# Are global investment fund markets heading towards efficient markets?

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## Abstract

We examine the American, European, Japanese, Chinese and Polish stock markets. We show that the forecasted, formed portfolios of a representative investor of the mutual fund market generate returns in light of the Merton's ICAPM, and the use of fundamental analysis does not allow for obtaining above-average returns. We also present that the structure of past long-term financial results can be modeled by the proposed fundamental functional *FUN*. We express the presumption that the average returns on the U.S. markets since 2007, and in European market since 2010 are in line with the ICAPM, which confirms the market efficiency. This can be a significant indication, especially for investment fund managers. We conclude that returns on the Japanese and Chinese markets do not confirm the efficiency of these markets.

***Keywords:*** forecast of future states of the economy, ICAPM, market efficiency, investment fund, portfolio analysis

JEL: G11, G12, G14, G15

## Introduction

The issue of market efficiency has been discussed in the literature for over half a century. There is a large variation in the results obtained due to the use of different methods and the analyzed periods. Moreover, universal methods to verify the hypothesis of market efficiency have not yet been developed. Our research differs from the previous ones and concerns the behavior of a representative investor of the analyzed market. Such an investor may make various decisions based on the received information. However, it always follows the rationality of the market. Our considerations assume the existence of a non-representative investor whose decisions may be based on the use of specific information, published or confidential, enabling economic benefits to be obtained. The decisions of such an investor may also take into account the behavioral perception of the market.

Our research work boils down to conditional market efficiency tests from the point of view of the dominant role of a representative investor. The representative investor forecasts the risk factors that model the state variables generating future returns on investment.

*Related literature.* The efficient market hypothesis (EMH hereafter) has been presented in the literature since the late 1950s by Roberts (1959), Fama (1965a, 1965b) and Samuelson (1965a). The EMH concept stems from an attempt to explain the random nature of prices as a consequence of a rational market. In order to demonstrate the indicated randomness, various probabilistic models are used, i.e., Random Walk Model (Fama (1965a)) or Martingale Model (Samuelson (1965a)). Samuelson (1965b) and Mandelbrot (1966) point out that random price fluctuations take place in a market that provides equal access to information to all its participants (Boldt and Arbit (1984)). Fama (1970) defines an efficient market as a market in which the price fully reflects the available information. Such a market fulfills three assumptions: no transaction costs, all available information is freely available to all market participants, and there is a relationship between the available information and the current price and the distribution of future prices. On the other hand, Malkiel (2003, p. 60) defines efficient financial markets as such markets that ‘do not allow investors to earn above-average returns without accepting above-average risks’. However, it is indicated that in reality the market cannot be perfectly efficient, because in such a situation there would be no incentives for professional entities to collect information reflected in market prices (Malkiel (2003, p. 80)). The existence of an efficient market, in a situation where obtaining information involves certain costs, would lead to the collapse of competitive markets, because every rational investor would resign from paying for information (Grossman and Stiglitz (1980, p. 404)).

Jensen (1978) points out that the efficient market hypothesis is an extension of the zero-profit competitive equilibrium condition of the classic price theory. It assumes that the market is efficient in terms of a specific set of information if it is not possible to achieve economic benefits based on this set of information.

Fama (1970) distinguishes three hypotheses regarding market efficiency: strong, semi-strong, and weak.

In the case of weak market efficiency, the set of information is only historical prices. Therefore, building a strategy based on technical analysis does not allow the investor to achieve above-average profits (Malkiel (1989)).

The semi-strong form of market efficiency assumes that prices effectively adjust to other information that is publicly available, for example announcements of annual profits. It is indicated that the marginal benefits of acting on information, i.e., the profits to be achieved, do not exceed the marginal costs. Therefore, the use of ex-post fundamental analysis by the investor does not allow for above-average profits (Malkiel (2016, p. 141)). The indicated form of hypothesis is treated as an accepted paradigm (Jensen (1978)). Another approach is to use the forecast error. In the case of efficient market there should be no systematic relationship between the current and previous forecast errors that make up the information set (Hansen and Hodrick (1980)). Yet another approach involves the use of forecasting models (Goss et al. (1992)).

The correctness of the semi-strong form of EMH is demonstrated by McKenzie (2008). The author shows that future prices continue to react to the release of USDA reports. Fama et al. (1969, p. 20), using companies listed on the NYSE, show that on the average market's judgments concerning the information of the split are fully reflected in the stock price. The split causes price adjustments only to the extent that it is associated with changes in the expected level of future dividends. Reilly and Drzycimski (1981, pp. 71-72) show that stock prices adjust before or shortly after the public announcement of a split, and that above-average returns are not available to the general public or professionals who bear normal transaction costs. In turn, Groenewold (1997) demonstrate market efficiency of the Statex Actuaries' Price Index for Australia and the NZSE-40 Index for New Zealand. Chu and Lim (1998), based on a regression study using Anderson and Peterson's modified efficiency results, show that stock prices in the Singapore market reflect profit efficiency more than cost efficiency, which means that this market is efficient. Agrawal (2007), using the study of events, i.e., monetary policy announcements, demonstrates the inefficiency of the Indian stock market CNX Nifty. Khan and Ikram (2010) demonstrate that the Indian capital market (represented by the National Stock Exchange and the Bombay Stock Exchange) is effective in the sense of the semi-strong form, due to the significant influence of foreign investors. On the other hand, the lack of effectiveness in this form is demonstrated, among others, by Basu (1977). The author shows that the prices of securities on the NYSE do not fully reflect the information on the P/E ratio, which indicates the inefficiency of this market. Leuthold and Hartmann (1979, 1980) demonstrate Hog Future Market inefficiency. Rose and Selody (1984), on the basis of the created test involving inter-forward market trade, show the lack of effectiveness on the Canada-USA market. In turn, Goss et al. (1992) using a forecasting model show no grounds for rejecting the semi-strong market efficiency hypothesis for the U.S. Oats Market.

The presented studies testing semi-strong market efficiency give ambiguous results. The reason for this is the different research methods that take into account narrow information ranges. This fact can be used by a non-representative investor who takes into account the insignificant market disturbances. It seems, therefore, that it would be more appropriate to use commonly available information affecting the future returns of stocks listed on the market. Assuming that the analyzed market is at least partially complete, taking into account the instruments hedging the future states of the economy would allow to obtain significant additional benefits. The use by a representative investor of such a hedging instrument resulting in obtaining economic benefits would contradict the hypothesis of market efficiency.

The strong form of market efficiency, on the other hand, focuses on price-relevant information to which individual investors or groups of investors have monopolistic access (Fama (1970, p. 388)). In the paper of Jensen (1978) such variant is treated as extreme only in the theoretical form. Fuller and Farrell (1987) distinguish two strong forms of EMH – the super-strong form and near-strong form. The super-strong form of EMH stipulates that the price of the security includes all information available only to insiders and exchange specialists. The near-strong form of EMH, on the other hand, states that the price of a security includes all private estimates based on public information (Mallikarjunappa and Manjunatha (2009, pp. 43-44)). Hypothesis verification tests are referred to by Fama (1991, p. 1577) as tests for private information. The most commonly used tests compare the performance of mutual funds with a benchmark portfolio like a stock market index. The correctness of the strong form of EMH is demonstrated by, among others, Gupta et al. (2008). They show that fund managers with better information, including insider information, do not to outperform randomly selected stock portfolios.

The tested strong form of market efficiency focuses on any information, including confidential information, to which not all investors have access. Therefore, this form could be studied in an extreme and theoretical form so far. However, it should be noted that market confidential information, e.g., financial results of companies before the official date of their publication, becomes public after they are made available by legal market regulations.

Among the research indirectly related to the topic of efficient markets, the work of Asness et al. (2019) deserves attention. Research conducted on the American market and 24 global market countries shows insignificantly higher stock prices characterized by high quality indicators. The authors find that high-quality stocks are underpriced and low-quality stocks are overpriced. The authors explain this fact by the influence of an unexplored risk factor or an existing anomaly. In our opinion, the cited research results do not contradict the assumptions made about the effectiveness of selected stock markets.

In this paper, we examine changes in return/risk profiles of stocks traded on the American, European, Japanese, Chinese and emerging markets (represented by the Polish market). The studies verify the commonly accepted paradigm of Jensen (1978), raised by Malkiel (2016), stating that the use of ex-post fundamental analysis does not allow for above-average investment returns. We define above-average investment returns as returns on fundamentally designed portfolios compared to returns on benchmark market portfolios.

The aim of this work is to show whether the designed fundamental stock portfolios are competitive with market portfolios simulated by benchmark indexes.

**The research conducted in this work differs from the previous ones in the assumption that investors form portfolios that take into account the structure of past financial results in relation to the company's equity. According to the work of Fama and French (1995), the analysis of the structure of past financial results in relation to the company's equity allows forecasting future asset prices and, therefore, future returns on investments.**

We assume that the representative investor is the investment fund manager.

In Section I, dealing with theoretical considerations, we present a fundamental model of building investment portfolios. The portfolios are formed in accordance with the research of Fama and French's (1995) results. If an investor, forms portfolios based on the structure of past financial results and opening investment positions before the official announcement of the results for the last reporting period does not allow for above-average returns, then the market is effective in a strong form. If, on the other hand, opening investment positions after the announcement of financial results for the last reporting period does not allow obtaining above-average returns, then the market is effective in a semi-strong form. Here we explain the relationship between market efficiency and the correctness of the pricing in light of the ICAPM.[1] Based on this consideration, we design tests verifying the conjecture that the average return and risk of generated portfolios are competitive with benchmark portfolios.

In Section II, we present the data needed for the calculations and the scope of the conducted research. Section III presents the results of the calculations and statistical analysis of the observed changes for each market studied. In Section IV, we discuss financial interpretations of the obtained results. The final section contains a summary and conclusions.

## I. Theoretical considerations

According to the ICAPM application, proposed by Fama and French (1993), future returns on stocks are generated by state variables representing the book value to market value of the company, and its capitalization. Based on the adopted state variables, Fama and French (1993) define commonly known risk factors: *HML* and *SMB*. Further research by Fama and French (1995) shows that the factors that directly generate returns are not *HML* and *SMB* but the

[1] According to Merton's ICAPM, future asset prices are determined by future states of the economy. A correct forecast of risk factors modeling future states of the economy will allow for the estimation of future returns.

structure of past long-term financial results. However, in Fama and French (1995) an explicit form of such risk factors is not presented. Attempts to define such factors have been made by Urbański (2012), and Urbański and Zarzecki (2022). **In this research it is shown that the structure of past long-term financial results can be modeled by the fundamental functional *FUN***, represented by equation (1). The conducted research also shows a significant relationship between the *FUN* values and future average returns on stock investments (see Section II). Therefore, it can be concluded that the state variable generating future returns can be modeled based on the values of the *FUN* functional (Urbański and Zarzecki (2022)).

$$FUN = \frac{NUM}{DEN} = \frac{nor(ROE)\times nor(ASB)\times nor(APOB)\times nor(APNB)}{nor(ME)\times nor(MB)}, \tag{1}$$

where

$$ROE = F_1; \tag{2}$$

$$ASB = F_2 = \frac{\{\sum_{t=1}^{i}[S(Q_t)]\}/BV(Q_t)}{\sum_{t=1}^{i}\overline{SBV(nQ_t)}}; \tag{3}$$

$$APOB = F_3 = \frac{\{\sum_{t=1}^{i}[PO(Q_t)]\}/BV(Q_t)}{\sum_{t=1}^{i}\overline{POBV(nQ_t)}}; \tag{4}$$

$$APNB = F_4 = \frac{\{\sum_{t=1}^{i}[PN(Q_t)]\}/BV(Q_t)}{\sum_{t=1}^{i}\overline{PNBV(nQ_t)}}; \tag{5}$$

$$ME = F_5; \tag{6}$$

$$MB = F_6. \tag{7}$$

$F_j$ ($j$=1,…,6) are transformed to the normalized areas $<a_j\,;b_j>$ according to Eq. (8):

$$nor(F_j) = [a_j + (b_j - a_j)\times \frac{F_j - c_j\times F_j^{min}}{d_j\times F_j^{max} - c_j\times F_j^{min} + e_j}]. \tag{8}$$

*ROE* is return on book value; $\sum_{t=1}^{i} S(Q_t), \sum_{t=1}^{i} PO(Q_t),$ and $\sum_{t=1}^{i} PN(Q_t)$ are respectively the net sales revenue (*S*), operating profits (*PO*) and net profits (*PN*) that are accumulated from the beginning of the year to the end of '*i*' quarter ($Q_i$); $\sum_{t=1}^{i}\overline{SBV(nQ_t)}$, $\sum_{t=1}^{i}\overline{POBV(nQ_t)}$, and $\sum_{t=1}^{i}\overline{PNBV(nQ_t)}$ are respectively the average *S/BV*, *PO/BV* and *PN/BV* that are accumulated from the beginning of the year to the end of $Q_i$ over the last *n* years (the present research assumes that *n* ranges from 1 to 3, which depends on the availability of data); *BV* is the book value; *ME* is the market-to-earnings (for the last four quarters) ratio; *MB* is the market-to-book value ratio; and $a_j, b_j, c_j, d_j$, and $e_j$ are the variation parameters. In the present study we assume the following values of parameters: $a_j = 1, b_j = 2, c_j = 1$ , $d_j = 1$, and $e_j = 0$.[2]

Functional *FUN* has a clear economic interpretation and can be used as a criterion for selecting securities for the portfolio. Function *NUM* represents an investor building a portfolio of the best fundamental companies. Function *DEN* represents an investor building a portfolio of undervalued stocks. Thus, *FUN* represents an investor building a portfolio which consists of the best fundamental and simultaneously undervalued stocks (in the case of long investments). The investment is more attractive if the *FUN* value is greater.

**In light of the research of Fama and French (1995) and the above considerations, the investment portfolio of a representative investor should be formed on the basis of the structure of past long-term financial results. For that purpose *FUN* maximizations can be used. In other words, in the analyzed markets, all known and unknown investment strategies of all investors can be based on *FUN*. Forming an investment portfolio on *FUN*, as in the case of forming *BV/MV* portfolios in the Fama-French model (1993) is a sufficient condition for returns to be consistent with the ICAPM. However, this is not a necessary condition. It is possible that another portfolio formed on another unknown functional *FUN_x* could provide similarly good or better results. Such strategy is meant in the sense of better modeling past financial results and generating future returns in light of the ICAPM at a higher significance level. Therefore, if a representative investor knows that future returns are generated by the past structure of financial results, they will look for such a known or unknown *FUN_x*, which models the past structure of profits (in relation to the company's equity) in the best possible way.**[3]

If, in turn, the representative investor is the dominant investor in the market, then no other investment strategy can beat the market and the market is efficient. Thus, market efficiency tests can be reduced to examining whether a representative investor builds portfolios whose returns are consistent with the ICAPM. Such research can be conducted in two directions:

[2] For such adopted values of variation parameters, all indices $F_j$ are transposed into <1; 2> intervals.
[3] Similarly, in the case of a CAPM investor, all investment strategies in risky assets (if there is a risk-free asset in the economy) are determined by the portfolio located on the tangent of the Capital Market Line and the Markowitz minimum risk set.

Direction A - portfolios formed on the basis of the structure of past financial results generate/do not generate significantly higher returns than returns in the market portfolio. According to Fama's effective markets theory, conclusions cited above (Fama (1970) and Malkiel's (2003, 2016)), the tested market is efficient if no investment strategy beats the market.

Direction B - requires using ICAPM multifactor-efficiency tests of formed portfolios (Chou and Zhou (2006)). In this approach the following tests are used: Wald test, Gibbons, Ross and Shanken test (1989), and correct pricing tests (Shanken (1985), Roll (1985)).

In this work we limit ourselves to direction A. In our future work, we plan to investigate direction B.

Based on the above considerations, we expect that the following conjectures are true:

**Conjecture 1.**

The average returns generated by selected portfolios formed on *FUN* values are different than the average returns generated by the portfolio modeled with the benchmark index of the analyzed market.

**Conjecture 2.**

The values of the Sharpe ratio (*SR* hereafter) of selected portfolios formed on *FUN* values are different than the *SR* of the portfolio modeled with the benchmark index of the analyzed market.

**Conjecture 3.**

The differences between *FUN* generated average returns and benchmark generated returns are due to differences in portfolio capitalization. These differences are caused by presence of small-cap companies in *FUN*-formed portfolios compared to the big-cap companies in the benchmark portfolio.

If Conjectures 1 and 2 are true, it can be presumed that the tested market is not efficient. If Conjectures 1 and 2 are not true, there is no basis to conclude that the tested market is not an efficient market.

Confirmation of the validity of Conjecture 3 allows us to conclude that the validity/incorrectness of Conjecture 1 also results from the relation of capitalization of *FUN* and benchmark portfolios.

In order to test the correctness of the adopted Conjectures, from the perspective of the proposed topic of the work, the returns and values of the *SR* of selected portfolios formed on *FUN* are estimated. Details on the scope of the research are presented in Section II.

## II. Data and scope of research

We analyze the monthly returns of the stocks listed on the American, European, Japanese and Chinese markets. The Polish capital market is the largest and most developed among CEE emerging markets. Therefore, we also analyze the Polish market, which may be an alternative investment opportunity to highly developed markets (see: Table I).[4]

**Table I**

Number of listed companies and values of capitalization of tested stock indexes in 2021.

| No. | Stock index | Number of listed companies | Value of capitalization [USD billion] |
|---|---|---|---|
| 1 | *S&P500* | 505 | 41696 |
| 2 | *BE500* | 497 | 13321 |
| 3 | *TOPIX500* | 496 | 5625 |
| 4 | *SSE380* | 380 | 1163 |
| 5 | *WIG* | 430 | 324 |

*Source*: Bloomberg database and www.gpw.pl

The companies are selected for the created portfolios from the benchmark indexes and listed in the following historical periods:

- Standard & Poor's 500 Index (*S&P500*); from January 1995 through March 2022; 311 monthly investment periods,
- Bloomberg Europe 500 Index (*BE500*); from January 2000 through March 2022; 251 monthly investment periods,
- Tokyo Stock Price Index 500 (*TOPIX500*); from January 2003 through March 2022; 215 monthly investment periods,
- Shanghai Stock Exchange 380 Index (*SSE380*); from January 2002 through March 2022; 227 monthly investment periods,

[4] Financial data of companies needed to calculate *FUN* are taken from the Bloomberg Terminal. Data to determine the structure of investors in the analyzed markets are made available by the EquityRT service through Notoria Serwis SA and S&P Global Market Intelligence, Morningstar, Factset Estimates by Xignite, Reuters, local exchanges and other local data providers. The author of the www.equityrt.com website is RasyOnet. The data providers for the service are: S&P Global Market Intelligence, Morningstar, Factset Estimates by Xignite, Reuters, local exchanges and other local data providers.

- Warsaw Stock Exchange WIG Index (*WIG*); from January 1995 through March 2022; 311 monthly investment periods.[5]

Calculation of *FUN* requires collecting minimum 12 consecutive months of data. Based on these 12 months of observations, *FUN* data is available for 4 months afterwards, resulting minimum 16 months of delay in the adopted investment period. The first hypothetical investment can be made after the company publishes its financial results for the first quarterly period.

We calculate *FUN* values for each company at the beginning of each monthly period. Stock portfolios of 5, 7, 10 and 15 companies are formed based on the maximum values of the *FUN* functional. Stock shares in the portfolios are linearly weighted. For such portfolios, returns are calculated in each monthly period. The analysis of change in returns on hypothetical investments in the analyzed markets is conducted on the basis of average returns in rolling 96- and 60- month time windows. The choice of 96- and 60- month historical periods results from the adopted assumptions about the length of the investment period of the hypothetical participant purchasing the assets of the investment fund. In this article, we present only the results for 96-month windows due to the volume of work.[6]

Rolling average returns from portfolios formed on *FUN* and from the benchmark index portfolio, with one month step, are calculated for each investigated market over the entire tested investment period.

In order to confirm the validity of the adopted assumptions, we examine the dynamics of differences between the return on benchmark portfolio and portfolio formed on *FUN*. Similarly, we also examine the evolution of the differences between the *SR* of *FUN*-formed portfolio and benchmark portfolio.

The following test procedures are adopted:

a) Modeling of returns and *SRs* differences.

We focus on inference regarding the above mentioned differences, measuring either out- or underperformance of our fundamental portfolios with respect to the given stock index. We

[5] Different historical periods are due to availability of data in the Bloomberg database.

[6] The results for 60-month windows are similar and can be made available upon request.

intend to track the temporal dynamics of the differences using time series methods. We estimate the value of the risk-free rate (*RF*) for all the analyzed markets on the basis of U.S. 3-month Treasury bills.

b) Time series forecasting.

