predictive power. In any case, whether choosing the initial training period of ten years or thirty years, using the dependent variable is market excess return or real return, when the variance of market returns $(V_{re}$ or V_{rr}) as a predictor in an unrestricted model, the $\frac{MSE_u}{MSE_r}$ is always greater than one, suggesting that the mean-squared forecasting errors of the variance of market returns-augmented model

is always higher than that in any benchmark models. The adjusted R^2 values are negative. Both MSE-F and ENC-NEW tests cannot reject the null that the variance of market returns contains no information about future excess returns or real returns, suggesting that the variance of market returns cannot be used to improve upon the return predictability from the constant mean benchmark or AR(1) benchmark. Regarding pe ratio, in any case, $\frac{MSE_u}{MSE_r}$ is greater than one (except for the case that market real return as the dependent variable and the initial training period is ten years), MSE-F test cannot be rejected while ENC-NEW test is statistically rejected, and the $OOS-R^2$ is negative. The poor performance of variance of market returns and pe is consistent with [Goyal and Welch](#page-26-4) [\(2008\)](#page-26-4) and [Goyal et al.](#page-26-6) [\(2021\)](#page-26-6).

In contrast, whether employing the initial estimation period of ten years or thirty years, and using the constant mean model or $AR(1)$ model as a benchmark model, when predicting excess return and V_{pe} is used in the unrestricted model, the $\frac{MSE_u}{MSE_r}$ is always less than one, suggesting that the mean-squared forecasting errors of V_{pe} -augmented model is always lower compared to the one in a benchmark model. The MSE-F test strongly rejects the null of equal forecast accuracy between the benchmark model and V_{pe} -augmented model at least at 5% significant level. The ENC-NEW test strongly rejects the null that V_{pe} has no predictive power for excess returns or real returns at least at 1% significant level. The OOS- R^2 is always positive. For example, when predicting excess return, using the constant mean model as a restricted model and V_{pe} -augmented model as the unrestricted model, $\frac{MSE_u}{MSE_r}$ is 0.972, MSE-F test statistic is 2.338, ENC-NEW test statistic is 4.666, and OOS- R^2 is 0.016 with ten-year initial estimation period, $\frac{MSE_u}{MSE_r}$ is 0.978, MSE-F test statistic is 1.823, ENC-NEW test statistic is 4.479, and $OOS-R^2$ is 0.010 with thirty-year initial estimation period. The ENC-NEW and MSE-F tests both strongly reject the null that V_{pe} does not provide any information about future excess returns or real returns that could be used to improve the benchmark models. The results presented in Table 7 indicate that V_{pe} has displayed statistically significant out-of-sample predictive power for market excess returns and contains information that is not included in the constant mean model or $AR(1)$ model. In addition, V_{pe} shows superior performance of OOS tests over pe and variance of excess returns. When predicting real returns, we find similar results although they are slightly weaker.

To put it briefly, we present novel evidence from in-sample and OOS predictive tests

suggesting that V_{pe} consistently and significantly predicts positive future market excess return and real returns, consistent with the prediction in leading asset pricing models with stochastic volatility (e.g., the habit model of [Campbell and Cochrane](#page-24-9) [\(1999\)](#page-24-9), long-run risk model of [Bansal and Yaron](#page-24-10) [\(2004\)](#page-24-10), and the rare disaster model of [Barro](#page-24-13) [\(2006\)](#page-24-13) and [Gabaix](#page-25-9) [\(2012\)](#page-25-9)) which anticipate a positive relationship between past realized volatility and future expected returns.

4.3 Predicting macroeconomic activities

Rational asset pricing literature exhibits that expected excess returns on common stocks are related to business conditions and vary countercyclically, indicating that risk premiums tend to be higher during economic recessions than in periods of expansion (e.g., [Fama and](#page-25-10) [French,](#page-25-10) [1989,](#page-25-10) [Ferson and Harvey,](#page-25-11) [1991,](#page-25-11) [Lettau and Ludvigson,](#page-27-9) [2001\)](#page-27-9). If the return predictability of V_{pe} reveals the rational response of investors to the business conditions, for example, time-varying investment opportunities, uncertainty, and risk aversion (e.g., [Sun](#page-28-11)[daresan,](#page-28-11) [1989,](#page-28-11) [Campbell and Cochrane,](#page-24-9) [1999,](#page-24-9) [Lettau and Ludvigson,](#page-27-9) [2001,](#page-27-9) and [Bansal and](#page-24-10) [Yaron,](#page-24-10) [2004\)](#page-24-10), we expect that V_{pe} , given its countercyclical nature, should predict lower future macroeconomic activities. Intuitively, high uncertainty of expected returns may reveal bad future states of economy. [Fama and French](#page-25-12) [\(1993,](#page-25-12) [1995,](#page-25-13) [1996\)](#page-25-14) argue that value factor, HML, and size factor, SMB, act as state variables in the context of [Merton](#page-27-13) [\(1973\)](#page-27-13) ICAPM, suggesting a risk-based explanation of value factor and size factor.^{[10](#page-0-0)} We posit that V_{pe} captures economic uncertainty related to business conditions if V_{pe} is a good candidate for a state variable within ICAPM, and predicts macroeconomic growths which are potential proxies for future investment opportunity set. We consider the following four macroeconomic activity series: growth rate in Gross Domestic Product (GDP), growth rate in Personal Consumption Expenditures index (PCE), growth rate in Corporate Profits (PRO) and growth rate in Net Cash Flow (NCF).

Panel A of Table 8 reports the univaraite regression results showing that V_{pe} predicts all four measures of macroeconomic growth with a negative coefficient, consistent with the

 10 Empirically [Liew and Vassalou](#page-27-14) [\(2000\)](#page-27-14) and [Vassalou](#page-28-12) [\(2003\)](#page-28-12) show that HML and SMB contain information on future economic growth.

prediction in Equation [\(9\)](#page-7-2) and a countercyclical nature of V_{pe} . In forecasting framework spanning from one year to ten years, V_{pe} exhibits a statistically significant predictability of GDP growth and PCE growth in the horizons of three years, five years, seven years, and ten years, while V_{pe} displays a statistically significant predictability of PRO growth and NCF growth in horizons of five years, seven years, and ten years. Our results suggest that high uncertainty of expected returns reveals low future macroeconomic activities, future bad states of economy in medium and long term.

The literature has documented that Fama-French three factors contain information about future macroeconomic growth (e.g., [Fama,](#page-25-15) [1981,](#page-25-15) and [Liew and Vassalou,](#page-27-14) [2000\)](#page-27-14). To examine whether the predictive power of V_{pe} for future macroeconomic activities is subsumed by Fama-French three factors, we run the macroeconomic growth predictive regressions of V_{pe} including Fama-French three factors. To save space, we report only the predictive coefficients, statistics of V_{pe} and adjusted R^2 in Panel B of Table 8, and other detailed results are reported in Table B.4. Controlling for Fama-French three factors, V_{pe} exhibits a similar pattern of predictability of macroeconomic growth, suggesting that the negative relationship between V_{pe} and future macroeconomic growth is not subsumed by known relation between Fama-French three factors and future macroeconomic growth.

In sum, We provide empirical results that V_{pe} significantly predicts high future market returns and low future macroeconomic growth, consistent with our second-order dynamic price-earnings ratio model, supporting the positive risk-return trade off theory and the rela-tionship between time-varying returns and business conditions.^{[11](#page-0-0)}

5 Understanding the volatility of log price-earnings ratio

To gain deeper understanding of the superior and significant predictive relation between V_{pe} and future market returns as well as macroeconomic activities, we conduct two additional tests, cross correlation analysis and predictive analysis of V_{pe} to investigate whether the predictability of V_{pe} arises from cash flow shocks, discount rate shocks, or volatility shocks.

 $^{11}\rm{We}$ obtain similar findings with quarterly data. The results are available upon request.

5.1 Cross correlation analysis

We perform cross correlation analysis of V_{pe} with three sets of risk measures. The first set of risk measures is related to market and macroeconomic risk, including the variance of market excess returns(V_{re}), variance of market real returns(V_{rr}), variance of Gross Domestic Product growth (V_{GDP}) , variance of Personal Consumption Expenditures index growth (V_{PCE}) , variance of Corporate Profits growth (V_{PRO}) and variance of Net Cash Flow growth (V_{NCF}) . In a dynamic asset pricing model with stochastic volatility, [Bansal et al.](#page-24-1) [\(2014\)](#page-24-1) and [Campbell et al.](#page-24-2) [\(2018\)](#page-24-2) demonstrate that stochastic volatility based on either conditional volatility of macroeconomic activities or conditional volatility of market returns yields an important and separate risk that significantly affects the macroeconomic activities and asset prices. The conditional market volatility and macroeconomic volatility based on the vector autoregressive model (e.g., [Bansal et al.,](#page-24-1) [2014\)](#page-24-1) are included in the second set of risk measures. [Bansal and Yaron](#page-24-10) [\(2004\)](#page-24-10) emphasize the importance of long-run risk in consumption growth for explaining the equity premium and the dynamic dependencies in returns over long multi-year horizons. Considering the influence of long-run risk, the third set of risk measures is related to long-run risk and risk aversion, including long-run cash flow risk(CF) proposed by [Hansen et al.](#page-26-15) [\(2008\)](#page-26-15), rational risk aversion(RA) based on the specification of external habit persistence in [Campbell and Cochrane](#page-24-9) [\(1999\)](#page-24-9) and economic uncertainty(EU) measured as the conditional volatility of consumption growth following [Bansal and Yaron](#page-24-10) $(2004).¹²$ $(2004).¹²$ $(2004).¹²$ $(2004).¹²$ The cross correlation analysis displays contemporaneous and dynamic (lead and lag) relation between V_{pe} and other risk measures.

Table 9 displays the cross correlation relations between V_{pe} and various risk measures. V_{pe} and V_{rr} are significantly and positively correlated with a contemporaneous correlation of 0.320 at the 0.01 level. V_{pe} is also significantly and positively correlated with variance of PRO growth and variance of NCF growth with contemporaneous correlation of 0.403 and 0.321 , respectively, at least at the 0.05 level. V_{pe} seems to have no contemporaneous correlation with the variance of GDP growth and variance of PCE growth. Regarding lead-

¹²Besides the long-run cash flow risk, we also examine the long-run consumption risk proposed by [Hansen et al.](#page-26-15) [\(2008\)](#page-26-15) and find similar results. We do to tabulate the results to save space, and they are available upon request.

lag relations, V_{re} and V_{rr} tends to be positively correlated with subsequent V_{pe} , while V_{pe} is negatively correlated with subsequent V_{re} or V_{rr} . In contrast, variance of macroeconomic activities (V_{GDP} , V_{PCE} , V_{PRO} , and V_{NCF}) tends to negatively correlate to subsequent V_{pe} , while V_{pe} tends to correlate to subsequent variance of macroeconomic activities positively. We also examine the relation between V_{pe} and conditional variance of market return (V_m) and conditional variance of macroeconomic growth (V_e) based on the vector autoregressive model (e.g., [Bansal et al.](#page-24-1) [\(2014\)](#page-24-1) and [Campbell et al.](#page-24-2) [\(2018\)](#page-24-2)). We find a similar lead-lag relation in the sense that V_{pe} is negatively correlated with the subsequent V_m and positively associated with V_e . Regarding the long-run risk and risk aversion measures, we find that V_{pe} are positively and significantly correlated with future RA and EU , while being positively and significantly correlated with past CF.

In sum, the cross correlation analysis shows that high V_{pe} is associated with high future macroeconomic volatility, risk aversion, and long-run risks. This positive dynamic correlation echoes the rational explanation for the positive return predictability and negative macroeconomic growth predictability of V_{pe} in the previous section.

5.2 Sources of return predictability

In the previous section, we provide the evidence that V_{pe} displays significant and consistent predictability to future market excess returns and real returns, macroeconomic growth as well as various risk measures. In this subsection, we delve into whether the return predictability of V_{pe} is due to its connection to cash flow shocks, discount rate shocks, or volatility shocks (e.g., [Bansal et al.,](#page-24-1) [2014](#page-24-1) and [Campbell et al.,](#page-24-2) [2018\)](#page-24-2). Additionally, we explore whether these factors can be further associated with the price of risk or the quantity of risk (e.g., [Campbell](#page-24-9) [and Cochrane,](#page-24-9) [1999,](#page-24-9) [Bansal and Yaron,](#page-24-10) [2004,](#page-24-10) and [Bao et al.,](#page-24-14) [2023\)](#page-24-14). We consider one-year ahead Gross Domestic Product growth (G_{GDP}) , Corporate Profits growth (G_{PRO}) , Net Cash Flow growth(G_{NCF}) as proxies for cash flow shocks. We proxy discount rate shocks using market excess returns, dividend-price ratio, and default spreads. The volatility of market excess returns(V_{re}), the volatility of Gross Domestic Product growth (V_{GDP}), the volatility of Corporate Profits growth (V_{PRO}) and the volatility of Net Cash Flow growth (V_{NCF}) are used as proxies for volatility shocks.

Table 10 presents the results of multivariate predictive regressions with controls. In addition, for comparison, we also report the univariate regression results since the data sample lengths of controls in each panel vary. The univariate regression results in each panel once again highlight the robust and significant predictive power of V_{pe} for future excess returns across horizons from one to ten years ahead, with significance levels of at least 5%. In Panel A of Table 10, we examine the predictive power of V_{pe} for future excess returns controlling for future cash flow shocks. We find that the return predictability of V_{pe} remains unexplained by these cash flow shocks. V_{pe} continuously predict future returns for all horizons considered after controlling for cash flow shocks. For instance, after controlling for cash flow shocks, an one percent increase in V_{pe} predicts the future excess returns of 1.436\%, 4.007\%, 7.581%, 10.239% and 16.471% for one-year, three-year, five-year, seven-year and ten-year ahead horizons, respectively. In comparison, the univariate regression results yield 1.338%, 4.292%, 7.875%, 10.743% and 16.828%, respectively.

Penal B of Table 10 presents results when we control for discount rate shocks. V_{pe} continuously predict future returns for all horizons considered after controlling for discount rate shocks. The predictive coefficients of V_{pe} change slightly in one-year, three-year and five-year ahead horizons, while the predictive coefficients decrease moderately in seven-year and ten-year horizons after controlling or discount rate shocks. For example, the predictive coefficients of V_{pe} for one-year, three-year, and five-year ahead horizons are 1.347, 4.731, and 7.034, respectively, after controlling for discount rate shocks, compared to 1.362, 4.452, and 7.241 in univariate regressions. The predictive coefficients for seven-year and ten-year ahead horizons are 8.843 and 15.957 respectively, compared to 10.340 and 18.408 in univariate regressions. In addition, the adjusted R^2 s increase sizably after controlling for discount rate shocks. Our results suggest that the return predictability in long-run is partially attributable to the correlation between V_{pe} and discount rate shocks. On the other hand, V_{pe} and discount rate shocks may predict different components in long-run returns.

The predictability may arise because V_{pe} signals periods of increased risk and uncertainty in the economy and market. In essence, investors typically demand higher returns when they perceive a greater level of risk. We examine the return predictability of V_{pe} controlling

for volatility shocks in Panel C of Table 10. V_{pe} continuously predicts future returns for all horizons considered after controlling for volatility shocks. The predictive coefficients decrease moderately in seven-year and ten-year ahead horizons, resembling the pattern observed in Panel B. Nevertheless, the adjusted R^2 s show only slight increases compared to those in Panel B, suggesting the return predictability of V_{pe} encompasses the bulk return predictability attributed to the quantity of risk.