Here we use both Holt-Winters double exponential smoothing approach and SARIMA model for these two types of differences. Let us start with a short description of the Holt-Winters model. Given an empirical data $\{X_t\}_{t=1}^{n}$, standard smoothing is carried out recursively with the level $\{l_t\}$ and trend $\{m_t\}$ for $1 \le t < n$:

$$l_t = \alpha X_{t-1} + (1-\alpha)(l_{t-1} + m_{t-1}), \tag{9}$$

$$m_t = \beta(l_t - l_{t-1}) + (1-\beta)m_{t-1}, \tag{10}$$

$$\hat{X}_{t+1} = l_t + m_t. \tag{11}$$

The smoothing parameters $\propto, \beta \in (0; 1]$ are either fixed or optimized with respect to specific criteria, e.g., minimizing mean-squared error (Chatfield, 1978). Using equations (11), forecasts are defined as:

$$\hat{X}_{n+h} = l_n + h \cdot m_n, \quad h = 1, 2, \ldots. \tag{12}$$

The second proposed model is SARIMA(*p*,*d*,*q*)(*P*,*D*,*Q*)[*S*]. We say that time series $X_t$ is modeled with SARIMA (*p*,*d*,*q*)(*P*,*D*,*Q*)[*S*] model:

$$\Phi_P(B^S)\phi(B)\nabla_S^D\nabla^d x_t = \delta + \Theta_Q(B^S)\theta(B)\omega_t\ , \tag{13}$$

where:

$\omega_t$ is the usual Gaussian white noise process;

$\nabla_S^D = (1 - B^S)^D$ is seasonal difference component;

$\nabla^d = (1 - B)^d$ is ordinary difference component;

$\delta = \mu(1 - \phi_1 - \cdots - \phi_p)$ and $\mu = E(\nabla^d x_t)$;

$\Phi_P(B^S) = 1 - \Phi_1 B^S - \Phi_2 B^{2S} - \cdots - \Phi_P B^{PS}$ is the seasonal autoregressive component of order *P*;

$\phi(B) = 1 - \phi_1 B^1 - \phi_2 B^2 - \cdots - \phi_p B^p$ is ordinary autoregressive component of order *p*;

$\Theta_Q(B^S) = 1 - \Phi_1 B^S - \Phi_2 B^{2S} - \cdots - \Phi_Q B^{QS}$ is the seasonal moving average component of order *Q*;

$\theta(B) = 1 - \theta_1 B^1 - \theta_2 B^2 - \cdots - \theta_q B^q$ is ordinary moving average component of order *q*;
*p, d, q* are ARIMA parameters; *P, D, Q* are SARIMA parameters;
*S* is season lag and *B* is the backshift operator (Shumway and Stoffer 2011).

Models for two variables are developed. The first variable (defined above) is the difference between the average return (from 96-month periods) on portfolio formed on *FUN* and benchmark portfolio. The second variable (defined above) is the difference between the *SR* value of *FUN*-formed portfolio and benchmark portfolio.

In each case, forecasts are made for 12- month time horizon. In order to check the statistical legitimacy of Conjectures 1 and 2, a 95% confidence interval is constructed. If the confidence interval for the forecasted differences contains zero, then the differences investigated in Conjectures 1 and 2 are statistically insignificant. This implies that hypothesis of market efficiency can’t be rejected. Our 12- month forecasts are updated every 12 months starting with 97th monthly observations. This is done for each tested market.

c) Rolling dynamic correlations, using 36-month window, between *FUN* portfolio and benchmark index returns.

Our cross-correlations are based on the intervals of the length *h* selected from both time series $R_t^B$ and $R_t^{FUN}$, where $R_t^B$ is the return on the benchmark index, while $R_t^{FUN}$ corresponds to the portfolio formed on *FUN* in period *t*. Given $\{(R_t^B, R_t^{FUN})\}_{t=T-h+1}^{T}$ we calculate the Pearson cross-correlation coefficient.

$$\hat{\rho}(T) = \frac{\hat{\gamma}(T)}{\hat{\sigma}^B \hat{\sigma}^{FUN}}, \tag{14}$$

where

$$\hat{\gamma}(T) = \frac{1}{h} \sum_{s=T-h+1}^{T} \left(R_s^B - \overline{R}^B\right)\left(R_s^{FUN} - \overline{R}^{FUN}\right), \tag{15}$$

$$\bar{R}^B = \frac{1}{h} \sum_{s=T-h+1}^{T} R_s^B, \tag{16}$$

$$\bar{R}^{FUN} = \frac{1}{h} \sum_{s=T-h+1}^{T} R_s^{FUN}, \tag{17}$$

$$\hat{\sigma}^B = \sqrt{\frac{1}{h-1} \sum_{s=T-h+1}^{T} \left(R_s^B - \overline{R}^B\right)}, \tag{18}$$

$$\hat{\sigma}^{FUN} = \sqrt{\frac{1}{h-1} \sum_{s=T-h+1}^{T} \left(R_s^{FUN} - \overline{R}^{FUN}\right)}. \tag{19}$$

The dynamic cross-correlation profiles are obtained moving $T$ along the data thus producing a sequence $\{\hat{\rho}(T)\}_{T=h}^{n}$.

The average capitalization values of the portfolios formed on *FUN* and the 50 big-cap companies of the benchmark index portfolio are calculated to confirm Conjecture 3. Stocks of selected indexes are weighted by market capitalization. Thus, stocks with high capitalization values determine the returns of the index. On the other hand, if *FUN*-formed portfolios consist of big-cap stocks, the returns of *FUN* and benchmark portfolios will be similar.

To assess the economic significance of *FUN* without referring to risk changes or the correct pricing of capital, we define cross-sectional relations between the expected returns and modeled functional *FUN*. Eq.(12) shows the impact of the average values of *FUN* determined at the end of periods *t-1* (*t=2;n*) on the average value of the returns determined from periods *t* (*t=1,n-1*).

$$\bar{r}_i = a_0 + a_1\overline{FUN}_i + \varepsilon_i; \;\; i = 1, \dots, m. \tag{20}$$

Table II shows the dependence of the average quintile/decile returns of portfolios (formed from 100 companies with maximum *FUN* values) on the average *FUN* values of these portfolios.

**Table II**

Values of the cross-sectional regression coefficients of the dependence of the average returns on the *FUN* values determined by the classic method of least squares

$$\bar{r}_i = a_0 + a_1\overline{FUN}_i + \varepsilon_i; \;\; i = 1, \dots, 5\ (quintiles)\ /10\ (deciles)$$

| $a_0$ | serr($a_0$) | *p-value* % | $a_1$ | serr($a_1$) | *p-value* % | $R^2$ % |
|---|---|---|---|---|---|---|
| Panel A: *S&P500*; tested period: 1996-2022; 312 months | | | | | | |
| 0.009/ 0.009 | 0.003/ 0.002 | 0.37/ 0.00 | 0.003/ 0.003 | 0.001/ 0.001 | 1.59/ 0.05 | 60.59/ 57.71 |
| Panel B: *BM500*; tested period: 2001-2022; 252 months | | | | | | |
| 0.006/ 0.007 | 0.003/ 0.002 | 0.69/ 0.00 | 0.004/ 0.004 | 0.001/ 0.001 | 0.01/ 0.00 | 82.16/ 79.30 |
| Panel C: *TOPIX500*; tested period: 2004-2022; 216 months | | | | | | |
| 0.000/ 0.004 | 0.003/ 0.004 | 46.00/ 19.88 | 0.004/ 0.003 | 0.001/ 0.002 | 0.04/ 5.60 | 78.80/ 24.02 |
| Panel D: *SSE380*; tested period: 2003-2022; 227 months | | | | | | |
| 0.004/ 0.004 | 0.004/ 0.002 | 17.04/ 3.29 | 0.006/ 0.005 | 0.002/ 0.001 | 0.01/ 0.00 | 82.80/ 78.12 |
| Panel E: *WIG*; tested period: 1996-2022; 312 months | | | | | | |
| 0.004/ 0.003 | 0.004/ 0.005 | 19.70/ 28.33 | 0.006/ 0.006 | 0.002/ 0.002 | 0.23/ 0.23 | 72.75/ 50.09 |

*Notes:* The study concerns 100 companies from the tested indexes with the highest *FUN* values; serr – standard error, $\overline{FUN_i}$ - average *FUN* value over time for quintile/decile portfolio *i* determined at the end of periods *t-1*; $\overline{r_i}$ – average return of quintile/decile portfolio *i* determined from periods *t*. The portfolios are linearly weighted. Source: own calculations.

Calculations of cross-sectional regressions indicate a linear and positive dependence of average returns on the values of *FUN*.

Table III shows summary statistics for tested benchmark indexes. Table IV shows summary statistics for portfolios formed on tested benchmark indexes.

**Table III**

Summary statistics for tested indexes

| Index | Mean (min; max), % | Median % | Standard deviations % | t-Statistic | Kurtosis | Skewness |
|---|---|---|---|---|---|---|
| Panel A: Monthly returns on tested historical period | | | | | | |
| *WIG* | 0.72 (-29.6; 23.7) | 0.62 | 6.68 | 1.90 | 2.14 | -0.15 |
| *S&P500* | 0.88 (-16.8; 12.8) | 1.34 | 4.38 | 3.58 | 1.10 | -0.60 |
| *BE500* | 0.51 (-21.1; 17.5) | 0.77 | 5.35 | 1.52 | 1.41 | -0.45 |
| *TOPIX500* | 0.43 (-14.4; 12.1) | 0.60 | 4.17 | 1.52 | 1.08 | -0.30 |
| *SSE380* | 1.48 (-29.2; 37.8) | 1.34 | 9.08 | 2.38 | 1.81 | 0.05 |
| Panel B: Moving average monthly returns on 96 months | | | | | | |
| *WIG* | 0.79 (-0.05; 1.76) | 0.78 | 0.47 | 16.5 | -0.91 | 0.30 |
| *S&P500* | 0.69 (-0.29; 1.47) | 0.65 | 0.39 | 26.3 | -0.62 | -0.09 |
| *BE500* | 0.57(0.08; 1.15) | 0.59 | 0.23 | 31.7 | 0.07 | 0.02 |
| *TOPIX500* | 0.44 (-0.04; 0.86) | 0.55 | 0.26 | 18.5 | -1.58 | -0.17 |
| *SSE380* | 1.41 (0.21; 2.59) | 1.28 | 0.66 | 24.8 | -1.22 | 0.09 |

*Notes*: Tested periods: *S&P500*: from May 1996 through March 2022; *BE500*: from May 2001 through March 2022; *TOPIX500*: from May 2004 through March 2022; *SSE380*: from May 2003 through March 2022; *WIG*: from May 1996 through March 2022. Source: own calculations.

**Table IV**

Summary statistics for portfolio formed on tested indexes

| Index | Mean (min; max), % | Median % | Stand. dev. % | t-Statistic | Kurtosis | Skewness |
|---|---|---|---|---|---|---|
| Panel A: Monthly returns on tested historical period | | | | | | |
| *AvPOR WIG* | 2.67 (-28.2; 53.2) | 1.93 | 10.39 | 4.53 | 2.34 | 0.52 |
| *AvPOR S&P500* | 2.04 (-15.9; 28.5) | 2.02 | 6.27 | 5.75 | 1.59 | 0.28 |
| *AvPOR BE500* | 1.95 (-28.6; 28.6) | 2.23 | 6.88 | 4.46 | 2.27 | -0.39 |
| *AvPOR TOPIX500* | 1.30 (-15.4; 15.6) | 1.47 | 5.37 | 3.54 | 0.44 | 0.05 |
| *AvPOR SSE380* | 2.38 (-27.8; 44.6) | 1.62 | 10.11 | 3.56 | 1.99 | 0.44 |
| Panel B: Moving average monthly returns on 96 months | | | | | | |
| *AvPOR WIG* | 2.96 (0.35; 6.22) | 2.89 | 1.60 | 18.17 | -1.19 | 0.28 |
| *AvPOR S&P500* | 1.96 (1.15; 2.95) | 1.87 | 0.46 | 62.35 | -0.97 | 0.39 |
| *AvPOR BE500* | 1.69 (1.01; 2.52) | 1.80 | 0.37 | 56.86 | -0.94 | -0.10 |

| | | | | | | |
|---|---|---|---|---|---|---|
| *AvPOR TOPIX500* | 1.31 (0.85; 1.76) | 1.30 | 0.20 | 71.46 | -0.74 | 0.20 |
| *AvPOR SSE380* | 2.28 (1.53; 3.02) | 2.27 | 0.36 | 73.90 | -1.02 | 0.00 |

*Notes*: *AvPOR TOPIX500*, *AvPOR BE500*, *AvPOR S&P500*, *AvPOR WIG* and *AvPOR SSE380* are linearly weighted portfolio consisting of 15, 10, 7 and 5 stock portfolios formed on *TOPIX500, BE500, S&P500, WIG* and *SSE380* indexes. Linearly weighted portfolios of 15, 10, 7 and 5 stocks are formed based on the maximization of the *FUN* of all stock of tested indexes. Tested periods *AvPOR S&P500*: from May 1996 through March 2022; *AvPOR BE500*: from May 2001 through March 2022; *AvPOR TOPIX500*: from May 2004 through March 2022; *AvPOR SSE380*: from May 2003 through March 2022; *AvPOR WIG*: from May 1996 through March 2022. Source: own calculations.

Tables III and IV represent basic summary statistics for 5 indexes and for 5 index portfolios. We can see that both in Table III and Table IV averaging the returns over 96 months leads to greater regularity. Kurtosis and skewness for averaged data for the most cases are closer to zero than for the raw data. The averaged portfolios are more regular than the raw ones.

## III. Results of calculation and analysis

In this section, we present an analysis of changes in the averaged rolling returns and rolling *SRs* of the examined markets. For the convenience of the reader we recall that each point of the figures below represents the start of 96- month investment period.

**U.S. Market**

Figure 1 shows rolling average returns (a), rolling correlations between *S&P500* formed portfolio and *S&P500* average returns (b), rolling *SRs* on 96 months of *S&P500* average portfolio (formed on 5, 7, 10 and 15 maximum values of *FUN*) and *S&P500* index (c) as well as rolling correlations between *SRs* of *S&P500* portfolios and *S&P500* index (d). The correlations are calculated using 36-month windows.

Fig. 1 a)

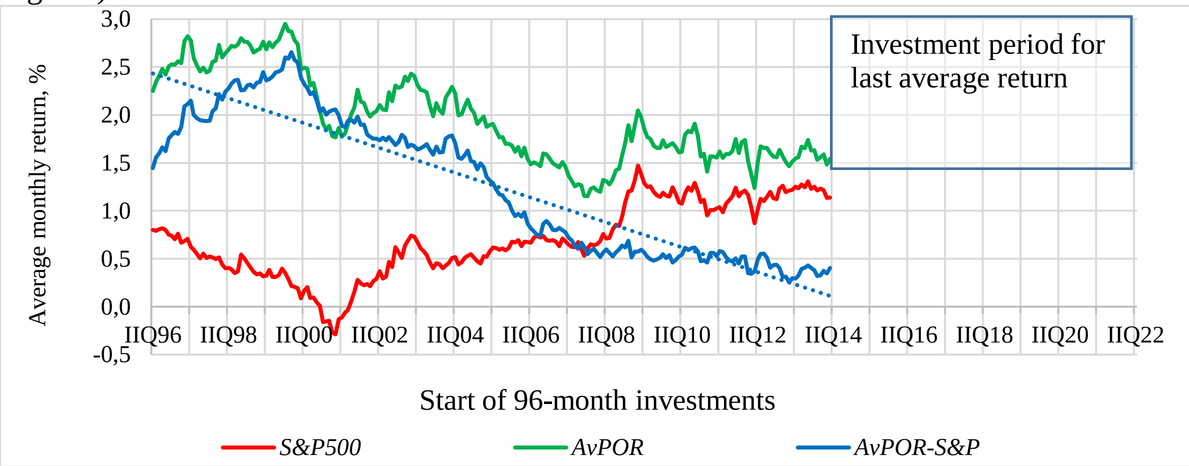


Fig. 1 b)

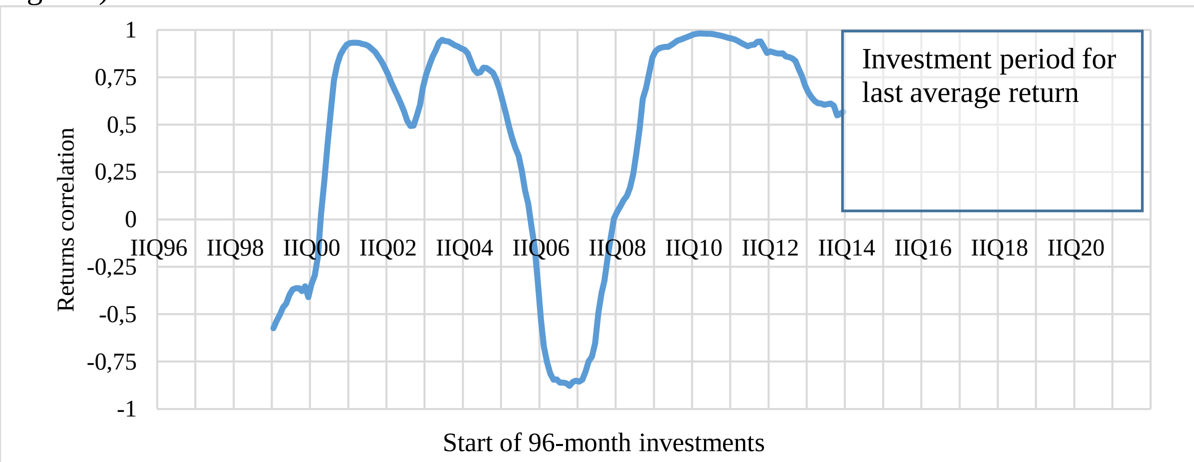


Fig. 1 c)

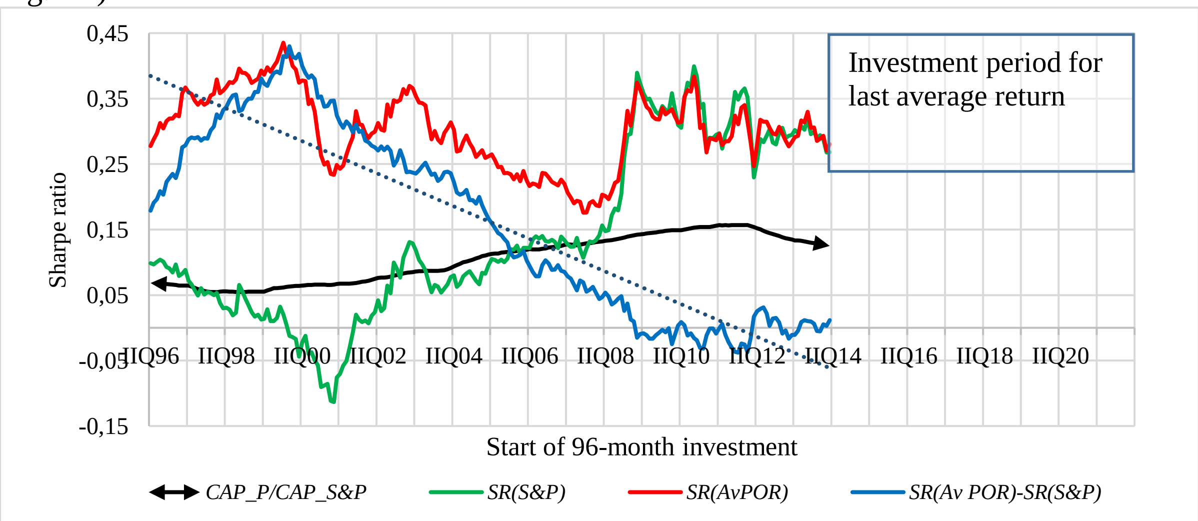

Fig. 1 d)

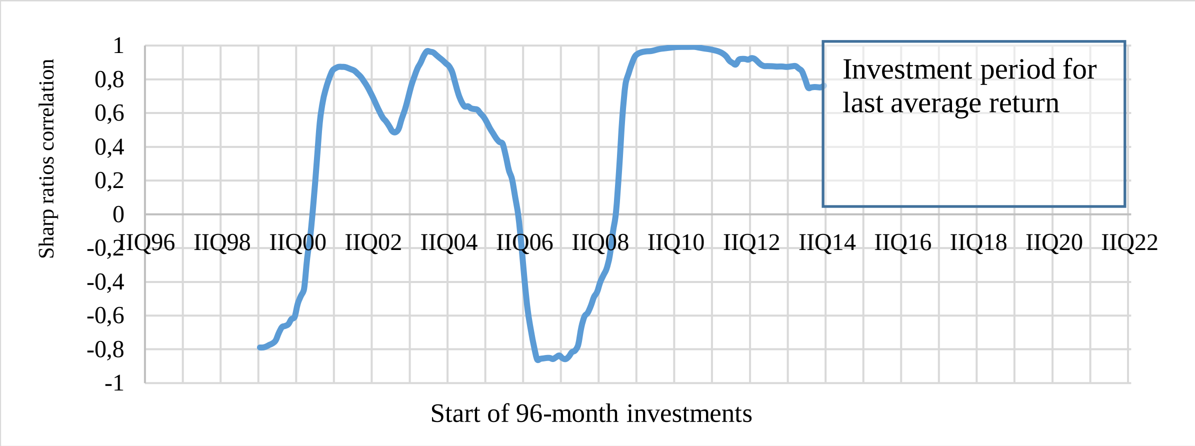


Fig. 1. Rolling a) average returns, b) correlations between *S&P500* formed portfolio and *S&P500* index average returns, c) *SR* on 96 months: of *S&P500* formed portfolio and *S&P500* index, d) correlations between the *SR* of *S&P500* formed portfolio and *S&P500* index.