Penal D of Table 10 displays the predictive results when controlling for all cash flow shocks, discount rate shocks, and volatility shocks. The predictive power of V_{pe} remains strong and significant for all horizons considered. The predictive coefficients of V_{pe} experience a substantial decrease, especially in medium and long-run, and the adjusted R^2 s increase sizably. For example, when controlling for all shocks, the predictive coefficients of V_{pe} are 1.157, 3.165, 5.898, 5.152, and 9.949 for one-year, three-year, five-year, seven-year, and tenyear ahead horizons, respectively, compared to 1.385, 4.221, 7.710, 10.286, and 16.409 in the univariate regressions, respectively. Additionally, the adjusted R^2 s are 6.3%, 40.9%, 40.4%, 59.6%, and 66.2% for one-year, three-year, five-year, seven-year, and ten-year ahead horizons, respectively, compared to 4.5%, 16.2%, 23.6%, 24.6%, and 30.3% in univariate regressions, respectively. Our results in Table 10 exhibit that the cash flow shocks, discount rate shocks, and volatility shocks can only account for a moderated portion of overall return predictability of V_{pe} . This evidence, along with the properties of V_{pe} discussed earlier, suggests that the return predictability of V_{pe} partially stems from cash flow shocks, discount rate shocks and volatility shocks. During economic contractions, as indicated by elevated V_{pe} levels, investors perceive a prospect of reduced future economic growth, and demand higher compensation to offset the escalated risks and uncertainties inherent in such challenging economic downturns. It is worth noting that when controlling for changes in cash flows, discount rates and volatilises, the return predictability of V_{pe} remains, although weaker. This evidence suggests additional perspectives are needed to understand our findings.

6 Conclusion

We propose a novel second-order dynamic price-earnings ratio model, allowing the second moment of log price-earnings ratio to play an important role in comprehending the dynamic accounting identity. We argue that both the level and the variance of price-earnings ratio serve as the optimal forecast of future market returns, cash flow growths, and volatility risk. We show that the variance of log price-earnings ratio, is useful in capturing time variation in expected market risk premium and macroeconomic activities.

We provide novel evidence that the volatility of log price-earnings ratio significantly predicts positive future market returns and negative macroeconomic growths. The predictability is robust across various sample periods, frequencies, and control variables. Further investigation reveals that the superior return and macroeconomic growth predictability cannot be totally attributed to the cash flow shocks, discount rate shocks and volatility shocks.

References

- Ai, H., Han, L. J., and Xu, L. (2022). Information-driven volatility. Available at SSRN 3961096.
- Ang, A. and Bekaert, G. (2007). Stock return predictability: Is it there? The Review of Financial Studies, 20(3):651–707.
- Asness, C. S. (2000). Stocks versus bonds: explaining the equity risk premium. Financial Analysts Journal, 56(2):96–113.
- Bali, T. G., Nichols, D. C., and Weinbaum, D. (2023). Inferring aggregate market expectations from the cross-section of stock prices. Journal of Financial and Quantitative Analysis, pages 1–57.
- Bansal, R., Kiku, D., Shaliastovich, I., and Yaron, A. (2014). Volatility, the macroeconomy, and asset prices. The Journal of Finance, 69(6):2471–2511.
- Bansal, R. and Yaron, A. (2004). Risks for the long run: A potential resolution of asset pricing puzzles. The journal of Finance, 59(4):1481–1509.
- Bao, J., Hou, K., and Zhang, S. (2023). Systematic default and return predictability in the stock and bond markets. Journal of Financial Economics, 149(3):349–377.
- Barro, R. J. (2006). Rare disasters and asset markets in the twentieth century. The Quarterly Journal of Economics, 121(3):823–866.
- Bloom, N. (2009). The impact of uncertainty shocks. Econometrica, 77(3):623–685.
- Bollerslev, T., Tauchen, G., and Zhou, H. (2009). Expected stock returns and variance risk premia. The Review of Financial Studies, 22(11):4463–4492.
- Campbell, J. Y. and Cochrane, J. H. (1999). By force of habit: A consumption-based explanation of aggregate stock market behavior. Journal of Political Economy, 107(2):205– 251.
- Campbell, J. Y., Giglio, S., Polk, C., and Turley, R. (2018). An intertemporal capm with stochastic volatility. Journal of Financial Economics, 128(2):207–233.
- Campbell, J. Y. and Shiller, R. J. (1988). The dividend-price ratio and expectations of future dividends and discount factors. The Review of Financial Studies, 1(3):195–228.
- Campbell, J. Y. and Shiller, R. J. (2005). Valuation ratios and the long-run stock market outlook: An update. Advances in Behavioral Finance, 2:173–201.
- Campbell, J. Y. and Thompson, S. B. (2008). Predicting excess stock returns out of sample: Can anything beat the historical average? The Review of Financial Studies, 21(4):1509– 1531.
- Cederburg, S., Johnson, T. L., and O'Doherty, M. S. (2023). On the economic significance of stock return predictability. Review of Finance, 27(2):619–657.
- Clark, T. and McCracken, M. (2001). Tests of equal forecast accuracy and encompassing for nested models. Journal of Econometrics, 105(1):85–110.
- Claus, J. and Thomas, J. (2001). Equity premia as low as three percent? evidence from analysts' earnings forecasts for domestic and international stock markets. The Journal of Finance, 56(5):1629–1666.
- Cochrane, J. H. (2008). The dog that did not bark: A defense of return predictability. The Review of Financial Studies, 21(4):1533–1575.
- Dickey, D. A. and Fuller, W. A. (1979). Distribution of the estimators for autoregressive time series with a unit root. Journal of the American Statistical Association, 74(366a):427–431.
- Easton, P. D. (2004). Pe ratios, peg ratios, and estimating the implied expected rate of return on equity capital. The accounting review, 79(1):73–95.
- Fama, E. F. (1981). Stock returns, real activity, inflation, and money. The American economic review, 71(4):545–565.
- Fama, E. F. and French, K. R. (1989). Business conditions and expected returns on stocks and bonds. Journal of financial economics, 25(1):23–49.
- Fama, E. F. and French, K. R. (1993). Common risk factors in the returns on stocks and bonds. Journal of Financial Economics, 33(1):3–56.
- Fama, E. F. and French, K. R. (1995). Size and book-to-market factors in earnings and returns. The Journal of Finance, 50(1):131–155.
- Fama, E. F. and French, K. R. (1996). Multifactor explanations of asset pricing anomalies. The Journal of Finance, 51(1):55–84.
- Fama, E. F. and French, K. R. (2001). Disappearing dividends: changing firm characteristics or lower propensity to pay? Journal of Financial economics, 60(1):3–43.
- Ferson, W. E. and Harvey, C. R. (1991). The variation of economic risk premiums. Journal of political economy, 99(2):385–415.
- Gabaix, X. (2012). Variable rare disasters: An exactly solved framework for ten puzzles in macro-finance. The Quarterly journal of economics, 127(2):645–700.
- Gao, C. and Martin, I. W. (2021). Volatility, valuation ratios, and bubbles: An empirical measure of market sentiment. *The Journal of Finance*, 76(6):3211-3254.
- Ghysels, E., Santa-Clara, P., and Valkanov, R. (2005). There is a risk-return trade-off after all. Journal of financial economics, 76(3):509–548.
- Glosten, L. R., Jagannathan, R., and Runkle, D. E. (1993). On the relation between the expected value and the volatility of the nominal excess return on stocks. The Journal of Finance, 48(5):1779–1801.
- Goyal, A. and Welch, I. (2008). A comprehensive look at the empirical performance of equity premium prediction. The Review of Financial Studies, 21(4):1455–1508.
- Goyal, A., Welch, I., and Zafirov, A. (2021). A comprehensive look at the empirical performance of equity premium prediction ii. Swiss Finance Institute Research Paper, (21-85).
- Granger, C. W. and Newbold, P. (1974). Spurious regressions in econometrics. Journal of econometrics, 2(2):111–120.
- Guo, H. (2006). On the out-of-sample predictability of stock market returns. The Journal of Business, 79(2):645–670.
- Guo, H. and Savickas, R. (2006). Idiosyncratic volatility, stock market volatility, and expected stock returns. Journal of Business $\mathcal B$ Economic Statistics, 24(1):43-56.
- Guo, H. and Whitelaw, R. F. (2006). Uncovering the risk–return relation in the stock market. The Journal of Finance, 61(3):1433–1463.
- Hansen, L. P., Heaton, J. C., and Li, N. (2008). Consumption strikes back? measuring long-run risk. Journal of Political economy, 116(2):260–302.
- Harvey, C. R. (2001). The specification of conditional expectations. *Journal of Empirical* Finance, 8(5):573–637.
- Harvey, D., Leybourne, S., and Newbold, P. (1997). Testing the equality of prediction mean squared errors. International Journal of forecasting, 13(2):281–291.
- Hodrick, R. J. (1992). Dividend Yields and Expected Stock Returns: Alternative Procedures for Inference and Measurement. The Review of Financial Studies, 5(3):357–386.
- Jiang, X. and Kang, Q. (2020). Cross-sectional peg ratios, market equity premium, and macroeconomic activity. Journal of Accounting, Auditing $\mathscr B$ Finance, 35(3):471–500.
- Jiang, X. and Lee, B.-S. (2014). The intertemporal risk-return relation: A bivariate model approach. Journal of Financial Markets, 18:158–181.
- Kelly, B. and Pruitt, S. (2013). Market expectations in the cross-section of present values. The Journal of Finance, 68(5):1721–1756.
- Koijen, R. S. and Van Nieuwerburgh, S. (2011). Predictability of returns and cash flows. Annu. Rev. Financ. Econ., 3(1):467–491.
- Konchitchki, Y. and Patatoukas, P. N. (2014). Taking the pulse of the real economy using financial statement analysis: Implications for macro forecasting and stock valuation. The Accounting Review, 89(2):669–694.
- Kostakis, A., Magdalinos, T., and Stamatogiannis, M. P. (2015). Robust econometric inference for stock return predictability. The Review of Financial Studies, 28(5):1506–1553.
- Lettau, M. and Ludvigson, S. (2001). Consumption, aggregate wealth, and expected stock returns. The Journal of Finance, 56(3):815–849.
- Lettau, M., Ludvigson, S. C., and Wachter, J. A. (2008). The declining equity premium: What role does macroeconomic risk play? The Review of Financial Studies, 21(4):1653– 1687.
- Lettau, M. and Van Nieuwerburgh, S. (2008). Reconciling the return predictability evidence: The review of financial studies: Reconciling the return predictability evidence. The Review of Financial Studies, 21(4):1607–1652.
- Lewellen, J. (2004). Predicting returns with financial ratios. Journal of Financial Economics, 74(2):209–235.
- Liew, J. and Vassalou, M. (2000). Can book-to-market, size and momentum be risk factors that predict economic growth? Journal of Financial Economics, 57(2):221–245.
- Lintner, J. (1956). Distribution of incomes of corporations among dividends, retained earnings, and taxes. The American Economic Review, 46(2):97–113.
- Lintner, J. (1963). The cost of capital and optimal financing of corporate growth. The Journal of Finance, 18(2):292–310.
- Lundblad, C. (2007). The risk return tradeoff in the long run: 1836–2003. Journal of Financial Economics, 85(1):123–150.
- Mark, N. C. (1995). Exchange rates and fundamentals: Evidence on long-horizon predictability. The American Economic Review, pages 201–218.
- Martin, I. (2017). What is the expected return on the market? The Quarterly Journal of Economics, 132(1):367–433.
- McCracken, M. (2007). Asymptotics for out of sample tests of granger causality. Journal of Econometrics, 140(2):719–752.
- Merton, R. C. (1973). An intertemporal capital asset pricing model. Econometrica: Journal of the Econometric Society, pages 867–887.
- Merton, R. C. (1980). On estimating the expected return on the market: An exploratory investigation. Journal of financial economics, 8(4):323–361.
- Nelson, C. R. and Kim, M. J. (1993). Predictable stock returns: The role of small sample bias. The Journal of Finance, 48(2):641–661.
- Nelson, W. R. (1999). The aggregate change in shares and the level of stock prices. Finance and Economic Discussion Series no. 1999- 08, Federal Reserve Board.
- Newey, W. K. and West, K. D. (1987). Hypothesis testing with efficient method of moments estimation. International Economic Review, pages 777–787.
- Penman, S. H. and Sougiannis, T. (1998). A comparison of dividend, cash flow, and earnings approaches to equity valuation. Contemporary accounting research, 15(3):343–383.
- Phillips, P. C. B. and Perron, P. (1988). Testing for a unit root in time series regression. Biometrika, 75(2):335–346.
- Polk, C., Thompson, S., and Vuolteenaho, T. (2006). Cross-sectional forecasts of the equity premium. Journal of Financial Economics, 81(1):101–141.
- Rapach, D. E., Strauss, J. K., and Zhou, G. (2010). Out-of-sample equity premium prediction: Combination forecasts and links to the real economy. The Review of Financial Studies, 23(2):821–862.
- Scruggs, J. T. (1998). Resolving the puzzling intertemporal relation between the market risk premium and conditional market variance: A two-factor approach. The Journal of Finance, 53(2):575–603.
- Sharpe, S. A. (2002). Reexamining stock valuation and inflation: The implications of analysts' earnings forecasts. Review of Economics and Statistics, 84(4):632–648.
- Stambaugh, R. F. (1999). Predictive regressions. Journal of financial economics, 54(3):375– 421.
- Sundaresan, S. M. (1989). Intertemporally dependent preferences and the volatility of consumption and wealth. *Review of financial Studies*, 2(1):73–89.
- Vassalou, M. (2003). News related to future gdp growth as a risk factor in equity returns. Journal of financial economics, 68(1):47–73.
- Vuolteenaho, T. (2002). What drives firm-level stock returns? The Journal of Finance, 57(1):233–264.
- Yang, W. (2023). Business cycles, regime shifts, and return predictability. *Journal of Fi*nancial and Quantitative Analysis, pages 1–48.
- Zhou, G. and Zhu, Y. (2015). Macroeconomic volatilities and long-run risks of asset prices. Management Science, 61(2):413–430.