*Notes*: *AvPOR* is a linearly weighted portfolio consisting of 15, 10, 7 and 5 stock portfolios formed on *S&P500* index. Each point of the chart represents the examined portfolio characteristics [average return, correlation between *AvPOR* and *S&P500* average returns, *SR*, correlation between *SR* of *AvPOR* and *SR* of *S&P500* index, and ratio of 15 companies portfolio capitalization (*CAP_P*) to the capitalization of 50 companies with the maximum capitalization of the index (*CAP_S&P*)] at the beginning of the 96-month investment period. *SR* of *AvPOR* is computed as the ratio of the average excess return on *AvPOR* over the average risk-free rate (*AvPOR-RF*) to the standard deviation of the excess return s(*AvPOR-RF*). *RF* is estimated based on U.S. 3-Month Treasury bills. *SR* of *S&P500* is computed as the ratio of the average excess return on *S&P500* over the average risk-free rate (*S&P500-RF*) to the standard deviation of the excess return s(*S&P500-RF*). Stock portfolios are formed based on the maximization of the *FUN* of all stock of *S&P500*. The dotted line in Figure a) shows the trend line of changes in the differences between the average return on *AvPOR* and the average return on the *S&P500* index. The dotted line in Figure c) shows the trend line of changes in the differences between the *SR* of *AvPOR* (*SR(AvPOR)*) and the *SR* of the *S&P500* index (*SR(S&P)*). The analyzed historical period from May 1996 through March 2022, 311 months. Source: own research.

The curves representing the time series of returns and *SRs* show changes over time of the formed portfolios and the entire index.

The maximum average returns on 96 monthly investments in the portfolios of 5, 7, 10 and 15 companies (with the highest *FUN* values) in the years 1996-2022 are respectively equal to: 45.9%, 43.3%, 39.2% and 38.7% (yearly).

Minimum average returns on 96 monthly investments in the portfolios of 5, 7, 10 and 15 companies (with the highest *FUN* values) in the years 1996-2022 are respectively equal to: 14.2%, 15.0%, 13.0% and 15.5% (yearly).

The maximum and minimum average return on 96 monthly investments in the *S&P500* index from 1996 to 2022 is equal to 19.2% and -3.4% (yearly).

Careful examination of the above figures indicates large differences between the return values corresponding to the formed portfolios and the index in the years 1996-2006. These differences are definitely decreasing since 2007, which is especially visible for the *SR*. The level of trend line of differences decreases from 2.5% to about 0% for average returns and from 0.4% to about 0% for *SRs* (see Figs. 1a and 1c).

The largest changes in the correlation coefficients of returns on formed portfolios and the *S&P500* index are found in the period 2005-2009. The correlation decreases monotonically from 0.8 in 2005 to -0.9 in 2007. Then it monotonically increases to values over 0.9 in 2009. In the following years, starting from 2010, the correlation reaches values close to 1. Similar changes in the correlation coefficient of the *SRs* of formed portfolios and the *S&P500* index are found throughout the analyzed period of 1996-2022 (see Figs. 1b, 1d).

Changes in the ratio of the capitalization of the formed portfolio of 15 companies to the capitalization of the 50 companies in the index (with maximum capitalization) indicate that the relative capitalization of the portfolio increases as of 2004, reaching the maximum of 15%. A slight decrease (about 2%) is observed after 2012 (see Fig. 1c). This fact may also partly explain the delayed occurrence of smaller differences between the returns of formed portfolios and the index after 2007.

In order to unambiguously assess the dynamics of time series related to formed portfolios and the *S&P500* index, we use other statistical methods. For this purpose, we study time series of the differences between the values of returns: *AvPOR-S&P* and the values of *SRs*: *SR*(*AvPOR*)-*SR*(*S&P*) of the formed portfolios and the values of the index.

Figure 2 shows SARIMA model in predicting rolling differences between the average returns and *SRs*, corresponding to the formed portfolios and the values of *S&P500* index.

Fig. 2 a)

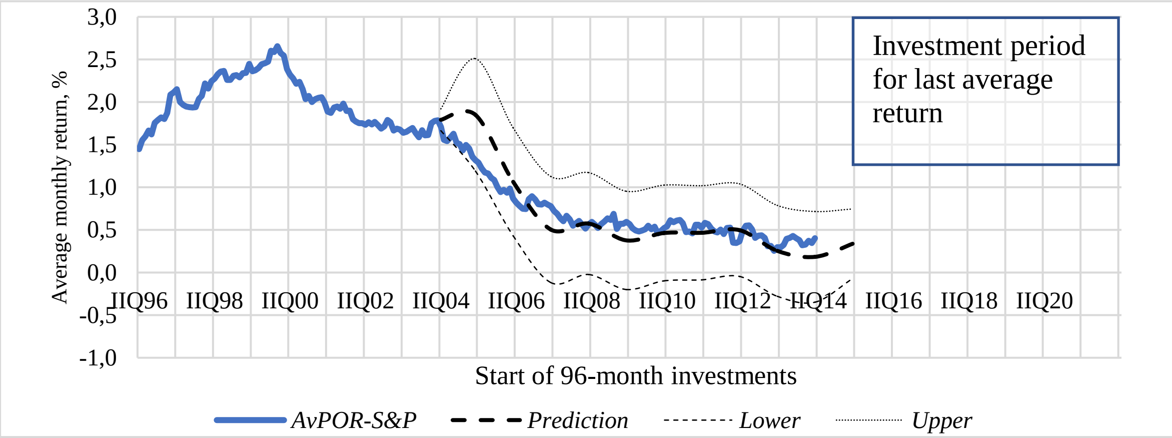


Fig. 2 b)

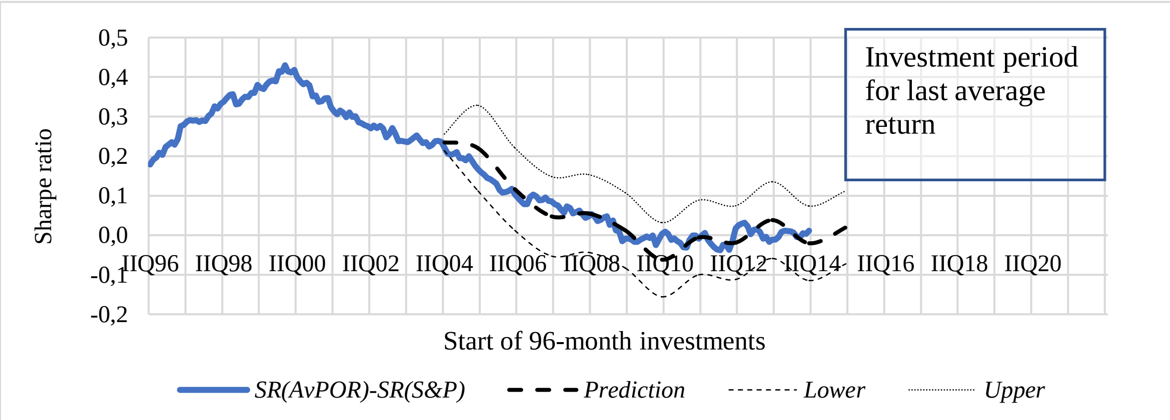


Fig. 2. SARIMA and prediction of rolling differences between a) average returns, b) *SR* on 96 months, corresponding to the formed portfolios and the values of *S&P500* index.

*Notes*: Each point of the chart presents the examined portfolio characteristics (average return and *SR*) at the beginning of the 96-month investment period. *SR* is computed as the ratio of the average excess return from the examined portfolio over the average risk-free rate (*Rp-RF*) to the standard deviation of the excess return s(*Rp-RF*). *RF* is estimated based on U.S. 3-Month Treasury bills. *AvPOR* is a linearly weighted portfolio consisting of 15, 10, 7 and 5 stock portfolios formed on *S&P500* index. Stock portfolios are formed based on the maximization of the *FUN* of all stock of *S&P500*. The dashed thin top and bottom lines are the upper and lower bounds of the forecast confidence intervals. The dashed thick line represents the forecast values of rolling return difference – in Figure a), and *SR* difference – in Figure b). The analyzed historical period from May 1996 through March 2022, 311 months. Source: Own research using R-Studio ver. 4.2.3.

The order of SARIMA model is chosen according to the AIC coefficient criterion (for more on AIC, see Shumway and Stoffer (2011)). The estimates of the parameters in this model are presented in Table V.

**Table V**

SARIMA model parameters, base period and prediction period for rolling differences between average returns and *SR* on 96 months, corresponding to the formed portfolios and the values of *S&P500* index.

| SARIMA parameters for season *S*=12 Months | | | | | | Base period | Prediction period | SARIMA coefficients | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| *p* | *d* | *q* | *P* | *D* | *Q* | | | const | $\varphi_1$ | $\theta_1$ | $\Phi_1$ | $\Phi_2$ |
| Average returns differences | | | | | | | | | | | | |
| 0 | 2 | 1 | | | | 05.96-04.04 | 05.04-04.05 | | | -0.9166*** | | |
| 0 | 2 | 1 | | | | 05.96-04.05 | 05.05-04.06 | | | -0.9299*** | | |
| 0 | 2 | 1 | | | | 05.96-04.06 | 05.06-04.07 | | | -0.9275*** | | |
| 0 | 2 | 1 | | | | 05.96-04.07 | 05.07-04.08 | | | -0.9326*** | | |
| 0 | 2 | 1 | | | | 05.96-04.08 | 05.08-04.09 | | | -0.9361*** | | |
| 0 | 2 | 1 | 2 | 0 | 0 | 05.96-04.09 | 05.09-04.10 | | | -0.9434*** | -0.0077 | -0.0396 |
| 1 | 2 | 1 | | | | 05.96-04.10 | 05.10-04.11 | | -0.0243 | -0.9372*** | | |
| 1 | 2 | 1 | | | | 05.96-04.11 | 05.11-04.12 | | -0.0199 | -0.9399*** | | |
| 1 | 2 | 1 | | | | 05.96-04.12 | 05.12-04.13 | | -0.0257 | -0.9434*** | | |
| 1 | 2 | 1 | | | | 05.96-04.13 | 05.13-04.14 | | -0.0034 | -0.9491*** | | |
| 0 | 1 | 0 | 2 | 0 | 0 | 05.96-04.14 | 05.14-04.15 | -0.005 | | | -0.0409 | 0.0096 |
| *SR* differences | | | | | | | | | | | | |
| 0 | 2 | 1 | | | | 05.96-04.04 | 05.04-04.05 | | | -0.9160*** | | |
| 0 | 2 | 1 | | | | 05.96-04.05 | 05.05-04.06 | | | -0.9209*** | | |
| 0 | 2 | 1 | | | | 05.96-04.06 | 05.06-04.07 | | | -0.9218*** | | |
| 0 | 2 | 1 | | | | 05.96-04.07 | 05.07-04.08 | | | -0.9235*** | | |
| 0 | 2 | 1 | | | | 05.96-04.08 | 05.08-04.09 | | | -0.9279*** | | |
| 0 | 2 | 1 | 1 | 0 | 0 | 05.96-04.09 | 05.09-04.10 | | | -0.9376*** | 0.0950 | |
| 0 | 2 | 1 | 1 | 0 | 0 | 05.96-04.10 | 05.10-04.11 | | | -0.9376*** | 0.0603 | |
| 0 | 2 | 1 | 1 | 0 | 0 | 05.96-04.11 | 05.11-04.12 | | | -0.9398*** | 0.0409 | |
| 0 | 2 | 1 | 1 | 0 | 0 | 05.96-04.12 | 05.12-04.13 | | | -0.9385*** | 0.0333 | |
| 0 | 2 | 1 | 1 | 0 | 0 | 05.96-04.13 | 05.13-04.14 | | | -0.9447*** | 0.0339 | |
| 0 | 2 | 1 | 1 | 0 | 0 | 05.96-04.14 | 05.14-04.15 | | | -0.9456*** | 0.0443 | |

*Notes*: Significance of coefficients: *** - p-value <0.01, ** - p-value<0.05, * - p-value<0.1. The parameters *p,d,q* correspond to ARIMA model while *P,D,Q* are related to its seasonal component. For more details see formula (5) and also formula (3.160) in Shumway and Stoffer (2011). Source: Own calculations using R-Studio ver. 4.2.3.

Available data allows for calculating forecasts and their confidence intervals for differences of returns and *SRs* since Q2 2004.

The SARIMA model shows that the confidence interval of the forecasted differences of *SRs* contains zero starting from Q2 2006 (see Fig. 2b). In our opinion this provides evidence that there is no statistically significant difference between the *SR* of *FUN* portfolio and *S&P500* benchmark portfolio. Therefore, we may suppose that Conjecture 2 should be rejected. Importantly, the latest forecast shows that the difference between the *SRs* of formed portfolios and the *S&P500* portfolio head towards zero.

The market's move towards efficiency is also illustrated by the differences of returns. The confidence interval of the forecasted return differences contains zero starting from Q4 2006 (see Fig. 2a). Therefore, we may suppose that Conjecture 1 should be rejected.

Figure 3 shows Holt-Winters method applied to predict rolling differences between the average returns and *SRs*, corresponding to the formed portfolios and the values of *S&P500* index.

Fig. 3 a)

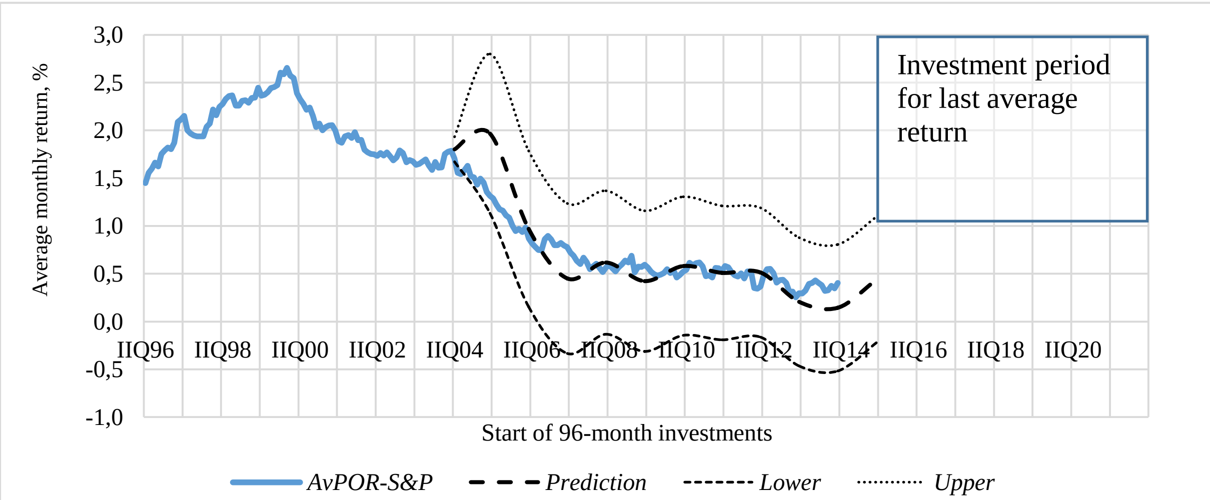

Fig. 3 b)

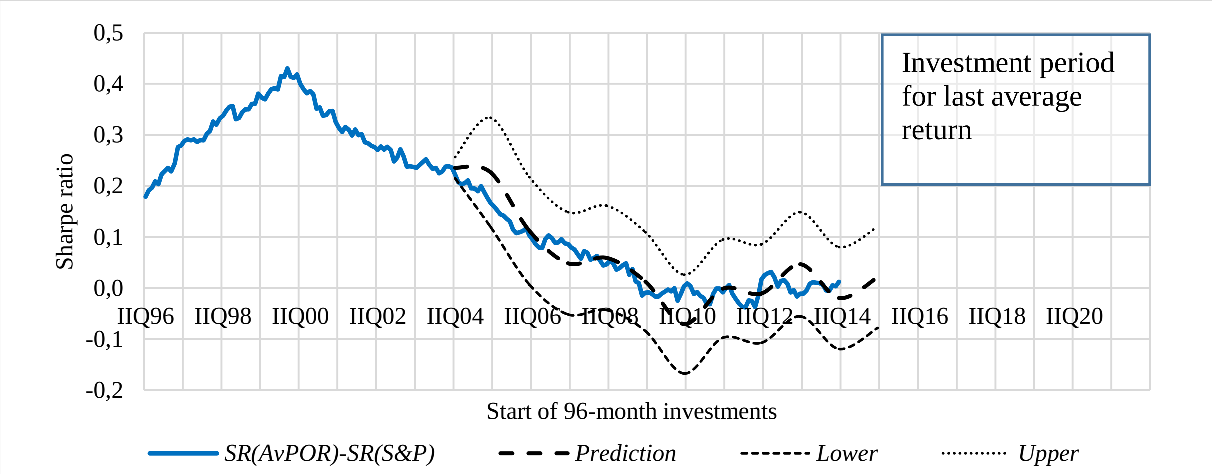


Fig. 3. Holt-Winters method to predict rolling differences between a) average returns, b) *SR* on 96 months, corresponding to the formed portfolios and the values of *S&P500* index.

*Notes*: Each point of the chart presents the examined portfolio characteristics (average return and *SR*) at the beginning of the 96-month investment period. *SR* is computed as the ratio of the average excess return from the examined portfolio over the average risk-free rate (*Rp-RF*) to the standard deviation of the excess return s(*Rp-RF*). *RF* is estimated based on U.S. 3-Month Treasury bills. *AvPOR* is a linearly weighted portfolio consisting of 15, 10, 7 and 5 stock portfolios formed on *S&P500* index. Stock portfolios are formed based on the maximization of the *FUN* of all stock of *S&P500*. The dashed thin top and bottom lines are the upper and lower bounds of the forecast confidence intervals. The dashed thick line represents the forecast values of rolling return difference – in Figure a), and *SR* difference – in Figure b). The analyzed historical period from May 1996 through March 2022, 311 months. Source: Own research using ITSM ver. 7.1.

In Table VI we present essential parameters of the applied Holt-Winters method.

**Table VI**

Holt-Winters method parameters, base period and prediction period for rolling differences between average returns and *SR* on 96 months, corresponding to the formed portfolios and the values of *S&P500* index

| Holt-Winters parameters | | Base period | Prediction period |
|---|---|---|---|
| *α* | *β* | | |
| Average returns differences | | | |
| 0.97 | 0.15 | 05.1996-04.2004 | 05.2004-04.2005 |
| 1 | 0.13 | 05.1996-04.2005 | 05.2005-04.2006 |
| 0.99 | 0.13 | 05.1996-04.2006 | 05.2006-04.2007 |
| 1 | 0.12 | 05.1996-04.2007 | 05.2007-04.2008 |
| 1 | 0.12 | 05.1996-04.2008 | 05.2008-04.2009 |
| 0.98 | 0.12 | 05.1996-04.2009 | 05.2009-04.2010 |
| 0.97 | 0.12 | 05.1996-04.2010 | 05.2010-04.2011 |
| 0.98 | 0.11 | 05.1996-04.2011 | 05.2011-04.2012 |
| 0.97 | 0.11 | 05.1996-04.2012 | 05.2012-04.2013 |
| 0.99 | 0.1 | 05.1996-04.2013 | 05.2013-04.2014 |
| 1 | 0.1 | 05.1996-04.2014 | 05.2014-04.2015 |
| *SR* differences | | | |
| 0.86 | 0.12 | 05.1996-04.2004 | 05.2004-04.2005 |
| 0.89 | 0.11 | 05.1996-04.2005 | 05.2005-04.2006 |
| 0.91 | 0.1 | 05.1996-04.2006 | 05.2006-04.2007 |
| 0.94 | 0.1 | 05.1996-04.2007 | 05.2007-04.2008 |
| 0.94 | 0.09 | 05.1996-04.2008 | 05.2008-04.2009 |
| 0.88 | 0.1 | 05.1996-04.2009 | 05.2009-04.2010 |
| 0.87 | 0.1 | 05.1996-04.2010 | 05.2010-04.2011 |
| 0.91 | 0.09 | 05.1996-04.2011 | 05.2011-04.2012 |
| 0.97 | 0.08 | 05.1996-04.2012 | 05.2012-04.2013 |
| 0.99 | 0.07 | 05.1996-04.2013 | 05.2013-04.2014 |
| 1 | 0.07 | 05.1996-04.2014 | 05.2014-04.2015 |

*Notes*: *α*, *β* – for more details see formula (3) and Chatfield (1978). Source: Own calculations using ITSM ver. 7.1.