Figure 1: Volatility of log price-earnings ratio and subsequent stock excess returns. The figure plots the time series of the volatility of log price-earnings ratio (dashed line) and subsequent one-year(R_1), five-year(R_5), seven-year(R_7) and ten-year(R_{10}) stock excess returns (solid line), normalized to have zero mean and unit variance. The shaded areas represent NBER recession dates.

Table 1: Descriptive statistics

This table reports summary statistics and unit-root test for excess market returns (R_e) , real market returns (R_r) , log price-earnings ratio(pe) from S&P 500 index, the variance of excess market return(V_{re}), the variance of real market return(V_{rr}) and the variance of log price-earnings ratio(V_{pe}). ρ is the firstorder autocorrelation coefficient. ADF and PP denote the augmented Dickey-Fuller test and the Philips-Perron test with four lags. The critical values for ADF and PP are -3.453, -2.871, -2.572 at 1%, 5%, and 10% significance level, respectively. The sample period is from 1937 to 2021.

Panel A: Summary statistics and unit-root test						
Series	Mean	Std	ρ	ADF	PP	
R_e	0.087	0.202	0.012	-5.875	-9.486	
R_r	0.090	0.202	-0.022	-5.456	-9.969	
pe	2.755	0.404	0.803	-2.403	-3.699	
V_{re}	0.040	0.026	0.897	-4.853	-2.925	
V_{rr}	0.041	0.022	0.905	-3.865	-2.508	
V_{pe}	0.073	0.032	0.834	-3.132	-3.259	
	Panel B: Correlation matrix					
	R_e	R_r	pe	V_{re}	V_{rr}	V_{pe}
R_e	1.000	0.982	0.120	-0.075	-0.019	0.247
R_r		1.000	0.157	-0.126	-0.084	0.170
pe			1.000	-0.280	-0.398	-0.323
V_{re}				1.000	0.969	0.165
V_{rr}					1.000	0.320
V_{pe}						1.000

Table 2: Univariate predictive regressions for market returns (1937-2021)

This table reports univariate long-horizon predictive regressions for compounded market excess returns(R_e) and real returns(R_r), at horizons of $k = 1, 3, 5, 7$ and 10 years ahead. The predictive variables are the log price-earnings ratio of $S\&P$ 500 index(pe), the variance of market excess returns(V_{re}), the variance of market real returns(V_{rr}), and the variance of log price-earnings ratio(V_{pe}), respectively. The table reports estimates of OLS regression, [Newey and West](#page-28-10) [\(1987\)](#page-28-10) corrected standard errors(with $k+1$ lags) in parentheses, [Hodrick](#page-26-12) [\(1992\)](#page-26-12) standard errors(with k lags) in brackets, bootstrapping p-value in curly brackets and adjusted R-squared in the bottom line of each prediction horizon. The annual sample period is from 1937 to 2021.

		$R_{e,t+k}$			$R_{r,t+k}$	
\boldsymbol{k}	pe	V_{re}	V_{pe}	pe	V_{rr}	V_{pe}
1	-0.075	-0.096	1.362	-0.073	-0.437	1.162
	(0.041)	(0.715)	(0.547)	(0.039)	(0.928)	(0.583)
	[0.047]	[0.687]	[0.685]	[0.046]	[0.891]	[0.699]
	${0.125}$	${0.909}$	${0.027}$	${0.119}$	${0.651}$	${0.058}$
	0.021	-0.012	0.052	0.018	-0.009	0.032
3	-0.246	0.306	4.452	-0.240	-1.103	3.761
	(0.109)	(1.929)	(1.196)	(0.085)	(2.093)	(1.289)
	[0.124]	[1.934]	[1.922]	[0.118]	[2.604]	[2.035]
	${0.005}$	${0.813}$	${0.000}$	${0.005}$	${0.495}$	${0.000}$
	0.088	-0.012	0.189	0.079	-0.007	0.125
5	-0.361	2.492	7.241	-0.345	-1.155	6.462
	(0.212)	(2.531)	(2.152)	(0.176)	(3.025)	(2.471)
	[0.193]	[2.763]	[2.904]	[0.190]	[3.548]	[3.098]
	${0.005}$	${0.248}$	${0.000}$	${0.007}$	${0.658}$	${0.000}$
	0.085	0.006	0.232	0.076	-0.010	0.180
7	-0.634	5.030	10.340	-0.641	-0.549	9.587
	(0.340)	(2.886)	(3.220)	(0.285)	(4.453)	(3.135)
	[0.267]	[3.525]	[3.662]	[0.266]	[4.878]	[3.867]
	${0.001}$	${0.049}$	${0.000}$	${0.001}$	${0.879}$	${0.000}$
	0.153	0.030	0.270	0.154	-0.013	0.228
10	-1.213	11.700	18.408	-1.287	1.721	17.218
	(0.531)	(4.589)	(5.293)	(0.355)	(7.406)	(4.073)
	[0.346]	[3.215]	[4.352]	[0.340]	[4.635]	[4.523]
	${0.001}$	${0.008}$	$\{0.000\}$	$\{0.001\}$	${0.721}$	${0.000}$
	0.223	0.078	0.340	0.289	-0.012	0.337

Table 3: Univariate predictive regressions for market returns (subsamples) Table 3: Univariate predictive regressions for market returns (subsamples) This table reports univariate long-horizon predictive regressions for compounded market excess returns(R_e) and real returns(R_r), at horizons of $k =$ 1, 3, 5, 7 and 10 years ahead. The predictive variables are the log price-earnings ratio of $S\&P$ 500 index (pe), the variance of market excess returns This table reports univariate long-horizon predictive regressions for compounded market excess returns(R_e) and real returns(R_r), at horizons of $k =$ 1, 3, 5, 7 and 10 years ahead. The predictive variables are the log price-earnings ratio of S&P 500 index (pe), the variance of market excess returns (V_{re}) , the variance of market real returns (V_{rr}) , and the variance of log price-earnings ratio (V_{pe}) . The table reports estimates of OLS regression,
Newey and West (1987) corrected standard errors (with k+1 lags) in pa (V_{re}) , the variance of market real returns (V_{rr}) , and the variance of log price-earnings ratio (V_{pe}) . The table reports estimates of OLS regression, [Newey](#page-28-10) and West [\(1987\)](#page-28-10) corrected standard errors (with k+1 lags) in parentheses, [Hodrick](#page-26-12) [\(1992\)](#page-26-12) standard errors(with k lags) in brackets, bootstrapping p-value in curly brackets and adjusted R-squared in the bottom line of each prediction horizon. The annual sample periods span from strapping p-value in curly brackets and adjusted R-squared in the bottom line of each prediction horizon. The annual sample periods span from $1027 \leftarrow 1000 \left(\frac{2}{1000} \right)$ and $\frac{2}{1000}$ from $\frac{2}{1000}$ from $\frac{2}{10$ 1937 to 1999, from 1937 to 2007, from 1937 to 2019, and from 1950 to 2021, respectively. 1937 to 1999, from 1937 to 2007, from 1937 to 2019, and from 1950 to 2021, respectively.

Table 4: Univariate predictive regressions for $S\&P$ 500 index real returns (1881-2021)						
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This table reports univariate long-horizon predictive regressions for compounded S&P 500 index real returns(R_r), at horizons of $k = 1, 3, 5, 7$ and 10 years ahead. The predictive variables are the log priceearnings ratio of $S\&P$ 500 index (pe), the variance of $S\&P$ 500 index real returns (V_{rr}) , and the variance of log price-earnings ratio (V_{pe}) . The table reports estimates of OLS regression, [Newey and West](#page-28-10) [\(1987\)](#page-28-10) corrected standard errors (with $k+1$ lags) in parentheses, [Hodrick](#page-26-12) [\(1992\)](#page-26-12) standard errors(with k lags) in brackets, bootstrapping p-value in curly brackets and adjusted R-squared in the bottom line of each prediction horizon. The annual sample period is from 1881 to 2021.

This table reports the estimates from kitchen sink long-horizon predictive regressions for compounded market excess returns, at horizons of $k = 1$, This table reports the estimates from kitchen sink long-horizon predictive regressions for compounded market excess returns, at horizons of $k = 1$, term spread (tms). The table reports estimates of OLS regression, Newey and West (1987) corrected standard errors (with $k+1$ lags) in parentheterm spread (tms). The table reports estimates of OLS regression, [Newey](#page-28-10) and West [\(1987\)](#page-28-10) corrected standard errors (with k+1 lags) in parentheses, Hodrick (1992) standard errors (with k lags) in brackets, bootstrapping p-value in curly brackets and adjusted R-squared (R^2) of each predic-ses, [Hodrick](#page-26-12) [\(1992\)](#page-26-12) standard errors (with k lags) in brackets, bootstrapping p-value in curly brackets and adjusted R-squared (R^2) of each predicdictors in Goyal and Welch (2008) available in the sample from 1937 to 2021, including the dividend-price ratio (dp) , the dividend yield (dy) , the dividend payout ratio (de), the relative T-bill rate (rtb), the Book-to-Market ratio (bm), default yield spread (dfy), the long term rate of returns 3, 5, 7 and 10 years ahead. The predictive variables are the log price-earning ratio(pe) and the variance of log price-earnings ratio (V_{pe}) and predictors in [Goyal](#page-26-4) and Welch [\(2008\)](#page-26-4) available in the sample from 1937 to 2021, including the dividend-price ratio (dp), the dividend yield (dy), the *(ltr)*, net equity expansion (ntis), the inflation rate (infl), percent equity issuing (eqis), stock variance (svar), default return spread (dfr), and 3, 5, 7 and 10 years ahead. The predictive variables are the log price-earning ratio(pe) and the variance of log price-earnings ratio (V_{pe}) and predividend payout ratio (de), the relative T-bill rate (rtb), the Book-to-Market ratio (bm), default yield spread (dfy), the long term rate of returns (*ltr*), net equity expansion (*ntis*), the inflation rate (*infl*), percent equity issuing (*eqis*), stock variance (*svar*), default return spread (*dfr*), and tion horizon. The sample period is from 1937 to 2021. tion horizon. The sample period is from 1937 to 2021.