The Holt-Winters method shows that the confidence interval of the forecasted differences of *SRs* contains zero continuously as of Q2 2006 (see Fig. 3b). Therefore, it should be stated that in these periods there are grounds for rejecting Conjecture 2. Importantly, the latest forecast shows that the differences between the *SRs* of formed portfolios and the *S&P500* portfolio tend towards zero. These results are very close to those using the SARIMA model.

Similarly, the market's move towards efficiency is illustrated by the return differences. The confidence interval of the forecasted return differences approaches zero starting from Q2 2006 (see Fig. 3a). This indicates the rejection of Conjecture 1. Also, these results are consistent with the results using the SARIMA model.

**European Market**

Figure 4 shows rolling average returns (a), rolling correlations between *BE500* formed portfolio and *BE500* average returns (b), rolling *SR* on 96 months of *BE500* average portfolio (formed on 5, 7, 10 and 15 maximum values of *FUN*) and *BE500* index (c) as well as rolling correlations between *SRs* of *BE500* portfolios and *BE500* index (d). The correlations are calculated using 36-month windows.

Fig. 4 a)

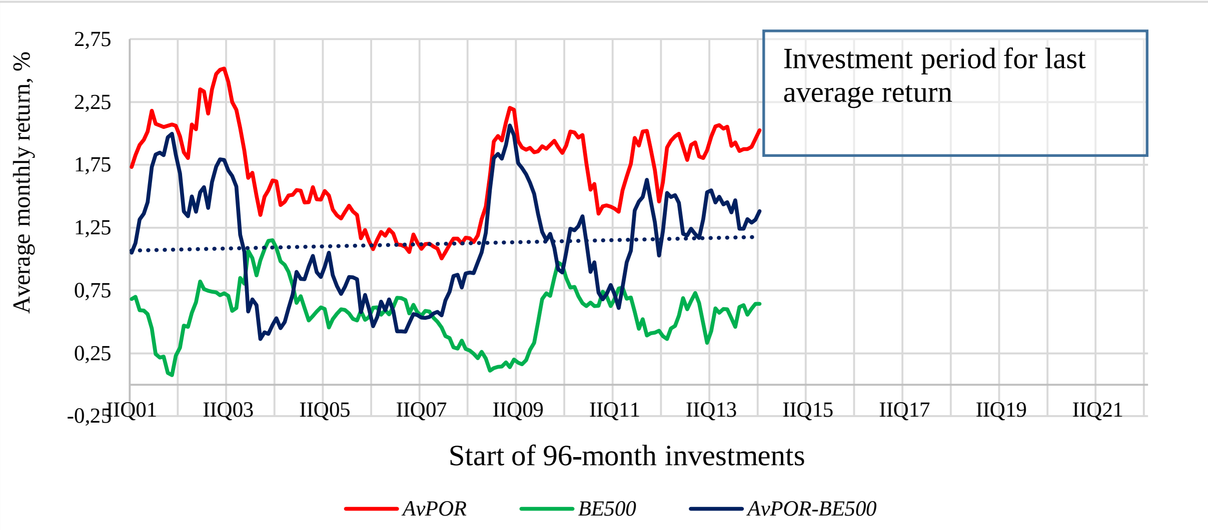


Fig. 4 b)

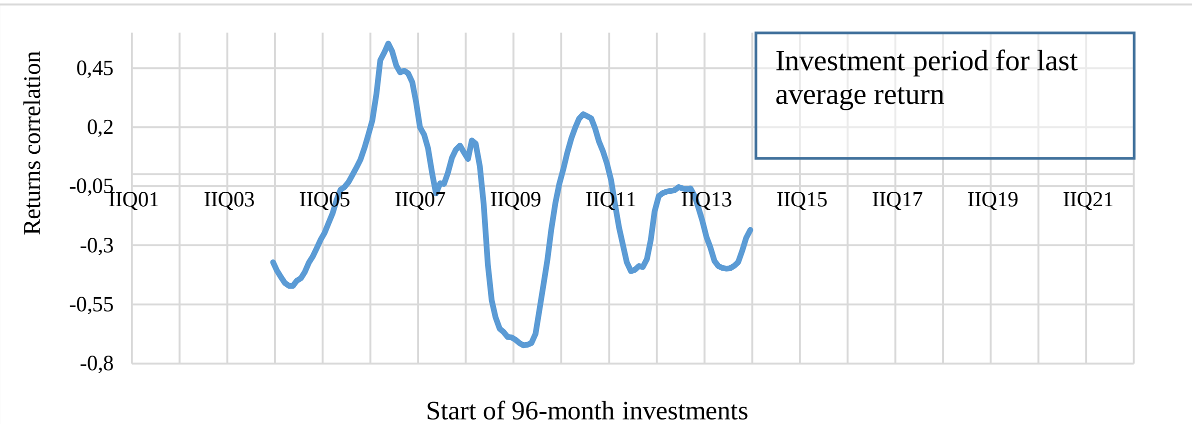

Fig. 4 c)

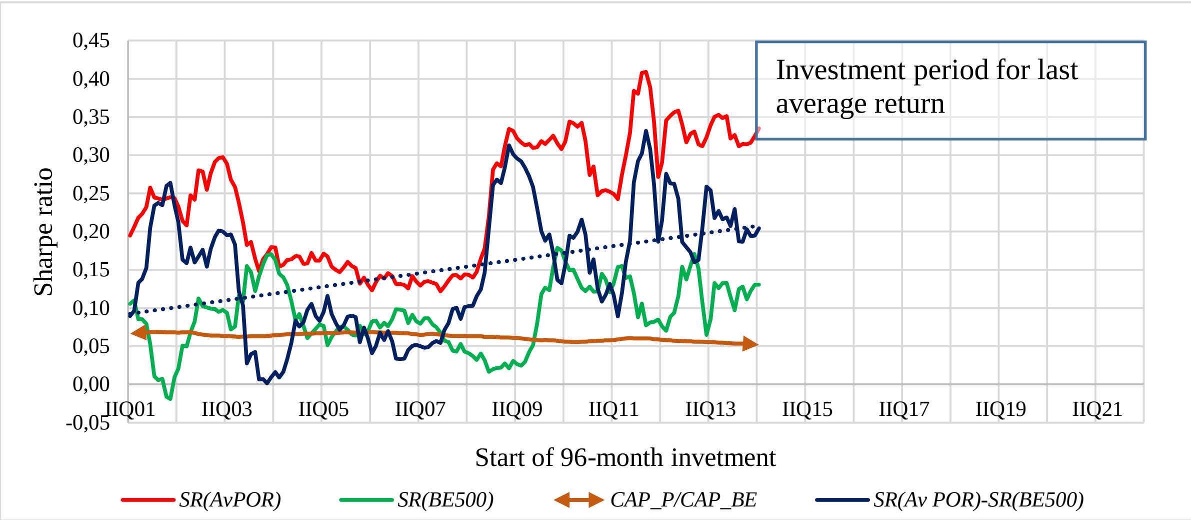


Fig. 4 d)

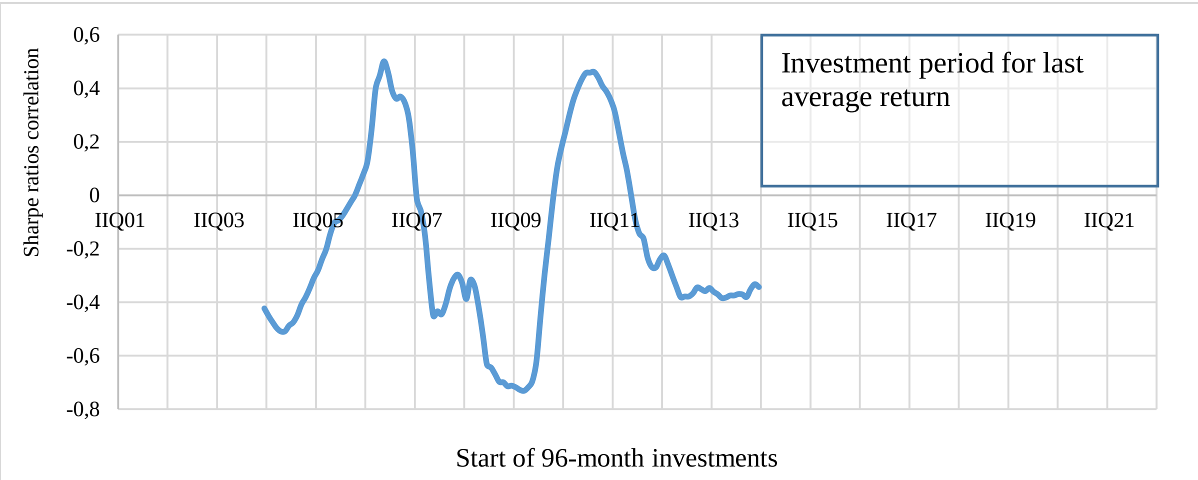


Fig. 4. Rolling a) average returns, b) correlations between *BE500* formed portfolio and *BE500* index average returns, c) *SR* on 96 months: of *BE500* formed portfolio and *BE500* index, d) correlations between *SR* of *BE500* formed portfolio and *BE500* index.

*Notes*: *AvPOR* is a linearly weighted portfolio consisting of 15, 10, 7 and 5 stock portfolios formed on *BE500* index. Each point of the chart represents the examined portfolio characteristics [average return, correlations between *AvPOR* and *BE500* average returns, *SR*, correlations between *SR* of *AvPOR* and *SR* of *BE500* index, and ratio of 15 companies portfolio capitalization (*CAP_P*) to the capitalization of 50 companies with the maximum capitalization of the index (*CAP_BE*)] at the beginning of the 96-month investment period. *SR* of *AvPOR* is computed as the ratio of the average excess return on *AvPOR* over the average risk-free rate (*AvPOR-RF*) to the standard deviation of the excess return s(*AvPOR-RF*). *RF* is estimated based on U.S. 3-Month Treasury bills. *SR* of *BE500* is computed as the ratio of the average excess return on *BE500* over the average risk-free rate (*BE500-*

*RF*) to the standard deviation of the excess return s(*BE500-RF*). Stock portfolios are formed based on the maximization of the *FUN* of all stock of *BE500*. The dotted line in Figure a) shows the trend line of changes in the differences between the average return on *AvPOR* and the average return on the *BE500* index. The dotted line in Figure c) shows the trend line of changes in the differences between the *SR* of *AvPOR* (*SR(AvPOR*)) and the *SR* of the *BE500* index (*SR(BE500*)). The analyzed historical period from May 2001 through March 2022, 251 months. Source: Own research.

The curves representing the time series of return and *SR* show changes over time of the formed portfolios and the entire index.

The maximum average returns on 96 monthly investments in the portfolios of 5, 7, 10 and 15 companies (with the highest *FUN* values) in the years 2001-2022 are respectively equal to: 35.2%, 34.5%, 36.0% and 33.6% (yearly).

Minimum average returns on 96 monthly investments in portfolios of 5, 7, 10 and 15 companies (with the highest *FUN* values) in the years 2001-2022 are respectively equal to: 12.2%, 12.7%, 14.6% and 11.6% (yearly).

The maximum and minimum average return on 96 monthly investments in the *BE500* index in the years 2001-2022 is equal to: 14.7% and 0.9% (yearly).

Careful examination of the above figures indicates large differences between the values of returns of the formed portfolios and the index portfolio. The characteristic trend line of differences in returns is practically constant and amounts to about 1% per month. For the *SR*, the trend line increases from 0.1 to 0.2 (see Figs. 4a and 4c).

Periodic changes in the correlation coefficient of returns of formed portfolios and the *BE500* index are observed as of 2005. The correlation coefficient decreases from about 0.6 in 2007 to -0.6 in 2009. These changes are opposite to the changes in the U.S. market. As of 2011, changes in the correlation decrease to a range of 0.2 to -0.4. Similarly, the largest changes in the correlation of the *SRs* of formed portfolios and the *BE500* index are found throughout the analyzed period of 2001-2022 (see Figs. 4b, 4d).

In contrast to the U.S. market, the relative capitalization of the *FUN*-formed portfolio is approximately constant at around 6% (see Fig 4c). This fact shows that the 15 companies with maximum *FUN* values are companies with relatively smaller capitalizations (in relation to the *BE500* index) compared to the U.S. market.

We study time series of the differences between the values of returns: *AvPOR-BE500*, and the values of *SR*: *SR*(*AvPOR*)-*SR*(*BE500*) of the formed portfolios and the values of the index.

Figure 5 shows SARIMA model in predicting rolling differences between the average returns and *SRs*, corresponding to the formed portfolios and the values of *BE500* index.

Fig. 5 a)

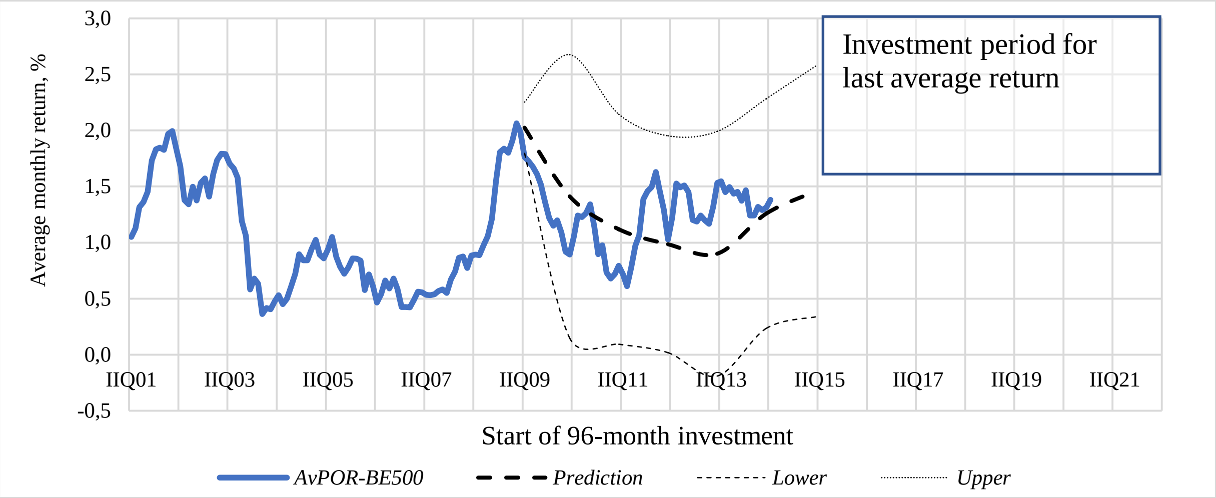


Fig. 5 b)

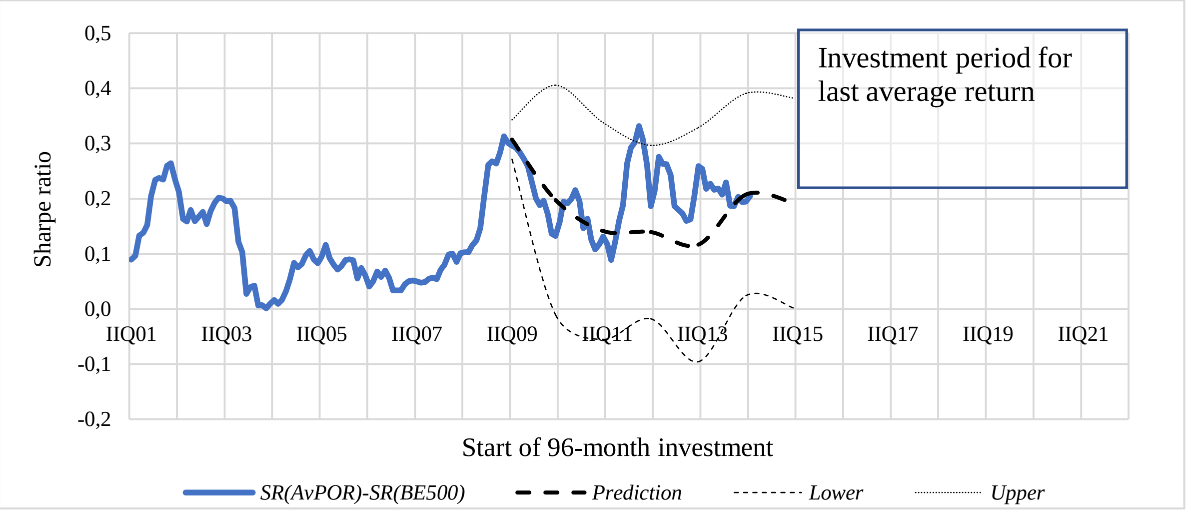


Fig. 5. SARIMA and prediction of rolling differences between a) average returns, b) *SR* on 96 months, corresponding to the formed portfolios and the values of *BE500* index.

*Notes*: Each point of the chart presents the examined portfolio characteristics (average return and *SR*) at the beginning of the 96-month investment period. *SR* is computed as the ratio of the average excess return from the examined portfolio over the average risk-free rate (*Rp-RF*) to the standard deviation of the excess return s(*Rp-RF*). *RF* is estimated based on U.S. 3-Month Treasury bills. *AvPOR* is a linearly weighted portfolio consisting of

15, 10, 7 and 5 stock portfolios formed on *BE500* index. Stock portfolios are formed based on the maximization of the *FUN* of all stock of *BE500*. The dashed thin top and bottom lines are the upper and lower bounds of the forecast confidence intervals. The dashed thick line represents the forecast values of rolling return difference – in Figure a), and *SR* difference – in Figure b). The analyzed historical period from May 2001 through March 2022, 251 months. Source: Own research using R-Studio ver. 4.2.3.

Available data is used to calculate confidence intervals for forecasts of differences in returns and *SRs* as of Q2 2009.

The SARIMA model shows that the confidence interval of the forecasted differences of *SRs* contains zero starting from Q2 2010 for the most cases (see Fig. 5b). Similarly to the U.S. market it should be stated that in these periods there are grounds for rejecting Conjecture 2 for the European market. Moreover, the market's move towards efficiency is also illustrated by the differences of returns (see Fig. 5a). This indicates the rejection of Conjecture 1.

Figure 6 shows Holt-Winters method in predicting rolling differences between the average returns and *SRs*, corresponding to the formed portfolios and the values of *BE500* index.