Table 7: Out-of-sample (OOS) predictability

This table reports the results of one-year-ahead nested prediction comparisons for compounded market excess returns and real returns. The predictive variables are the log price-earnings ratio of $S\&P$ 500 index (pe), the variance of market excess returns (V_{re}) , the variance of market real returns (V_{rr}) , and the variance of log price-earnings ratio (V_{pe}) , respectively. The benchmark (restricted) model is the constant mean model in Panel A and the first-order autoregressive model $(AR(1))$ in Panel B. MSE_u is the meansquared forecasting error from the relevant unrestricted model; MSE_r is the mean-squared error from the relevant restricted model. "MSE-F' the [McCracken](#page-27-15) [\(2007\)](#page-27-15) F-statistic; the null hypothesis is that the restricted and unrestricted models have equal mean-squared error (MSE); the alternative is that the restricted model has higher MSE. "ENC-NEW" gives the modified Harvey, Leybourne, and Newbold test statistic by [Clark and McCracken](#page-25-8) [\(2001\)](#page-25-8); the null hypothesis is that the restricted model encompasses the unrestricted model; the alternative is that the unrestricted model contains information that could be used to significantly improve the restricted model's forecast. The 95th percentile of the asymptotic distribution of the statistic as derived in [Clark and McCracken](#page-25-8) [\(2001\)](#page-25-8) is 1.518 for MSE-F and 2.085 for ENC-NEW. The initial estimation period is either twenty years, from 1937 to 1946 (T=10), or thirty years, from 1937 to 1966 (T=30). The model is recursively reestimated until 2021.

***,**,*indicate statistical significance at the 0.01,0.05, and 0.10 level.

Table 8: Predicting macroeconomic activities

This table reports univariate and multivariate long-horizon predictive regressions for compounded macroeconomic growths, at horizons of $k = 1, 3, 5, 7$ and 10 years ahead. We consider four measures of macroeconomic growth, the Gross Domestic Product growth (G_{GDP}) , Personal Consumption Expenditures index growth (G_{PCE}) , Corporate Profits growth (G_{PRO}) and Net Cash Flow growth (G_{NCF}) . In Panel A, we conduct univariate analysis using the predictor V_{pe} . In Panel B, we conduct multivariate analysis with the predictor of V_{pe} and Fama-French three factors. The table reports estimates of OLS regression, [Newey and](#page-28-10) [West](#page-28-10) [\(1987\)](#page-28-10) corrected standard errors (with $k+1$ lags) in parentheses, [Hodrick](#page-26-12) [\(1992\)](#page-26-12) standard errors(with k lags) in brackets, bootstrapping p-value in curly brackets and adjusted R-squared of each prediction horizon.

	Panel A: Univariate Analysis			Panel B: Multivariate Analysis				
\boldsymbol{k}	G_{GDP}	G_{PCE}	G_{PRO}	G_{NCF}	G_{GDP}	G_{PCE}	G_{PRO}	G_{NCF}
$\mathbf{1}$	-0.146	-0.187	-0.164	0.114	-0.205	-0.271	-0.316	0.151
	(0.152)	(0.202)	(0.557)	(0.269)	(0.171)	(0.202)	(0.605)	(0.294)
	[0.136]	[0.137]	[0.512]	[0.319]	[0.152]	[0.150]	[0.520]	[0.325]
	${0.238}$	${0.148}$	${0.758}$	${0.740}$	${0.138}$	${0.036}$	${0.536}$	${0.647}$
	0.003	0.018	-0.012	-0.012	0.074	0.099	-0.001	-0.033
3	-0.538	-0.725	-1.336	-0.323	-0.597	-0.934	-1.135	-0.061
	(0.437)	(0.667)	(0.956)	(0.761)	(0.536)	(0.752)	(1.294)	(0.762)
	[0.357]	[0.349]	[1.390]	[0.823]	[0.389]	[0.365]	[1.409]	[0.805]
	${0.066}$	${0.038}$	${0.130}$	${0.544}$	${0.077}$	${0.026}$	${0.307}$	${0.942}$
	0.036	0.064	0.021	-0.008	0.057	0.107	0.036	0.060
5	-1.292	-1.460	-4.245	-1.281	-1.773	-2.215	-5.347	-1.560
	(0.769)	(1.154)	(0.916)	(1.316)	(1.056)	(1.476)	(1.340)	(1.658)
	[0.508]	[0.538]	[1.985]	[1.236]	[0.544]	[0.574]	[2.004]	[1.218]
	${0.003}$	${0.011}$	${0.001}$	${0.048}$	${0.009}$	${0.007}$	${0.001}$	${0.056}$
	0.116	0.114	0.229	0.041	0.139	0.137	0.230	0.047
$\overline{7}$	-2.108	-2.284	-5.815	-1.970	-3.483	-4.024	-9.265	-3.214
	(1.092)	(1.594)	(0.855)	(1.644)	(1.741)	(2.287)	(1.657)	(2.470)
	[0.647]	[0.686]	[2.438]	[1.575]	[0.701]	[0.748]	[2.459]	[1.572]
	${0.001}$	${0.001}$	${0.001}$	${0.011}$	${0.001}$	${0.001}$	${0.001}$	${0.007}$
	0.181	0.158	$0.392\,$	0.083	0.201	0.182	0.368	0.086
10	-3.021	-3.373	-6.391	-3.558	-6.601	-7.795	-12.439	-7.457
	(1.489)	(2.120)	(0.698)	(1.988)	(2.991)	(3.766)	(1.882)	(3.932)
	[0.778]	[0.784]	[2.766]	[1.784]	[0.845]	[0.897]	[2.817]	[1.800]
	${0.001}$	${0.001}$	${0.001}$	${0.001}$	${0.001}$	${0.001}$	${0.001}$	${0.001}$
	0.206	0.188	0.429	0.199	0.231	0.206	0.409	0.230

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This table reports estimates from cross correlation analysis of the variance of log price-earnings ratio and various risk measures, including realized Cochrane (1999) and economic uncertainty (EU) measured as the conditional volatility of consumption growth following Bansal and Yaron (2004). This table reports estimates from cross correlation analysis of the variance of log price-earnings ratio and various risk measures, including realized [Cochrane](#page-24-9) [\(1999\)](#page-24-9) and economic uncertainty(EU) measured as the conditional volatility of consumption growth following [Bansal](#page-24-10) and Yaron [\(2004\)](#page-24-10). returns(V_m), conditional volatility of economy (V_e) based on the vector autoregressive model (VAR) following Bansal et al. (2014), long-run cash variance of market excess returns(V_{re}), realized variance of market real returns(V_{rr}), variance of GDP growth(V_{GDP}), variance of aggregate consumption growth(V_{PCE}), varaince of corporate profits growth(V_{PRO} returns(V_m), conditional volatility of economy (V_e) based on the vector autoregressive model (VAR) following [Bansal](#page-24-1) et al. [\(2014\)](#page-24-1), long-run cash flow risk(CF) following Hansen et al. (2008), rational risk aversion(RA) based on the specification of external habit persistence in Campbell and flow risk(CF) following [Hansen](#page-26-15) et al. [\(2008\)](#page-26-15), rational risk aversion(RA) based on the specification of external habit persistence in [Campbell](#page-24-9) and variance of market excess returns(V_{re}), realized variance of market real returns(V_{rr}), variance of GDP growth(V_{GDP}), variance of aggregate consumption growth(V_{PCE}), varaince of corporate profits growth(V_{PRO}), variance of net cash flow growth(V_{NCF}), conditional volatility of market The table reports cross correlation estimates at different lags and leads. The table reports cross correlation estimates at different lags and leads.

Table 10: Volatility of log price-earnings ratio, cash flow shocks, discount rate shocks, and volatility shocks.

This table reports results from long-horizon predictive regressions for compounded market excess returns, at horizons of $k = 1, 3, 5, 7$ and 10 years ahead. Panel A controls for future cash flow shocks using oneyear ahead Gross Domestic Product growth (G_{GDP}) , Corporate Profits growth (G_{PRO}) and Net Cash Flow growth(G_{NCF}). Panel B controls for the discount rate shocks using market excess return(R_e), dividend yield(dp), and default return spread(dfr). Panel C controls for the aggregate volatility shocks. V_{re} , V_{GDP} , V_{PRO} and V_{NCF} represent volatility of market excess return, the volatility of Gross Domestic Product growth, Corporate Profits growth, and Net Cash Flow growth, respectively. Panel D controls for all the cash flow shocks, discount rate shocks and volatility shocks. The table reports estimates of OLS regres-sion, [Newey and West](#page-28-10) (1987) corrected standard errors (with $k+1$ lags) in parentheses, [Hodrick](#page-26-12) (1992) standard errors in brackets, bootstrapping p-value in curly brackets and adjusted R-squared (R^2) in the bottom line of each prediction horizon.

Appendix A The second-order dynamic price-earnings ratio model

Market gross return can be written as:

$$
R_{t+1} = \frac{P_{t+1} + \Lambda_{t+1} E_{t+1}}{\Lambda_{t+1} E_{t+1}} \frac{\Lambda_{t+1} E_{t+1}}{\Lambda_t E_t} \frac{\Lambda_t E_t}{P_t}
$$
(A.1)

where P , E , Λ and R are price, earning, dividend payout ratio and gross return of the market, respectively. Take logarithm to both sides of equation $(A.1)$ yields

$$
r_{t+1} = \Delta e_{t+1} + \lambda_{t+1} - pe_t + \log(1 + e^{pe_{t+1} - \lambda_{t+1}})
$$
\n(A.2)

where $pe_t = p_t - e_t = logP_t - logE_t$, $\Delta e_{t+1} = e_{t+1} - e_t$, and $\lambda_t = log(\Lambda_t)$. For simplicity, we assume $\lambda_t = log(\bar{\Lambda}) + \gamma_t$.

Taking the second-order Taylor expansion, we obtain:

$$
log(1 + e^{pe_{t+1} - \lambda_{t+1}}) = k + \rho(pe_{t+1} - \lambda_{t+1} - \bar{pe} + \bar{\lambda}) + \frac{1}{2}\rho(1 - \rho)(pe_{t+1} - \lambda_{t+1} - \bar{pe} + \bar{\lambda})^2
$$
\n(A.3)

where $k = log(1 + e^{\bar{p}e - \bar{\lambda}}) = log(1 + e^{\bar{p}d})$ and $\rho = \frac{e^{\bar{p}e - \bar{\lambda}}}{1 + e^{\bar{p}e}}$ $\frac{e^{\bar{p}\bar{e}-\bar{\lambda}}}{1+e^{\bar{p}\bar{e}-\bar{\lambda}}}=\frac{e^{\bar{p}\bar{d}}}{1+e^{\bar{p}}}$ $\frac{e^{p\overline{d}}}{1+e^{p\overline{d}}} > 0$. \overline{pe} and $\overline{\lambda}$ are defined as the aggregate mean of log price-earnings ratio and log long-term target dividend payout ratio.

Then equation $(A.2)$ can be written as,

$$
r_{t+1} - \Delta e_{t+1} = k - \rho(\bar{p}e - \bar{\lambda}) - p e_t + \rho p e_{t+1} + (1 - \rho)\lambda_{t+1} - \rho(1 - \rho)(p e_{t+1} - \bar{p}e)(\lambda_{t+1} - \bar{\lambda})
$$

$$
+ \frac{1}{2}\rho(1 - \rho)(\lambda_{t+1} - \bar{\lambda})^2 + \frac{1}{2}\rho(1 - \rho)(p e_{t+1} - \bar{p}e)^2 \quad \text{(A.4)}
$$

Taking expectation based on the information set available at time t, we have

$$
pe_t - \rho E_t pe_{t+1} = k - \rho \bar{p}e + \bar{\lambda} + \frac{1}{2}\rho(1-\rho)\kappa^2 + E_t(\Delta e_{t+1} - r_{t+1}) + \frac{1}{2}\rho(1-\rho)E_t(pe_{t+1} - \bar{p}e)^2
$$
\n(A.5)

Note that $\lambda_{t+1} - \bar{\lambda} = \gamma_{t+1}$, a white noise.

Following [Campbell and Shiller](#page-24-3) [\(1988\)](#page-24-3) and [Gao and Martin](#page-25-0) [\(2021\)](#page-25-0), we derive a novel present value identify:

$$
pe_t = \delta^* + E_t \left[\sum_{j=0}^{\infty} \rho^j (\Delta e_{t+1+j} - r_{t+1+j}) \right] + \frac{1}{2} \rho (1-\rho) E_t \left[\sum_{j=0}^{\infty} \rho^j (pe_{t+1+j} - \bar{pe})^2 \right] \tag{A.6}
$$

where $\delta^* = \frac{1}{2}$ $\frac{1}{2}\rho\kappa^2+\frac{k-\rho\bar{p}e+\bar{\lambda}}{1-\rho}$ $\frac{\rho p e + \lambda}{1-\rho}$ contains all the constant items.

We also assume log price-earnings ratio follows AR(1) process. However, we relax the homoscedasticity assumption in [Gao and Martin](#page-25-0) (2021) and allow time-varying conditional volatility of a state variable in the economy. It is important to note that this conditional volatility will directly affect the level and the variance of log price-earnings ratio. Formally,

$$
pe_{t+1} - \bar{pe} = \phi(pe_t - \bar{pe}) + \psi \sigma_t u_{t+1}
$$
\n(A.7)

where $u_{t+1} \sim N i.i.d(0,1)$ and σ_{t+1} denotes the conditional volatility reflecting time-varying economic uncertainty.

With the assumption of equation $(A.7)$, we have

$$
E_t[(pe_{t+1+j} - \bar{p}e)^2]
$$

= $E_t[(\phi(pe_{t+j} - \bar{p}e) + \psi \sigma_{t+j}u_{t+1+j})^2]$
= $E_t[(\phi^{j+1}(pe_t - \bar{p}e) + \phi^j \psi \sigma_t u_{t+1} + \phi^{j-1} \psi \sigma_{t+1} u_{t+2} + ... + \psi \sigma_{t+j} u_{t+1+j})^2]$
= $\phi^{2(j+1)} E_t[(pe_t - \bar{p}e)^2] + E_t[\phi^{2j} \psi^2 \sigma_t^2 u_{t+1}^2 + \phi^{2(j-1)} \psi^2 \sigma_{t+1}^2 u_{t+2}^2 + ... + \psi^2 \sigma_{t+j}^2 u_{t+1+j}^2]$ (A.8)

Consider

$$
E_t[\sigma_{t+j}^2 u_{t+1+j}^2] = E_t[E_{t+j}[\sigma_{t+j}^2 u_{t+1+j}^2]] = E_t[\sigma_{t+j}^2 E_{t+j}[u_{t+1+j}^2]] = E_t[\sigma_{t+j}^2]
$$
(A.9)

Equation [\(A.8\)](#page-46-1) becomes

$$
E_t[(pe_{t+1+j} - \bar{p}e)^2]
$$

= $\phi^{2(j+1)}E_t[(pe_t - \bar{p}e)^2] + E_t[\phi^{2j}\psi^2\sigma_t^2u_{t+1}^2 + \phi^{2(j-1)}\psi^2\sigma_{t+1}^2u_{t+2}^2 + ... + \psi^2\sigma_{t+j}^2u_{t+1+j}^2]$
= $\phi^{2(j+1)}E_t[(pe_t - \bar{p}e)^2] + E_t[\phi^{2j}\psi^2\sigma_t^2 + \phi^{2(j-1)}\psi^2\sigma_{t+1}^2 + ... + \psi^2\sigma_{t+j}^2]$
= $\phi^{2(j+1)}E_t[(pe_t - \bar{p}e)^2] + \psi^2E_t\sum_{i=0}^j\phi^{2(j-i)}\sigma_{t+i}^2$ (A.10)

and

$$
E_{t}[\sum_{j=0}^{\infty} \rho^{j}(pe_{t+1+j} - \bar{p}\bar{e})^{2}]
$$

\n
$$
= E_{t}[\sum_{j=0}^{\infty} \rho^{j}\phi^{2(j+1)}[(pe_{t} - \bar{p}\bar{e})^{2}] + \psi^{2}E_{t}[\sum_{j=0}^{\infty} \rho^{j}\sum_{i=0}^{j}\phi^{2(j-i)}\sigma_{t+i}^{2}]
$$

\n
$$
= \frac{\phi^{2}}{1 - \rho\phi^{2}}E_{t}[(pe_{t} - \bar{p}\bar{e})^{2}]
$$

\n
$$
+ \psi^{2}E_{t}[\sigma_{t}^{2} + \rho(\phi^{2}\sigma_{t}^{2} + \sigma_{t+1}^{2}) + \rho^{2}(\phi^{4}\sigma_{t}^{2} + \phi^{2}\sigma_{t+1}^{2} + \sigma_{t+2}^{2}) + \rho^{3}(\phi^{6}\sigma_{t}^{2} + \phi^{4}\sigma_{t+1}^{2} + \phi^{2}\sigma_{t+2}^{2} + \sigma_{t+3}^{2}) + ...]
$$