Fig. 6 a)

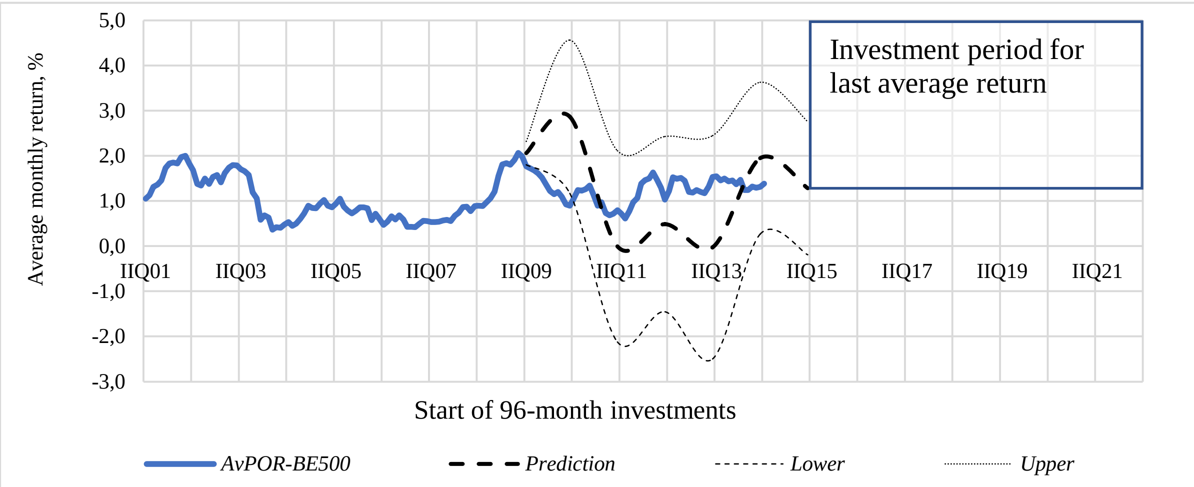

Fig 6 b)

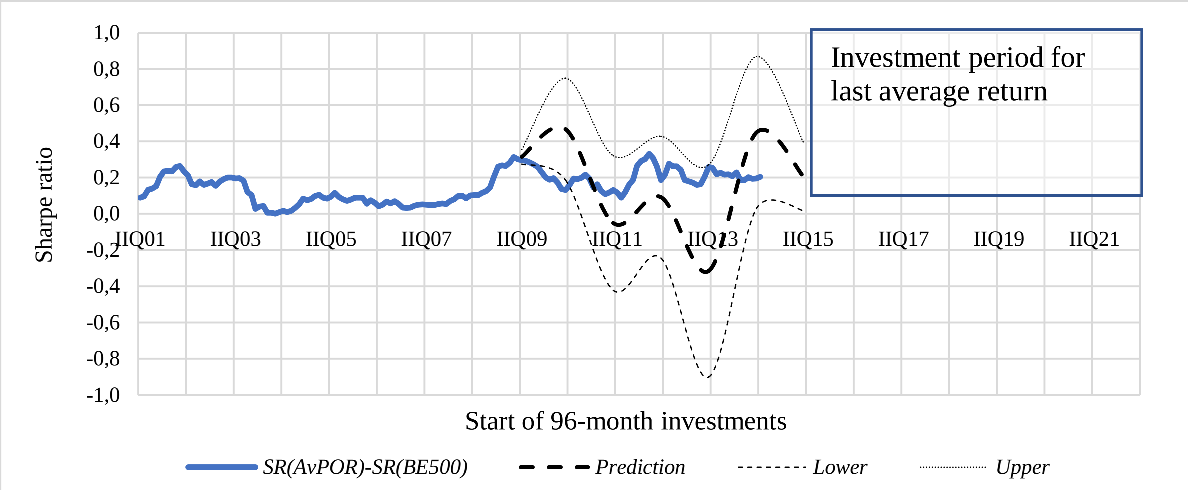


Fig. 6. Holt-Winters method to predict rolling differences between a) average returns, b) *SR* on 96 months, corresponding to the formed portfolios and the values of *BE00* index.

*Notes*: Each point of the chart presents the examined portfolio characteristics (average return and *SR*) at the beginning of the 96-month investment period. *SR* is computed as the ratio of the average excess return from the examined portfolio over the average risk-free rate (*Rp-RF*) to the standard deviation of the excess return s(*Rp-RF*). *RF* is estimated based on U.S. 3-Month Treasury bills. *AvPOR* is a linearly weighted portfolio consisting of 15, 10, 7 and 5 stock portfolios formed on *BE500* index. Stock portfolios are formed based on the maximization of the *FUN* of all stock of *BE500*. The dashed thin top and bottom line are the upper and lower bounds of the forecast confidence intervals. The dashed thick line represents the forecast values of rolling return difference – in Figure a), and *SR* difference – in Figure b). The analyzed historical period from May 2001 through March 2022, 251 months. Source: Own research using ITSM ver. 7.1.

The Holt-Winters method shows that the confidence interval of the forecasted differences of *SRs* contains zero from Q3 2010 for the most cases (see Fig. 6b). Conclusions are similar as in the case of SARIMA model for this market. This leads to rejecting Conjecture 2. Similarly, the market's move towards efficiency is illustrated by the differences of returns (see Fig. 6a). This indicates the rejection of Conjecture 1. These results are consistent with the results using the SARIMA model.

Essential parameters of the SARIMA models and the Holt-Winters method can be made available upon request.

**Japanese Market**

Figure 7 shows rolling average returns (a), rolling correlations between formed portfolio and *TOPIX500* average returns (b), rolling *SR* on 96 months of *TOPIX500* average portfolios (formed on 5, 7, 10 and 15 maximum values of *FUN*) and *TOPIX500* index (c) as well as rolling correlations between *SRs* of *TOPIX500* portfolios and *TOPIX500* index (d). The correlations are calculated using 36-month windows.

Fig. 7 a)

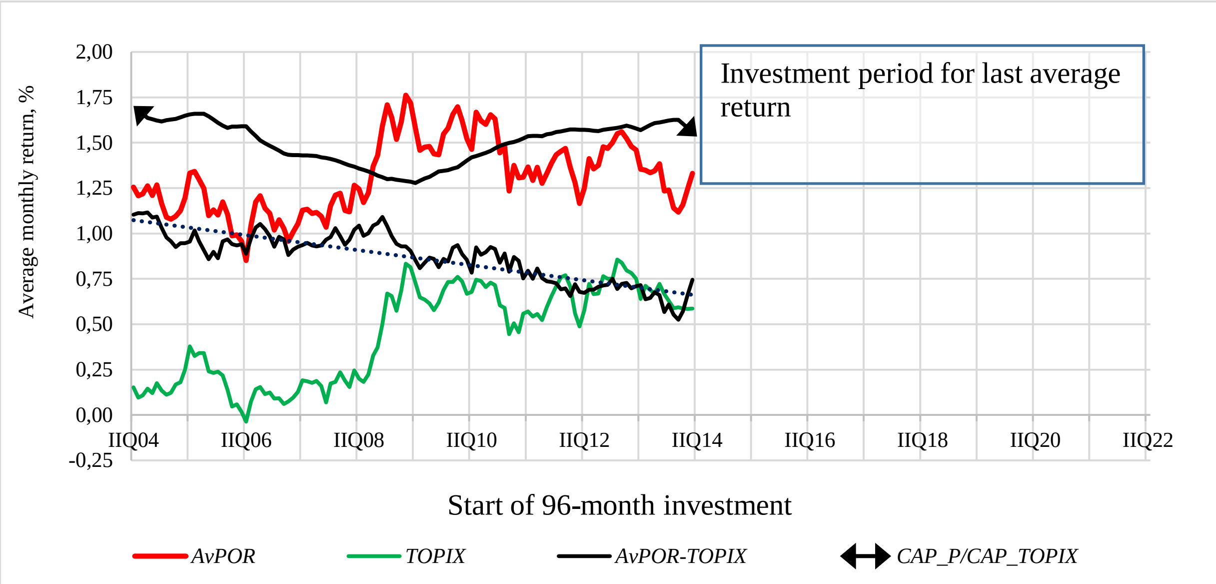


Fig. 7 b)

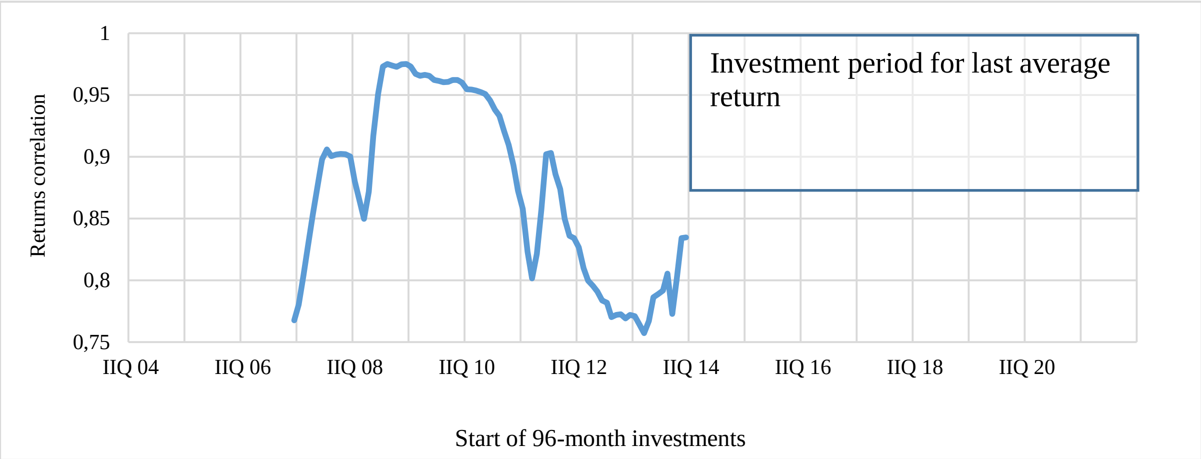

Fig. 7 c)

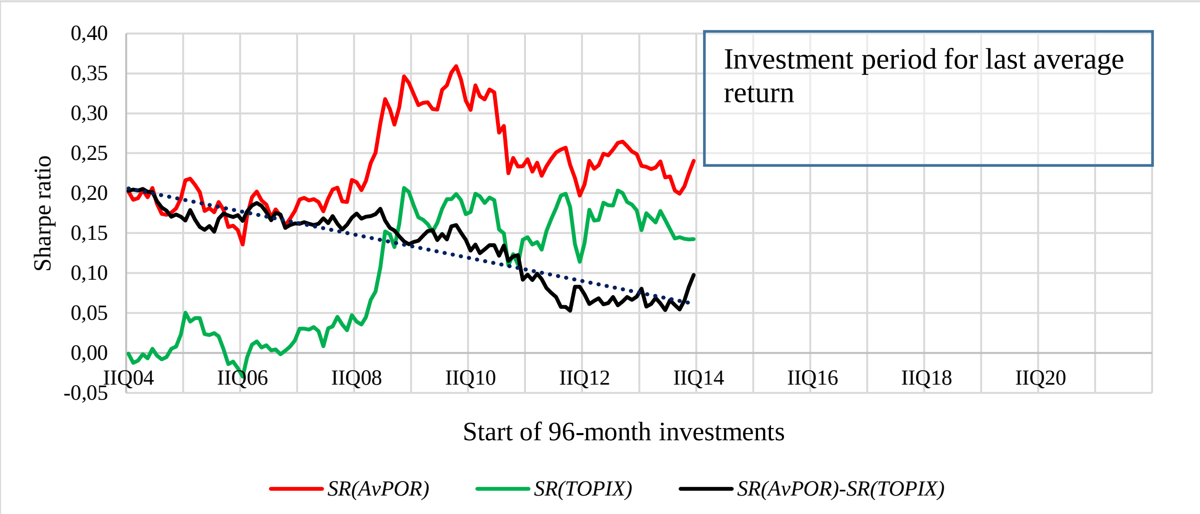


Fig. 7 d)

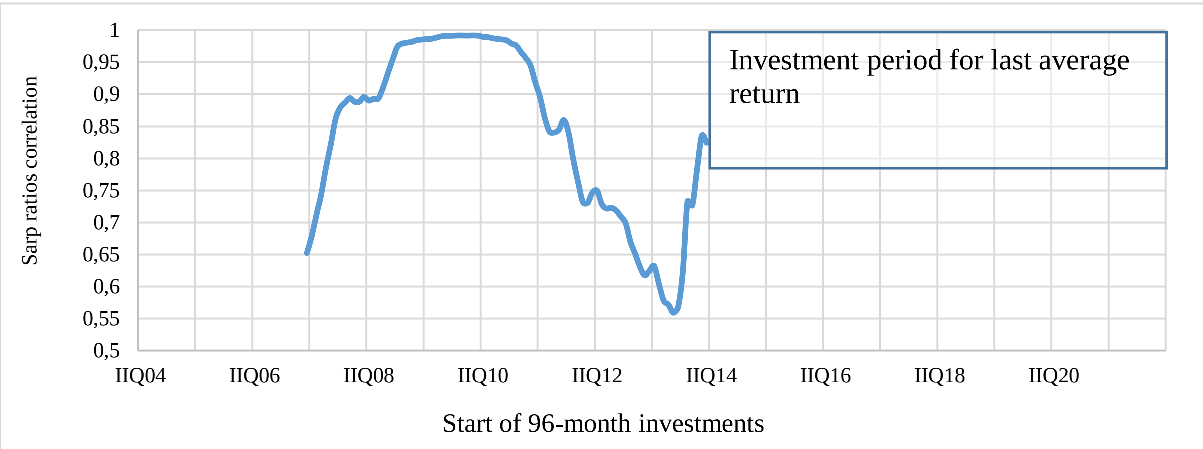


Fig. 7. Rolling a) average returns, b) correlations between *TOPIX500* formed portfolio and *TOPIX500* index average returns, c) *SR* on 96 months: of *TOPIX500* formed portfolio and *TOPIX500* index, d) correlations between *SR* of *TOPIX500* formed portfolio and *TOPIX500* index.

*Notes*: *AvPOR* is a linearly weighted portfolio consisting of 15, 10, 7 and 5 stock portfolios formed on *TOPIX500* index. Each point of the chart represents the examined portfolio characteristics [average return, correlations between *AvPOR* and *TOPIX500* averaged returns, *SR*, correlations between *SR* of *AvPOR* and *SR* of *TOPIX500* index, and ratio of 15 companies portfolio capitalization (*CAP_P*) to the capitalization of 50 companies with the maximum capitalization of the index (*CAP_ TOPIX*)] at the beginning of the 96-month investment period. *SR* of *AvPOR* is computed as the ratio of the average excess return on *AvPOR* over the average risk-free rate (*AvPOR-RF*) to the standard deviation of the excess return s(*AvPOR-RF*). *RF* is estimated based on U.S. 3-Month Treasury bills. *SR* of *TOPIX500* is computed as the ratio of the average excess return on *TOPIX500* over the average risk-free rate (*TOPIX500-RF*) to the standard deviation of the excess return s(*TOPIX500-RF*). Stock portfolios are formed based on the maximization of the *FUN* of all stock of *TOPIX500*. The dotted line in Figure a) shows the trend line of changes in the differences between the average return on *AvPOR* and the average return on the *TOPIX500* index. The dotted line in Figure c) shows the trend line of changes in the differences between the *SR* of *AvPOR* (*SR(AvPOR*)) and the *SR* of the *TOPIX500* index (*SR(TOPIX*)). The analyzed historical period from May 2004 through March 2022, 215 months. Source: Own research.

The curves representing the time series of return and *SR* show changes over time of the formed portfolios and the entire index.

The maximum average returns on 96 monthly investments in the portfolios of 5, 7, 10 and 15 companies (with the highest *FUN* values) in the years 2004-2022 are respectively equal to: 22.7%, 24.3%, 23.0% and 23.5% (yearly).

Minimum average returns on 96 monthly investments in portfolios of 5, 7, 10 and 15 companies (with the highest *FUN* values) in the years 2004-2022 are respectively equal to: 12.0%, 11.8%, 9.5% and 9.6% (yearly).

The maximum and minimum average return on 96 monthly investments in the *TOPIX500* index in the years 2004-2022 is equal to: 10.8% and -0.4% (yearly).

Careful examination of the above figures indicates large differences between the values of returns of the formed portfolios and the index portfolio. However, the characteristic trend line of differences in returns decreases from about 1% to 0.7% per month. Also, for the *SR* the trend line decreases from 0.22 to 0.05 (see Figs. 7a and 7c).

A strong increase in the correlation coefficient, of returns (of formed portfolios and the *TOPIX500*) from about 0.75 to 0.95 is observed in the years 2007-2009. Thereafter, the correlation remained stable until 2011. In the following years, the correlation decreases to about 0.8. Similarly, the largest changes in the correlation of the *SRs* (of formed portfolios and the *TOPIX500*) are found throughout the analyzed period of 2004-2022 (see Figs. 7b, 7d).

In contrast to the U.S. market, the relative capitalization of the *FUN*-formed portfolio varies in the range from 1.25% to 1.75%. This fact shows that the 15 companies with maximum *FUN* values are companies with relatively smaller capitalizations (in relation to the *TOPIX500* index) compared to the U.S. market.

Using similar methods as for U.S. and European Markets, we study time series of the differences between the values of returns: *AvPOR-TOPIX*, and the values of *SRs*: *SR*(*AvPOR*)-*SR*(*TOPIX*) of the formed portfolios and the values of the index.

Figure 8 shows SARIMA model in predicting rolling differences between the average returns and *SRs*, corresponding to the formed portfolios and the values of *TOPIX500* index.

Fig. 8 a)

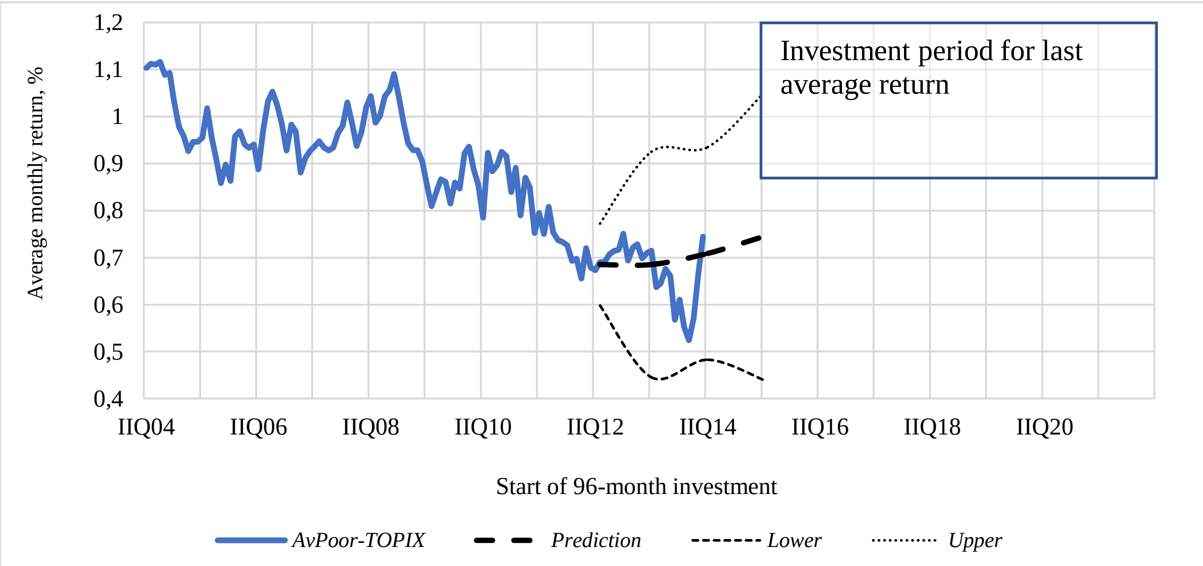


Fig. 8 b)

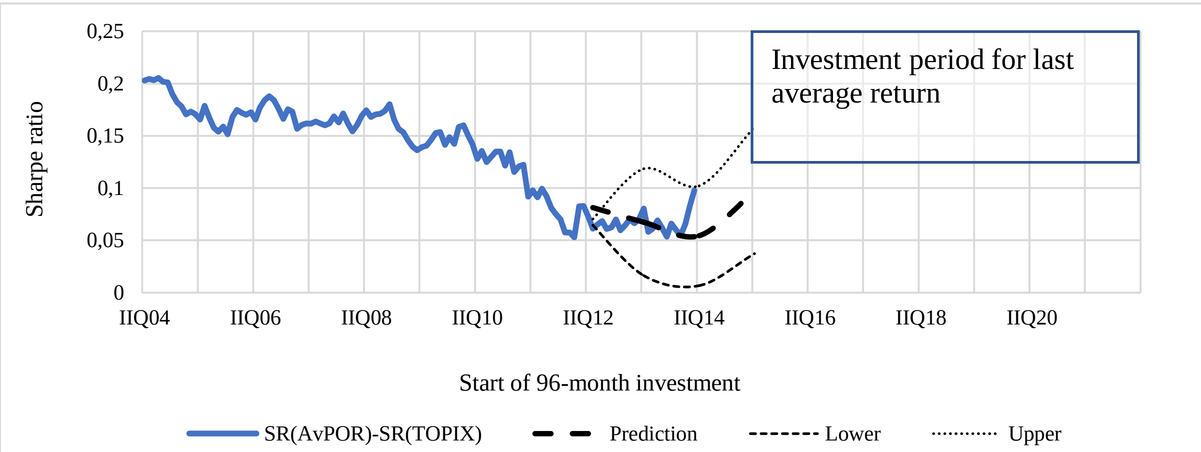


Fig. 8. SARIMA and prediction of rolling differences between a) average returns, b) *SR* on 96 months, corresponding to the formed portfolios and the values of *TOPIX500* index.

*Notes*: Each point of the chart presents the examined portfolio characteristics (average return and *SR*) at the beginning of the 96-month investment period. *SR* is computed as the ratio of the average excess return from the examined portfolio over the average risk-free rate (*Rp-RF*) to the standard deviation of the excess return s(*Rp-*

*RF*). *RF* is estimated based on U.S. 3-Month Treasury bills. *AvPOR* is a linearly weighted portfolio consisting of 15, 10, 7 and 5 stock portfolios formed on *TOPIX500* index. Stock portfolios are formed based on the maximization of the *FUN* of all stock of *TOPIX500*. The dashed thin top and bottom lines are the upper and lower bounds of the forecast confidence intervals. The dashed thick line represents the forecast values of rolling return difference – in Figure a), and *SR* difference – in Figure b). The analyzed historical period from May 2004 through March 2022, 215 months. Source: Own research using R-Studio ver. 4.2.3.

The majority of analyzed models exhibit parameters that are statistically significant. For Japanese Market available data allow for forecasting starting Q2 2012.

Despite the downward trend in *SR* differences in the entire tested period (see Fig. 7c), the confidence interval generated by the SARIMA model does not contain zero values (see Fig. 8b). A similar structure of changes applies to return differences (see Fig. 8a). Therefore, it can be assumed that in the entire period of 2004-2022, there are no grounds to reject Conjectures 2 and 1. This puts the studied Japanese Market in the stark contrast to the U.S. and European Markets.

Figure 9 shows Holt-Winters method in predicting rolling differences between the *SRs* and averaged returns, corresponding to the formed portfolios and the values of *TOPIX500* index.

Fig. 9 a)

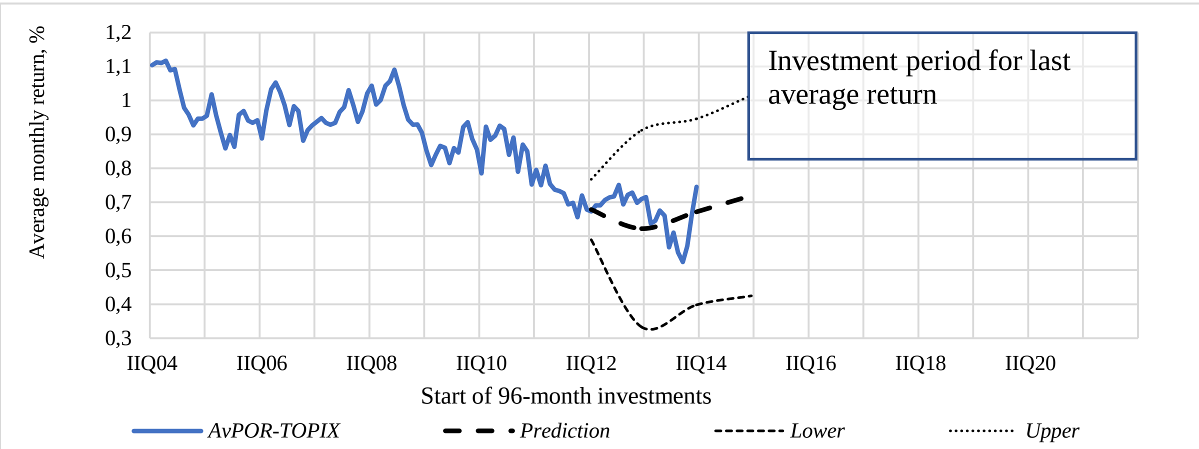

Fig. 9 b)

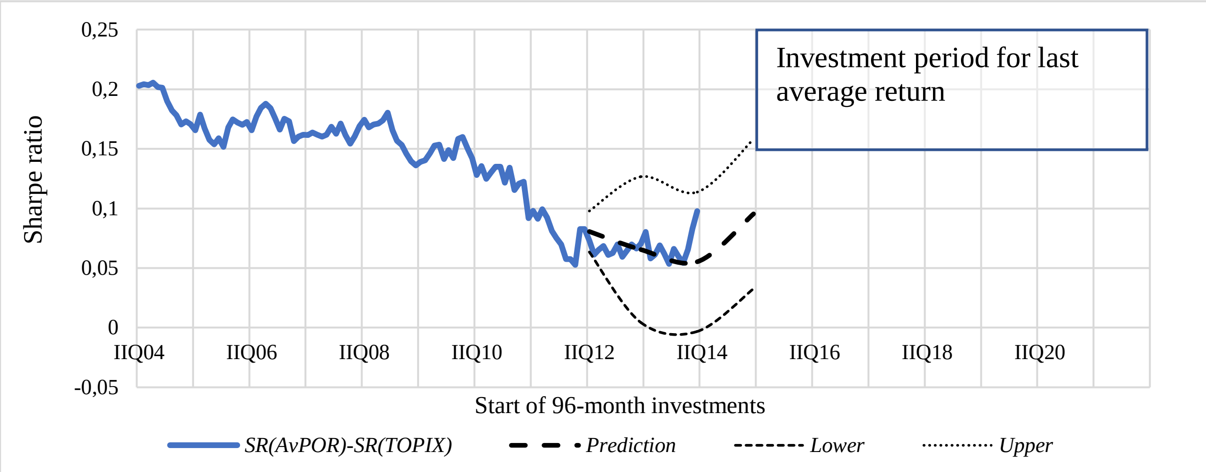


Fig. 9. Holt-Winters method to predict rolling differences between a) average returns, b) *SR* on 96 months, corresponding to the formed portfolios and the values of *TOPIX500* index.

*Notes*: Each point of the chart presents the examined portfolio characteristics (average return and *SR*) at the beginning of the 96-month investment period. *SR* is computed as the ratio of the average excess return from the examined portfolio over the average risk-free rate (*Rp-RF*) to the standard deviation of the excess return s(*Rp-RF*). *RF* is estimated based on U.S. 3-Month Treasury bills. *AvPOR* is a linearly weighted portfolio consisting of 15, 10, 7 and 5 stock portfolios formed on *TOPIX500* index. Stock portfolios are formed based on the maximization of the *FUN* of all stock of *TOPIX500*. The dashed thin top and bottom lines are the upper and lower bounds of the forecast confidence intervals. The dashed thick line represents the forecast values of rolling return difference – in Figure a), and *SR* difference – in Figure b). The analyzed historical period from May 2004 through March 2022, 215 months. Source: Own research using ITSM ver. 7.1.

From the Holt-Winters method we conclude that confidence intervals of the forecasted *SR* differences contain zero (for starting investment) in the period from Q2 2013 to Q2 2014 (see Fig. 9b). However, recent forecasts show that the confidence interval of differences between *SR* values (for starting investment) from Q2 2014 does not contain zero. Therefore, it can be assumed that in the entire period of Q2 2004-Q2 2022, there are no grounds to reject Conjecture 2.

In the case of average returns differences, the calculated confidence interval does not contain zero (for starting investment) in the period from Q2 2012 to Q2 2015. This also indicates that we do not reject Conjecture 1.

Essential parameters of the SARIMA models and the Holt-Winters method can be made available upon request.

**Chinese Market**

Figure 10 shows rolling average returns (a), rolling correlations between formed portfolio and *SSE380* average returns (b), rolling *SR* on 96 months of *SSE380* average portfolios (formed on 5, 7, 10 and 15 maximum values of *FUN*) and *SSE380* index (c) as well as rolling correlations between *SRs* of *SSE380* portfolios and *SSE380* index (d). The correlations are calculated using 36-month windows.

Fig. 10 a)

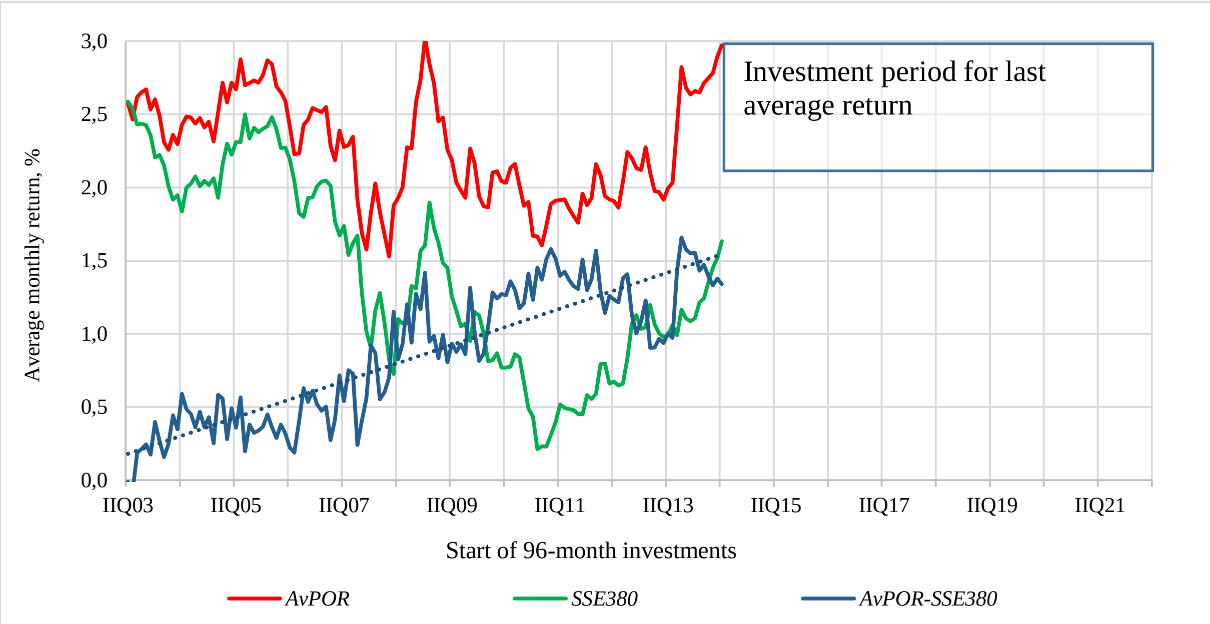


Fig. 10 b)

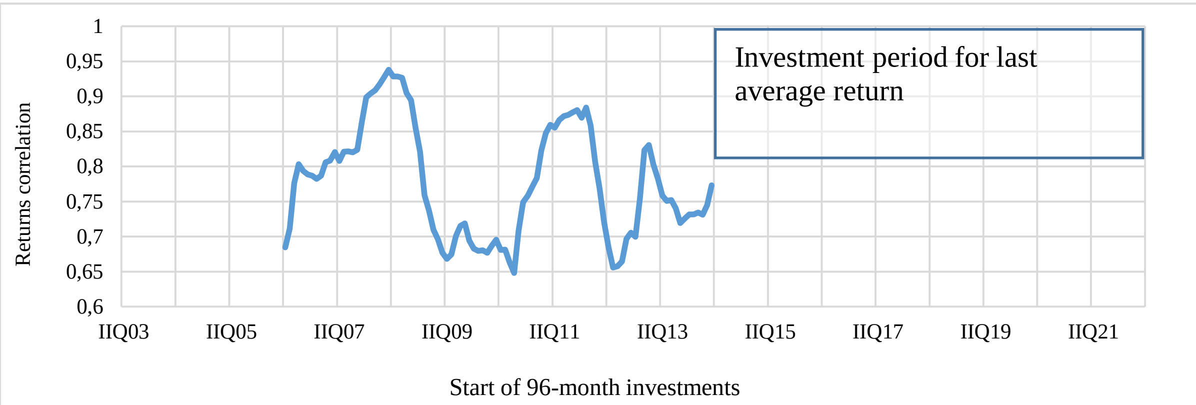

Fig 10 c)

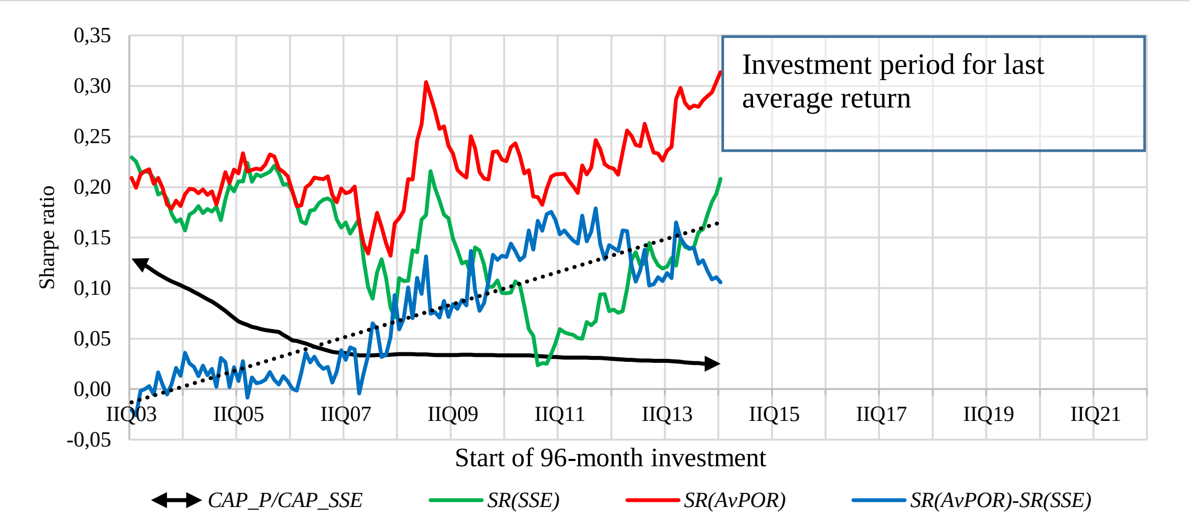


Fig. 12 d)

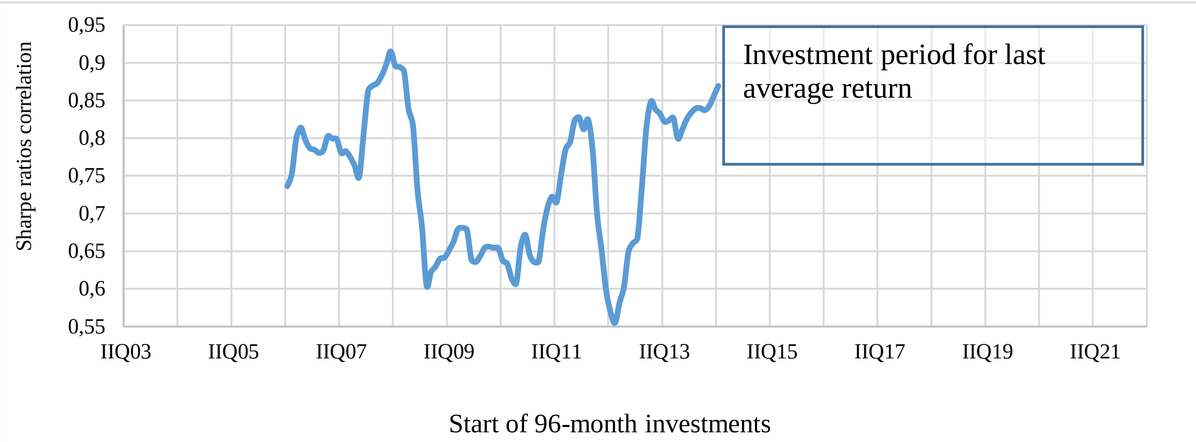


Fig. 10. Rolling a) average returns, b) correlations between *SSE380* formed portfolio and *SSE380* index average returns, c) *SR* on 96 months: of *SSE380* formed portfolio and *SSE380* index, d) correlations between *SR* of *SSE380* formed portfolio and *SSE380* index.

*Notes*: *AvPOR* is a linearly weighted portfolio consisting of 15, 10, 7 and 5 stock portfolios formed on *SSE380* index. Each point of the chart presents the examined portfolio characteristics [average return, correlations between *AvPOR* and *SSE380* average returns, *SR*, correlations between *SR* of *AvPOR* and *SR* of *SSE380* index, and ratio of 15 companies portfolio capitalization (*CAP_P*) to the capitalization of 50 companies with the maximum capitalization of the index (*CAP_SSE*))] at the beginning of the 96-month investment period. *SR* of *AvPOR* is computed as the ratio of the average excess return on *AvPOR* over the average risk-free rate (*AvPOR-RF*) to the standard deviation of the excess return s(*AvPOR-RF*). *RF* is estimated based on U.S. 3-Month Treasury bills. *SR* of *SSE380* is computed as the ratio of the average excess return on *SSE380* over the average risk-free rate (*SSE380-RF*) to the standard deviation of the excess return s(*SSE380-RF*). Stock portfolios are formed based on the maximization of the *FUN* of all stock of *SSE380*. The dotted line in Figure a) shows the trend line of changes in the differences between the average return on *AvPOR* and the average return on the *SSE380* index. The dotted line in Figure c) shows the trend lines of changes in the differences between the *SR* of *AvPOR* (*SR(AvPOR*)) and the *SR* of the *SSE380* index (*SR(SSE*)). The analyzed historical period from May 2003 through March 2022, 227 months. Source: Own research.

The curves representing the time series of return and *SR* show changes over time of the formed portfolios and the entire index.

The maximum average returns on 96 monthly investments in the portfolios of 5, 7, 10 and 15 companies (with the highest *FUN* values) in the years 2003-2022 are respectively equal to: 41.9%, 43.3%, 47.4% and 44.6% (yearly).

Minimum average returns on 96 monthly investments in portfolios of 5, 7, 10 and 15 companies (with the highest *FUN* values) in the years 2003-2022 are respectively equal to: 17.2%, 20.1%, 21.4% and 19.3% (yearly).

The maximum and minimum average return on 96 monthly investments in the *SSE380* index between 2003 and 2022 is equal to: 35.9% and 2.58% (yearly).

Careful examination of the above figures indicates large differences between the values of returns corresponding to the formed portfolios and the values of the index starting from 2007. The values of these differences are definitely smaller in the years 2003-2007, which is similarly visible for the *SR*. The characteristic trend line of differences increases from about 0% to about 1,5% for average returns and from 0 to about 0.15 for *SR* (see Figs 10a, 10c).

The greatest changes in the correlation coefficient of returns (of formed portfolios and the *SSE380* index) are observed in the years 2008-2012. In 2008, the correlation decreases from about 0.95 to 0.65, and in 2011 it increases to 0.9 and fell again to 0.65. Similar changes in the correlation of the *SRs* are found throughout the analyzed period of 2003-2022 (see Figs 10b, 10d).

Changes in the ratio of the average capitalization of the portfolio to the average capitalization of the 50 companies in the index (with maximum capitalization) indicate that the relative capitalization of the portfolio decreases from 2003 to 2007 with values varying from about 13% to 3%.This fact may also partly explain occurrence of smaller differences between the returns of formed portfolios and the index until 2007.

Using methods previously applied, we study time series of the differences between the values of returns: *AvPOR-SSE*, and the values of *SRs*: *SR*(*AvPOR*)-*SR*(*SSE*) of the formed portfolios and the values of the index.

Figure 11 shows SARIMA model in predicting rolling differences between the average returns and *SRs*, corresponding to the formed portfolios and the values of *SSE380* index.

Fig. 11 a)

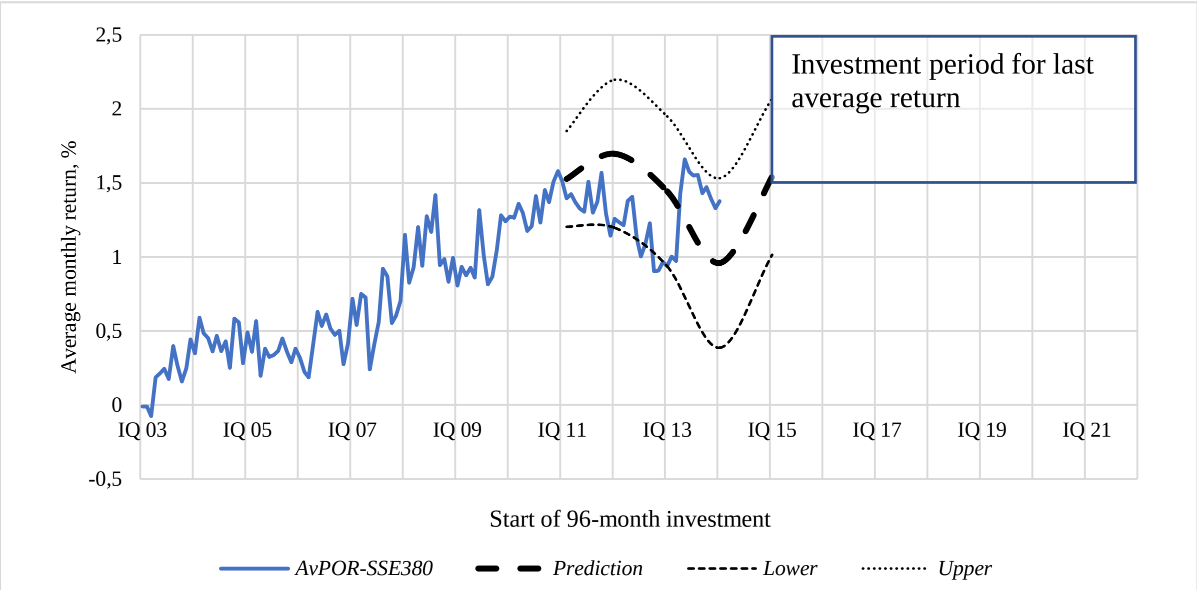


Fig. 11 b)

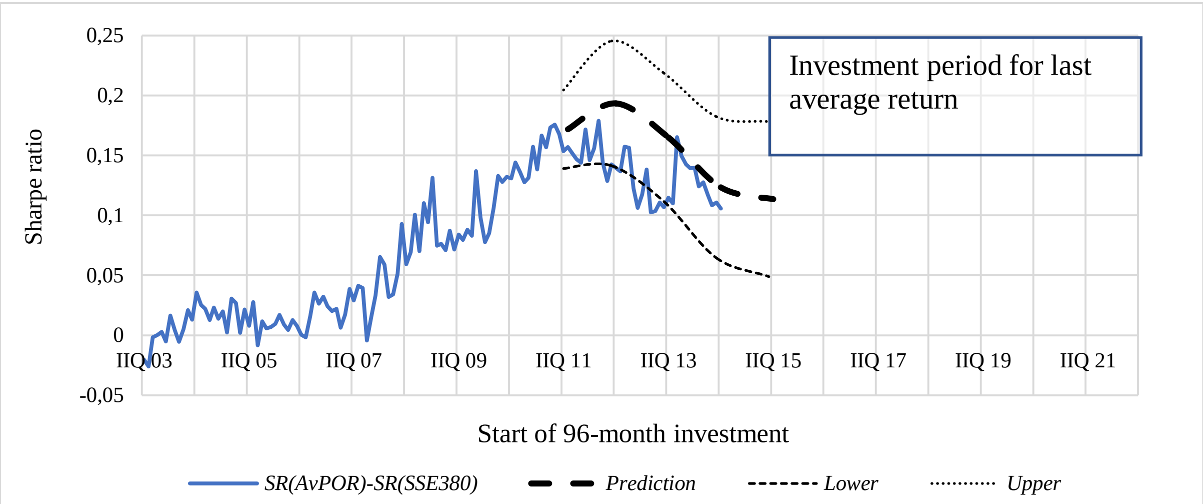


Fig. 11. SARIMA and prediction of rolling differences between a) average returns, b) *SR* on 96 months, corresponding to the formed portfolios and the values of *SSE380* index.