\n
$$
= \frac{\phi^{2}}{1 - \rho\phi^{2}}E_{t}[(pe_{t} - \bar{p}\bar{e})^{2}]
$$

\n
$$
+ \psi^{2}E_{t}[\frac{1}{1 - \rho\phi^{2}}\sigma_{t}^{2} + \frac{\rho}{1 - \rho\phi^{2}}\sigma_{t+1}^{2} + \frac{\rho^{2}}{1 - \rho\phi^{2}}\sigma_{t+2}^{2} + ...]
$$

\n
$$
= \frac{\psi^{2}\sigma^{2}}{1 - \rho\phi^{2}} + \frac{\phi^{2}}{1 - \rho\phi^{2}}E_{t}[(pe_{t} - \bar{p}\bar{e})^{2}] + \frac{\rho\psi^{2}}{1 - \rho\phi^{2}}E_{t}[\sum_{j=0}^{\infty} \rho^{j}\sigma_{t+1+j}^{2}]
$$

\n(A.11)

Substituting $(A.11)$ into $(A.6)$, we can derive

$$
pe_t - \pi E_t[(pe_t - \bar{pe})^2] = \alpha^* + E_t[\sum_{j=0}^{\infty} \rho^j(\Delta e_{t+1+j} - r_{t+1+j}) + \nu \sigma_{t+1+j}^2]
$$
(A.12)

where $\pi = \frac{\rho(1-\rho)\phi^2}{2(1-\rho\phi^2)}$ $\frac{\rho(1-\rho)\phi^2}{2(1-\rho\phi^2)}$, $\alpha^* = \frac{\psi^2\sigma^2}{1-\rho\phi^2} + \delta^*$, $\nu = \frac{\psi^2\rho(1-\rho)}{2(1-\rho\phi^2)}$ $\frac{\psi^2 \rho (1-\rho)}{2(1-\rho\phi^2)}$.

Appendix B Additional Tables

Table B.1: Univariate predictive regressions for market returns

This table reports univariate long-horizon predictive regressions for compounded market excess returns(R_e) and real returns(R_r), at horizons of $k = 1, 3, 5, 7$ and 10 years ahead. The predictive variables are the variance of log price-dividend ratio(V_{pd}), variance risk premium(vrp) in [Bollerslev et al.](#page-24-12) (2009) , and stock variance(svar) in [Guo and Whitelaw](#page-26-10) [\(2006\)](#page-26-10). The table reports estimates of OLS regression, [Newey and West](#page-28-10) [\(1987\)](#page-28-10) corrected standard errors(with k+1 lags) in parentheses, [Hodrick](#page-26-12) [\(1992\)](#page-26-12) standard errors(with k lags) in brackets, bootstrapping p-value in curly brackets and adjusted R-squared in the bottom line of each prediction horizon. The annual sample period is from 1937 to 2021.

		$R_{e,t+k}$			$R_{r,t+k}$	
\boldsymbol{k}	V_{pd}	vrp	svar	V_{pd}	vrp	svar
1	-0.221	0.398	0.526	-0.427	0.356	0.556
	(0.708)	(0.138)	(0.455)	(0.683)	(0.118)	(0.410)
	[0.688]	[0.426]	[0.703]	[0.699]	[0.373]	[0.676]
	${0.730}$	${0.328}$	${0.247}$	${0.513}$	${0.313}$	${0.200}$
	-0.010	0.004	0.005	-0.006	-0.002	0.007
3	0.022	0.230	1.783	-0.992	0.046	1.573
	(1.731)	(0.521)	(1.146)	(1.721)	(0.486)	(1.168)
	[1.830]	[0.519]	[1.489]	$[1.900]$	[0.507]	[1.429]
	${0.985}$	${0.699}$	${0.016}$	${0.375}$	${0.877}$	${0.039}$
	-0.012	-0.029	0.040	-0.003	-0.032	0.031
5	1.257	1.476	2.189	-0.971	1.041	1.575
	(3.002)	(0.843)	(1.081)	(2.975)	(0.848)	(1.189)
	[2.440]	[0.627]	[2.170]	[2.526]	[0.608]	[2.150]
	${0.491}$	${0.184}$	${0.052}$	${0.549}$	${0.453}$	${0.172}$
	-0.005	0.028	0.028	-0.008	-0.005	0.010
7	2.101	1.314	2.043	-1.535	0.723	1.022
	(4.854)	(0.893)	(1.385)	(4.565)	(0.943)	(1.467)
	[3.085]	[0.581]	[2.260]	[3.273]	[0.577]	[2.285]
	${0.314}$	${0.340}$	${0.141}$	${0.465}$	${0.637}$	${0.508}$
	-0.002	0.003	0.011	-0.007	-0.026	-0.006
10	5.811	3.010	1.968	-1.454	2.176	-0.989
	(7.793)	(0.382)	(2.679)	(6.876)	(0.561)	(2.498)
	[3.782]	[0.614]	[2.156]	[3.906]	[0.605]	[2.378]
	${0.115}$	${0.087}$	${0.416}$	${0.668}$	${0.300}$	${0.644}$
	0.021	0.061	-0.003	-0.011	0.005	-0.010

This table reports univariate estimates from the regression of compounded market excess returns(R_e) and real returns(R_r), at horizons of k = 1, 2, 3, 4, 12, 20, 28 and 40 quarters ahead. The predictive variables are the log price-earnings ratio of $S\&P$ 500 index(pe), the variance of market excess returns(V_{re}), the variance of market real returns(V_{rr}), and the variance of log price-earnings ratio(V_{pe}), respectively. The table reports estimates of OLS regression, [Newey and West](#page-28-10) [\(1987\)](#page-28-10) corrected standard errors(with $k+1$ lags) in parentheses, [Hodrick](#page-26-12) (1992) standard errors(with k lags) in brackets, bootstrapping p-value in curly brackets and adjusted R-squared in the bottom line of each prediction horizon. The sample period is from 1937:Q1 to 2021:Q4.

Table B.3: Multivariate forecast of quarterly stock returns

This table reports multivariate long-horizon predictive regressions for compounded market excess returns and real returns, at horizons of $k = 1, 2, 3, 4, 12, 20, 28$ and 40 quarters ahead. We consider three pairs of predictive variables, the log price-earnings ratio of $S\&P$ 500 index (pe) and the variance of market excess returns (V_{re}) or the variance of market real returns (V_{rr}) , the lagged log price-earnings ratio of $S\&P$ 500 index (pe) and the variance of log price-earnings ratio (V_{pe}) , and the variance of market returns (V_{re} or V_{rr}) and the variance of log price-earnings ratio (V_{pe}) , respectively. The table reports estimates of OLS re-gression, [Newey and West](#page-28-10) [\(1987\)](#page-28-10) corrected standard errors (with $k+1$ lags) in parentheses, [Hodrick](#page-26-12) [\(1992\)](#page-26-12) standard errors(with k lags) in brackets, bootstrapping p-value in curly brackets and adjusted R-squared $(R²)$ of each prediction horizon. The sample period is from 1937: Q1 to 2021:Q4.

		Table B.4: Predicting macroeconomic activities	
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This table reports estimates from regression of economic variables on the lagged volatility of price-earnings ratio(V_{pe}) in a yearly frequency while controlling market risk premium($R_m - R_f$), HML and SMB from Fama-French Research Factors. The dependent variables are the sum of k-period log Gross Domestic Product growth (G_{GDP}) , log Personal Consumption Expenditures index growth (G_{PCE}) , log Corporate Profits growth(G_{PRO}) and log Net Cash Flow growth(G_{NCF}). The table reports OLS estimates of regressors, [Newey and West](#page-28-10) [\(1987\)](#page-28-10) corrected standard errors(with k+1 lags) in parentheses, [Hodrick](#page-26-12) [\(1992\)](#page-26-12) standard errors(with k lags) in brackets, bootstrapping p-value in curly brackets and adjusted R-squared $(R²)$ of each prediction horizon.