*Notes*: Each point of the chart presents the examined portfolio characteristics (average return and *SR*) at the beginning of the 96-month investment period. *SR* is computed as the ratio of the average excess return from the examined portfolio over the average risk-free rate (*Rp-RF*) to the standard deviation of the excess return s(*Rp-RF*). *RF* is estimated based on U.S. 3-Month Treasury bills. *AvPOR* is a linearly weighted portfolio consisting of 15, 10, 7 and 5 stock portfolios formed on *SSE380* index. Stock portfolios are formed based on the maximization of the *FUN* of all stock of *SSE380*. The dashed thin top and bottom lines are the upper and lower bounds of the forecast confidence intervals. The dashed thick line represents the forecast values of rolling return difference – in Figure a), and *SR* difference – in Figure b). The analyzed historical period from May 2003 through March 2022, 227 months. Source: Own research using R-Studio ver. 4.2.3.

The majority of analyzed models exhibit parameters that are statistically significant.

The reader should be reminded that in the case of Chinese Market available data allow forecasting starting Q2 2011.

Unlike the U.S. and Japanese markets, the trend lines of return and *SR* differences have a positive slope. However, as in the Japanese market, the confidence intervals generated by the SARIMA model do not contain zero values (see Figs. 11a, 11b). Therefore, it can be assumed that there are no grounds to reject Conjectures 1 and 2.

Figure 12 shows Holt-Winters method in predicting rolling differences between the average returns and *SRs*, corresponding to the formed portfolios and the values of *SSE380* index.

Fig. 12 a)

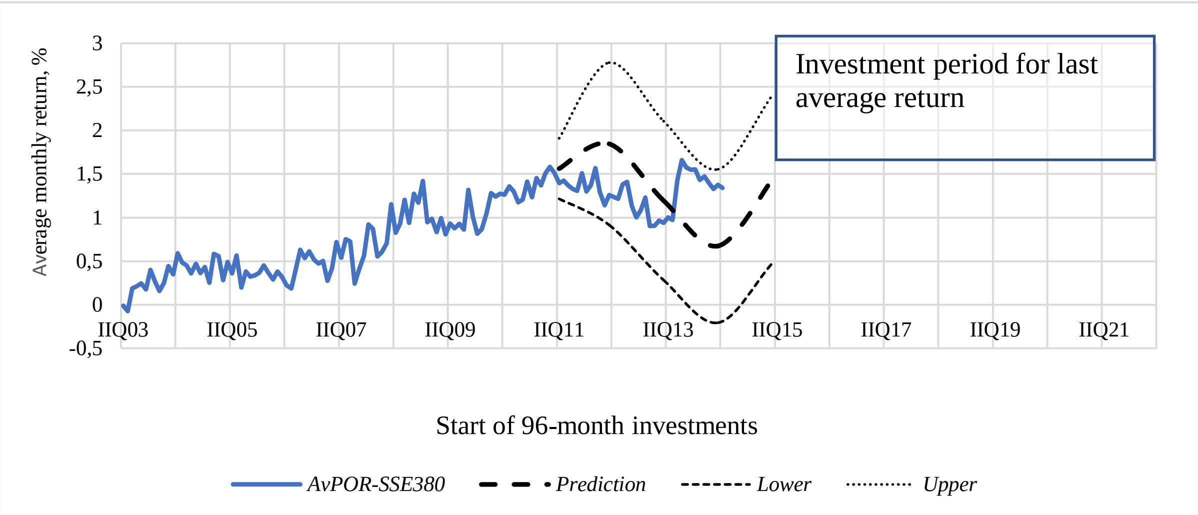

Fig. 12 b)

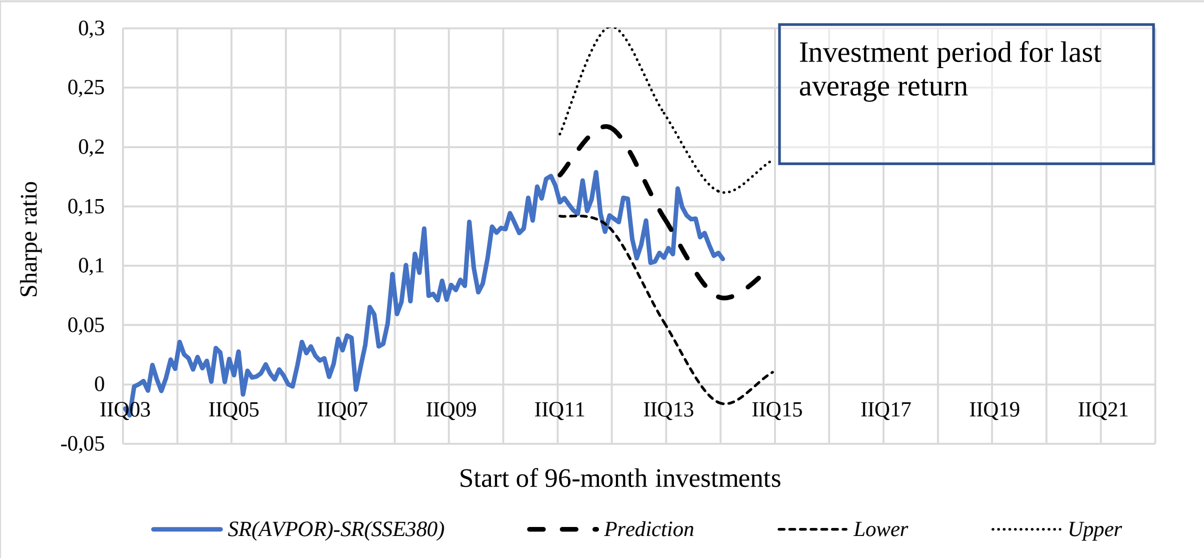


Fig. 12. Holt-Winters method to predict rolling differences between a) average returns, b) *SR* on 96 months, corresponding to the formed portfolios and the values of *SSE380* index.

*Notes*: Each point of the chart presents the examined portfolio characteristics (averaged return and *SR*) at the beginning of the 96-month investment period. *SR* is computed as the ratio of the average excess return from the examined portfolio over the average risk-free rate (*Rp-RF*) to the standard deviation of the excess return s(*Rp-RF*). *RF* is estimated based on U.S. 3-Month Treasury bills. *AvPOR* is a linearly weighted portfolio consisting of 15, 10, 7 and 5 stock portfolios formed on *SSE380* index. Stock portfolios are formed based on the maximization of the *FUN* of all stock of tested indexes. The dashed thin top and bottom lines are the upper and lower bounds of the forecast confidence intervals. The dashed thick line represents the forecast values of rolling return difference – in Figure a), and *SR* difference – in Figure b). The analyzed historical period from May 2003 through March 2022, 227 months. Source: Own research using ITSM ver. 7.1.

From the Holt-Winters method we conclude that confidence intervals of forecasted return and *SR* differences do not contain zero for most of the studied periods (see Figs. 12a, 12b). Therefore, it can be assumed that there are no grounds to reject Conjectures 1 and 2 in the investigated periods.

Essential parameters of the SARIMA models and the Holt-Winters method can be made available upon request.

**Polish Market**

Figure 13 shows rolling average returns a), rolling correlations between formed portfolio and *WIG* average returns (b), rolling *SRs* on 96 months of *WIG* average portfolios (formed on 5, 7, 10 and 15 maximum values of *FUN*) and *WIG* index (c) as well as rolling correlations between *SRs* of *WIG* portfolios and *WIG* index (d). The correlations are calculated using 36-month windows.

Fig. 13 a)

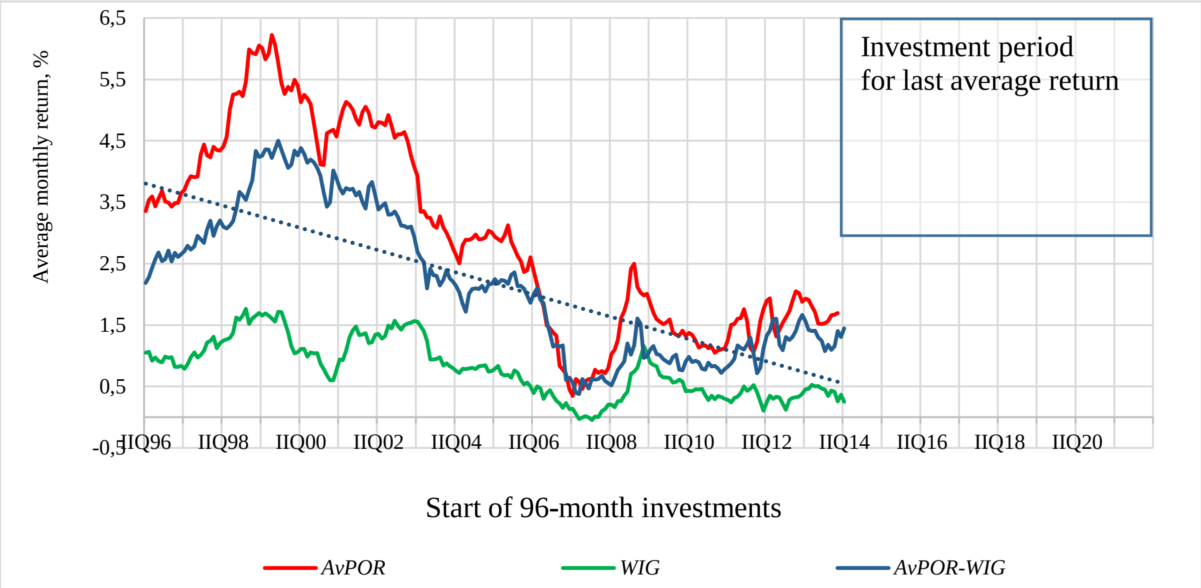


Fig. 13 b)

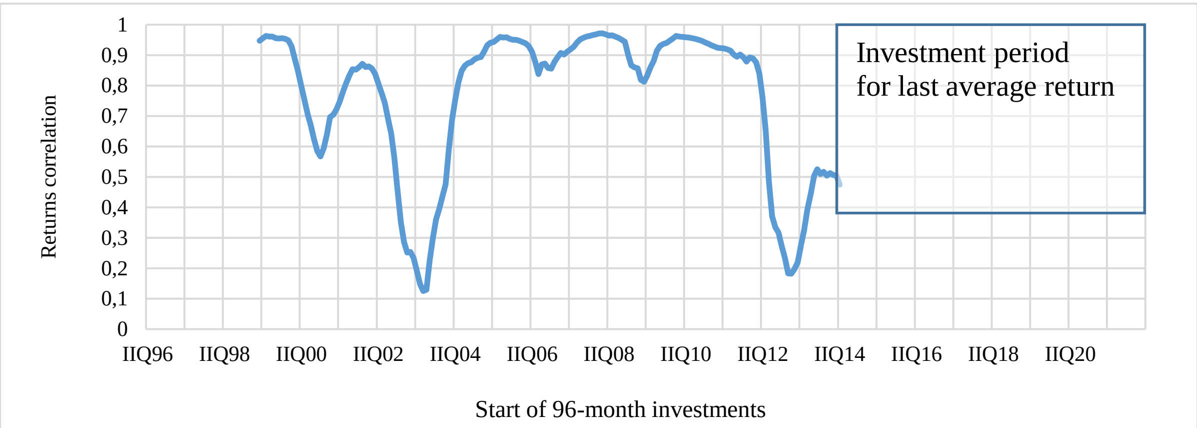

Fig. 13 c)

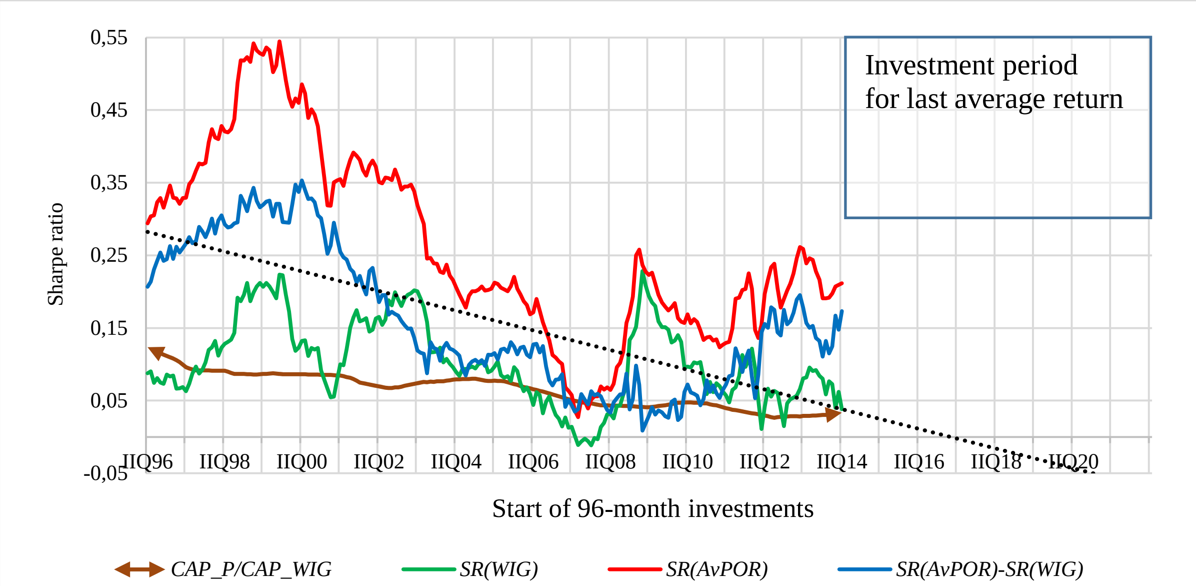


Fig. 13 d)

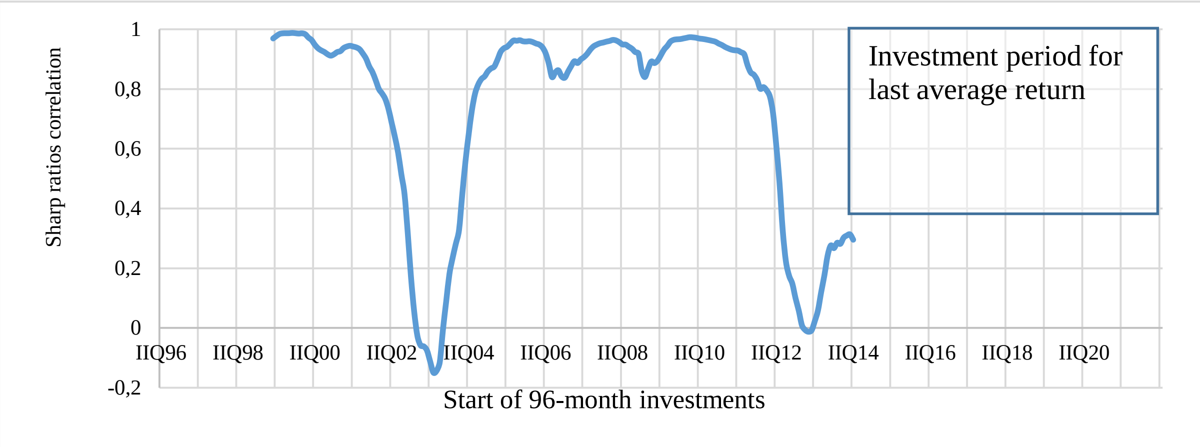


Fig. 13. Rolling a) average returns, b) correlations between *WIG* formed portfolio and *WIG* index average returns, c) *SR* on 96 months: of *WIG* formed portfolio and *WIG* index, d) correlations between *SR* of *WIG* formed portfolio and *WIG* index.

*Notes*: *AvPOR* is a linearly weighted portfolio consisting of 15, 10, 7 and 5 stock portfolios formed on *WIG* index. Each point of the chart presents the examined portfolio characteristics [average return, correlations between *AvPOR* and *WIG* average returns, *SR*, correlations between *SR* of *AvPOR* and *SR* of *WIG* index, and ratio of 15 companies portfolio capitalization (*CAP_P*) to the capitalization of 50 companies with the maximum capitalization of the index (*CAP_WIG*)] at the beginning of the 96-month investment period. *SR* of *AvPOR* is computed as the ratio of the average excess return on *AvPOR* over the average risk-free rate (*AvPOR-RF*) to the standard deviation of the excess return s(*AvPOR-RF*). *RF* is estimated based on U.S. 3-Month Treasury bills. *SR* of *WIG* is computed as the ratio of the average excess return on *WIG* over the average risk-free rate (*WIG-RF*) to the standard deviation of the excess return s(*WIG-RF*). Stock portfolios are formed based on the maximization of the *FUN* of all stock of *WIG*. The dotted line in Figure a) shows the trend line of changes in the differences between the average return on *AvPOR* and the average return on the *WIG* index. The dotted line in Figure c) shows the trend lines of changes in the differences between the *SR* of *AvPOR* (*SR(AvPOR)*) and the *SR* of the *WIG* index (*SR(WIG)*). The analyzed historical period from May 1996 through March 2022, 311 months. Source: Own research.

The curves representing the time series of return and *SR* show changes over time of the formed portfolios and the entire index.

The maximum average returns on 96 monthly investments in the portfolios of 5, 7, 10 and 15 companies (with the highest *FUN* values) in the years 1996-2022 are respectively equal to: 120.2%, 113.7%, 104.8% and 88.0% (yearly).

Minimum average returns on 96 monthly investments in portfolios of 5, 7, 10 and 15 companies (with the highest *FUN* values) in the years 1996-2022 are respectively equal to: 4.4%, 0.7%, 5.7% and 6.2% (yearly).

The maximum and minimum average return on monthly investments in the *WIG* index between 1996 and 2022 is equal to: 23.4% and -0.5% (yearly).

Careful examination of the above figures indicates large differences between the values of returns corresponding to the formed portfolios and the values of the index in the years 1996-2006. These differences definitely decrease as of 2007, which is also visible for the *SR*. The characteristic trend line of differences decreases from 3.6% to about 0.5% for average returns and from 0.3 to about 0.05 for *SR* (see Figs. 13a, 13c).

The largest changes in the correlation coefficients of returns on formed portfolios and the *WIG* index are found in the periods 2002-2005, and 2012-2014. The correlation decreases monotonically from about 0.9 in 2002 to 0.1 in 2003. Then it monotonically increases to values over 0.9 in 2006. In the following years 2006-2012 it reaches values close to 1. The correlation decreases again in the years 2012-2013 to the value of about 0.2 and then increases to 0.5 in 2014. Similar changes in the correlation coefficient of the *SRs* of formed portfolios and the *WIG* index are found throughout the analyzed period of 1996-2022 (see Figs. 13b, 13d).

Changes in the ratio of the average capitalization of the portfolio to the average capitalization of the 50 companies in the index (with maximum capitalization) indicate that the relative capitalization of the portfolio increases as of 2004, reaching the maximum of 10%. A slight decrease (about 2%) is observed after 2012. This fact may also partly explain the delayed occurrence of smaller differences between the returns of formed portfolios and the index after 2007.

Similarly as before, we study time series of the differences between the values of returns: *AvPOR-WIG*, and the values of *SRs*: *SR*(*AvPOR*)-*SR*(*WIG*) of the formed portfolios and the values of the index.

Figure 14 shows SARIMA modeling to predict rolling differences between the average returns and *SRs*, corresponding to the formed portfolios and the values of *WIG* index.

Fig. 14 a)

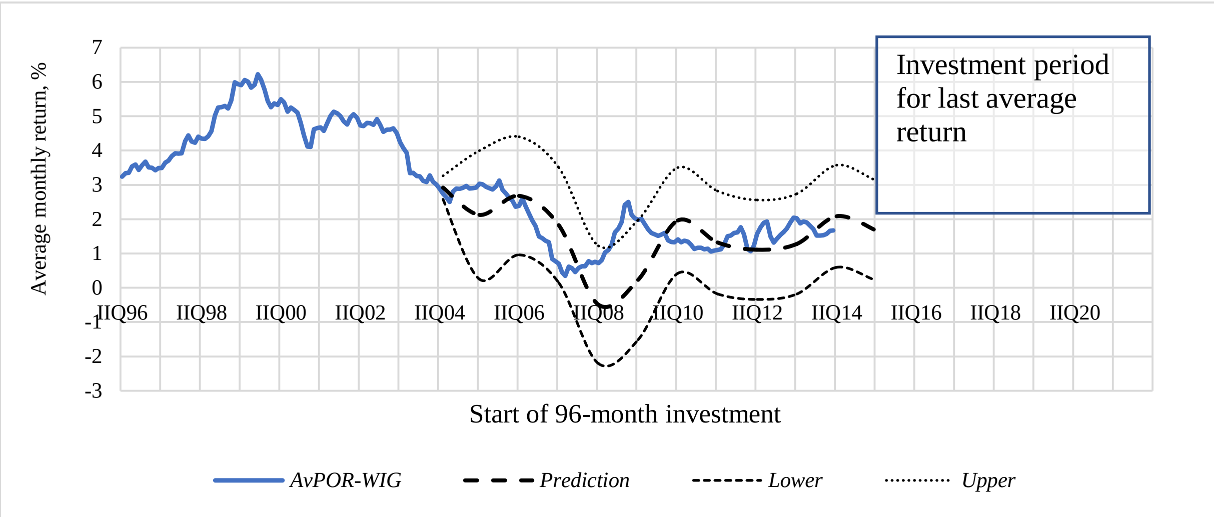

Fig. 14 b)

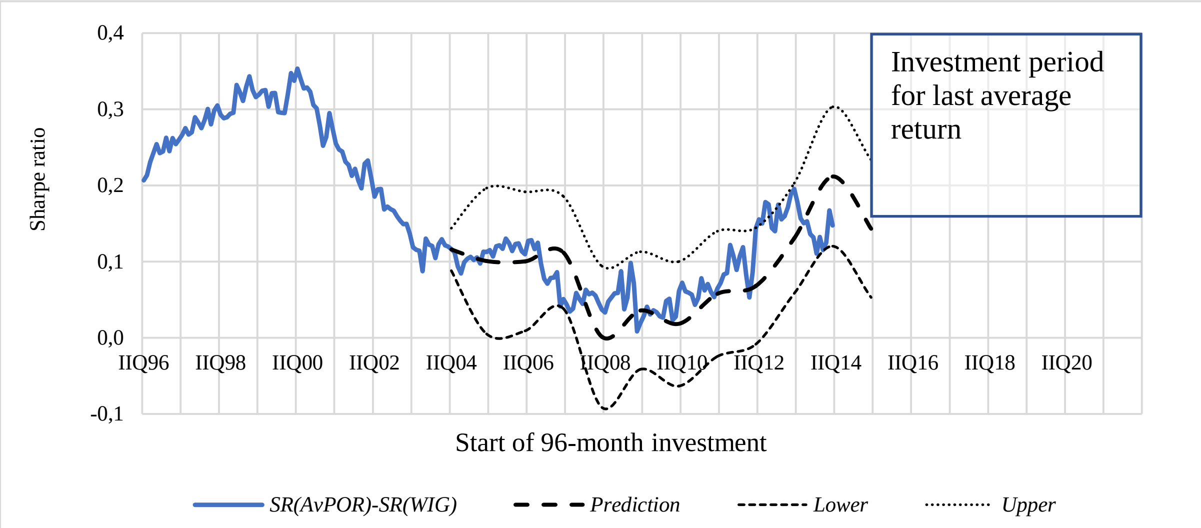


Fig. 14. SARIMA modeling to predict rolling differences between a) average returns, b) *SR* on 96 months, corresponding to the formed portfolios and the values of *WIG* index.

*Notes*: Each point of the chart presents the examined portfolio characteristics (average return and *SR*) at the beginning of the 96-month investment period. *SR* is computed as the ratio of the average excess return from the examined portfolio over the average risk-free rate (*Rp-RF*) to the standard deviation of the excess return s(*Rp-RF*). *RF* is estimated based on U.S. 3-Month Treasury bills. *AvPOR* is a linearly weighted portfolio consisting of 15, 10, 7 and 5 stock portfolios formed on *WIG* index. Stock portfolios are formed based on the maximization of the *FUN* of all stock of *WIG*. The dashed thin top and bottom lines are the upper and lower bounds of the forecast confidence intervals. The dashed thick line represents the forecast values of rolling return difference – in Figure a), and *SR* difference – in Figure b). The analyzed historical period from May 1996 through March 2022, 311 months. Source: Own research using R-Studio ver. 4.2.3.

Similarly as before, majority of applied models exhibit statistically significant parameters. The Polish Market data lead to deriving forecasts starting Q2 2004.

The SARIMA model shows that the confidence interval of the forecasted *SR* differences contains zero from Q3 2007 to Q4 2012 (see Fig. 14b). The confidence interval of the forecasted differences of returns contains zero for the most cases as of Q3 2007 (see Fig. 14a). The long-term forecasts indicate that both return and *SR* differences from created portfolios and the *WIG* portfolio head towards zero. This indicates the rejection of Conjectures 2 and 1.

Figure 15 shows Holt-Winters method to predict rolling differences between the average returns and *SRs*, corresponding to the formed portfolios and the values of *WIG* index.

Fig. 15 a)

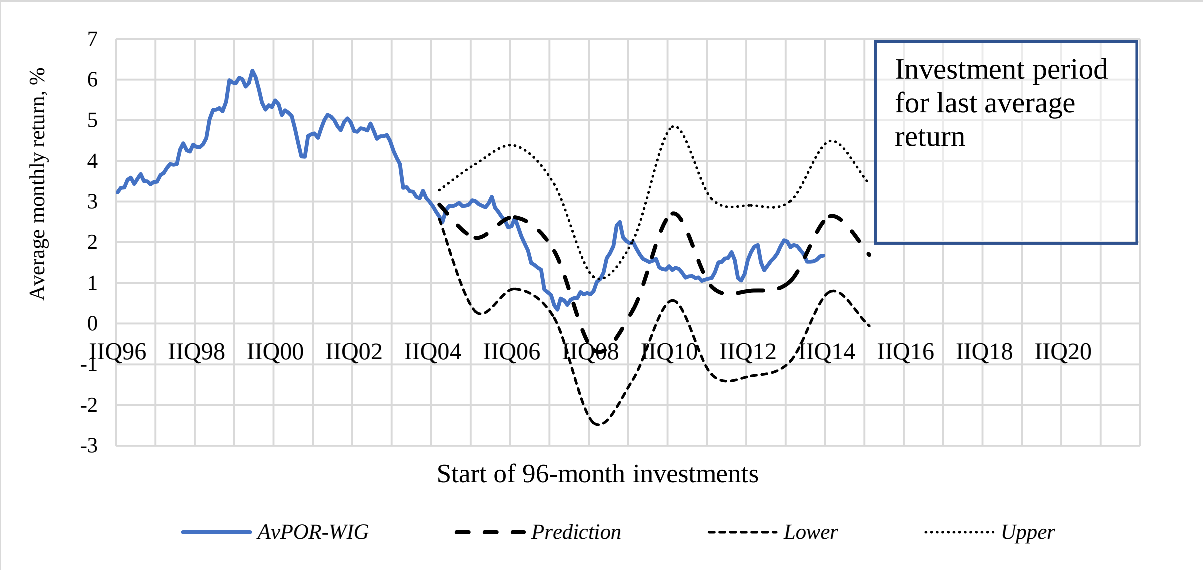


Fig. 15 b)

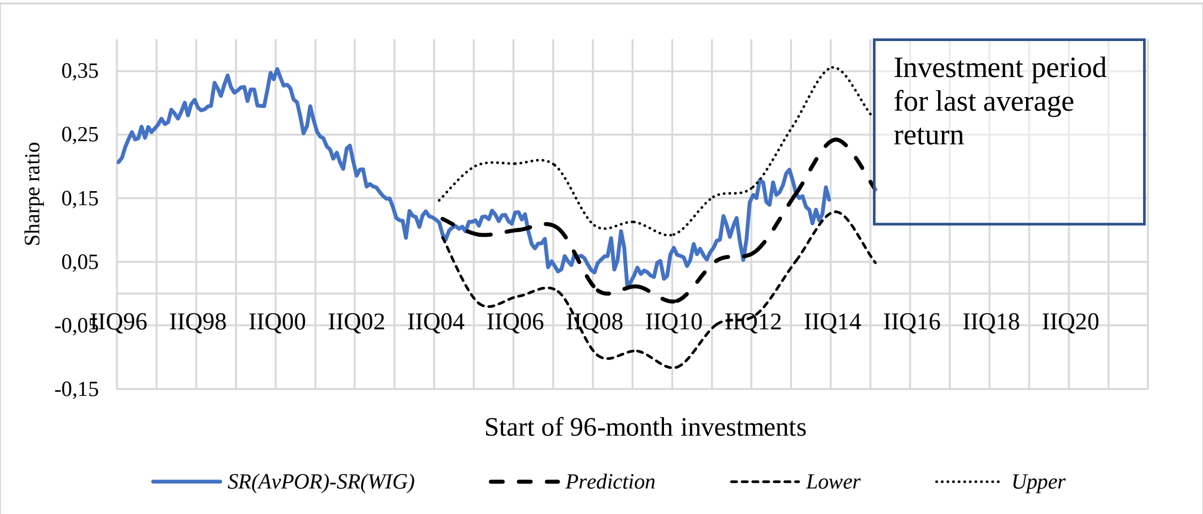


Fig. 15. Holt-Winters method to predict rolling differences between a) average returns, b) *SR* on 96 months, corresponding to the formed portfolios and the values of *WIG* index.

*Notes*: Each point of the chart presents the examined portfolio characteristics (average return and *SR*) at the beginning of the 96-month investment period. *SR* is computed as the ratio of the average excess return from the examined portfolio over the average risk-free rate (*Rp-RF*) to the standard deviation of the excess return s(*Rp-RF*). *RF* is estimated based on U.S. 3-Month Treasury bills. *AvPOR* is a linearly weighted portfolio consisting of 15, 10, 7 and 5 stock portfolios formed on *WIG* index. Stock portfolios are formed based on the maximization of the *FUN* of all stock of tested indexes. The dashed thin top and bottom lines are the upper and lower bounds of the forecast confidence intervals. The dashed thick line represents the forecast values of rolling return difference

– in Figure a), and *SR* difference – in Figure b). The analyzed historical period from May 1996 through March 2022, 311 months. Source: Own research using ITSM ver. 7.1.

According to the Holt-Winters method the predicted confidence intervals of *SR* differences contain zero starting from Q2 2005 to Q4 2012 (see Fig. 15b). A similar effect can be observed for SARIMA models (see Figs. 14a, 15a). The long-term forecasts indicate that both return and *SR* differences head towards zero. This also indicates the rejection of Conjectures 2 and 1.

Essential parameters of the SARIMA models and the Holt-Winters method can be made available upon request.

## IV. Discussion

The reader, while analyzing the research results presented in Section III and following the discussion below, should bear in mind that the values of the rolling returns are 96-month averages and are assigned to the beginning of the first month. In other words, each abscissa in the figures is the average of the values for the next 96 months. Also, the market efficiency tests presented in Section II are based on average returns from 96 months investment periods. Therefore, inference about market efficiency in the field of performed research includes the condition of investment in periods of 96 months. This condition is met in the case of a representative investor opening long-term positions in the assets of an investment fund.

In order to verify the conjectures made in Section I, we investigate the differences between the return values and *SR* values of portfolios formed on *FUN* and benchmarked portfolios. In particular, the results concerning the differences in the values of *SRs* are our priority (see Malkiel (2003, p. 60)).

The methodological considerations presented in this paper use the research of Fama and French (1995), who stated that the source factors generating future returns are the structure of past financial results and not *HML* and *SMB*.. Therefore, the three-factor Fama-French (1993) model could be modified by taking into account risk factors modeling the structure of past financial results. Such a modification, proposed by Urbański (2012), may constitute another ICAPM application. Thus, if the portfolio of a representative investor is based on the structure of past financial results, then the returns on investment in such a portfolio should be in line with the ICAPM. As a consequence, all investor portfolios generate similar returns (per risk unit) forming a market portfolio, thus the market is efficient.

Such conclusions fully reflect the changes on the American, Polish and partly Japanese markets after 2007. In the case of American, Polish and Japanese markets, the slope of the trend line of the differences between the returns on formed portfolios and tested market indexes is negative.

Changes in returns in the Chinese market are more difficult to explain. We observe the symptoms of the effectiveness of the Chinese market in the early years of the first decade of the 21st century. In the second decade changes in returns on formed portfolios and the *SSE380* index are opposite to changes in the other studied markets. The slope of the trend line of the differences between the returns on formed portfolios and the index is positive.

We find different changes in returns and *SRs* on formed portfolios and the *BE500* index compared to other tested markets. Throughout the tested period, we observe large and practically unchanging differences between the returns and *SRs* of formed portfolios and the index. However, SARIMA and Holt-Winters time-series forecasts of differences between returns and *SRs* show their insignificant values.

In order to confirm the impact of a representative investor on the studied market changes, we conduct research on changes in the structure of investors in the tested periods. Figures 16 to 20 show changes in the investor structure of shares and capitalization of stocks on the American, European, Japanese, Chinese and Polish markets in the first two decades of the 21st century. The share of the examined group of investors is calculated as the ratio of the capitalization of stocks held by a given group to the capitalization of all groups.

Fig. 16 a)

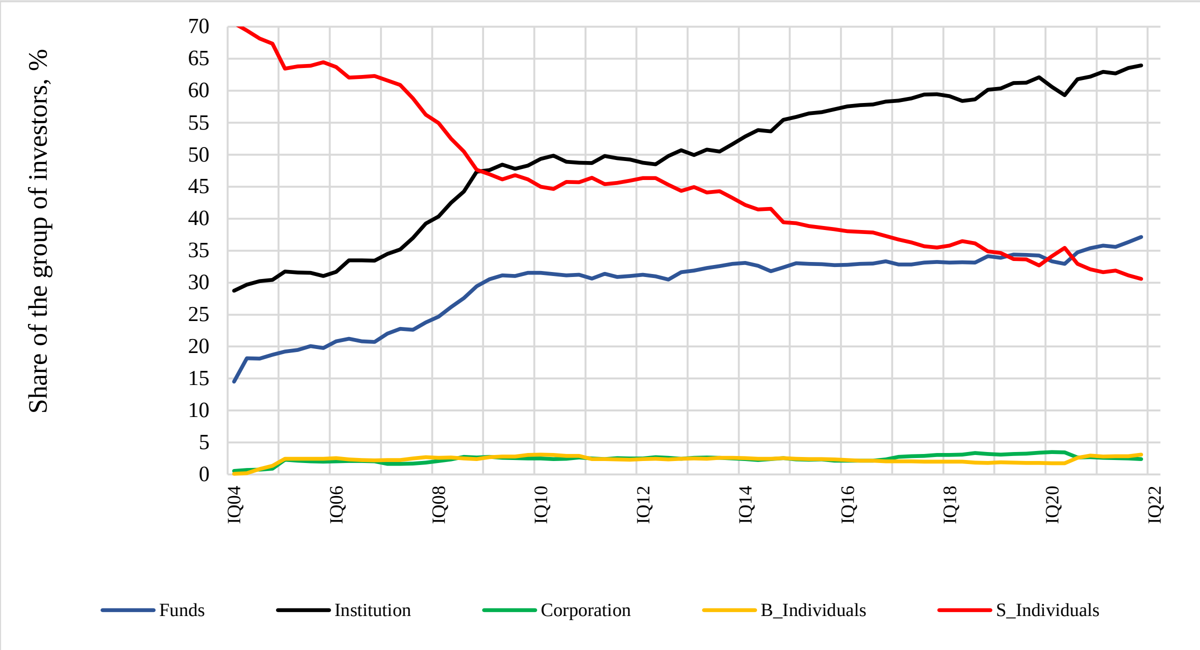


Fig. 16 b)

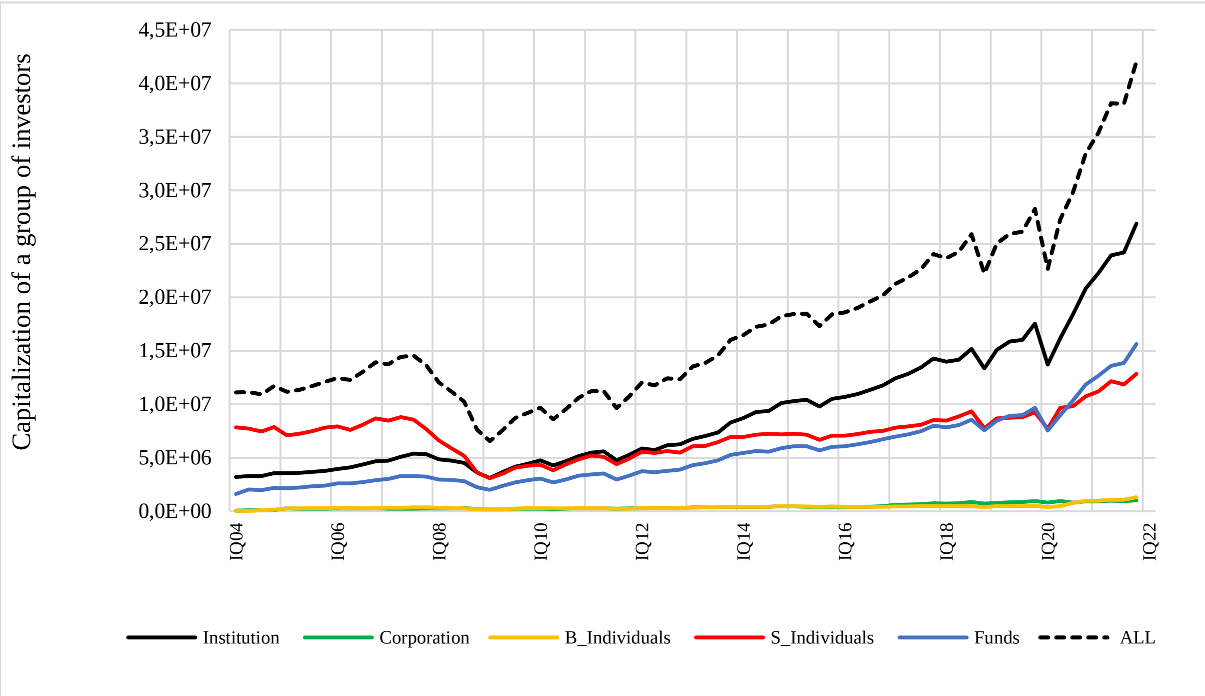


Fig. 16. The structure of individual investor groups investing in *S&P500* index companies; a) share of the group of investors at the end of the quarter periods; b) capitalization of the investor groups.

*Notes*: The share of the examined group of investors is calculated as the ratio of the capitalization of stocks held by a given group to the capitalization of all groups. 'Institution' includes public and private investment firms.

'Funds' includes private and public funds. The Funds are included in the Institution group. 'Corporation' includes all company types excluding the public and private investment firms. 'B_Individuals' includes individuals listed by the stock exchange as shareholders. 'S_Individuals' incudes small individuals investors. 'ALL' includes all firms of *S&P500* index. Source: own elaboration using the EquityRT database.

Fig. 17 a)

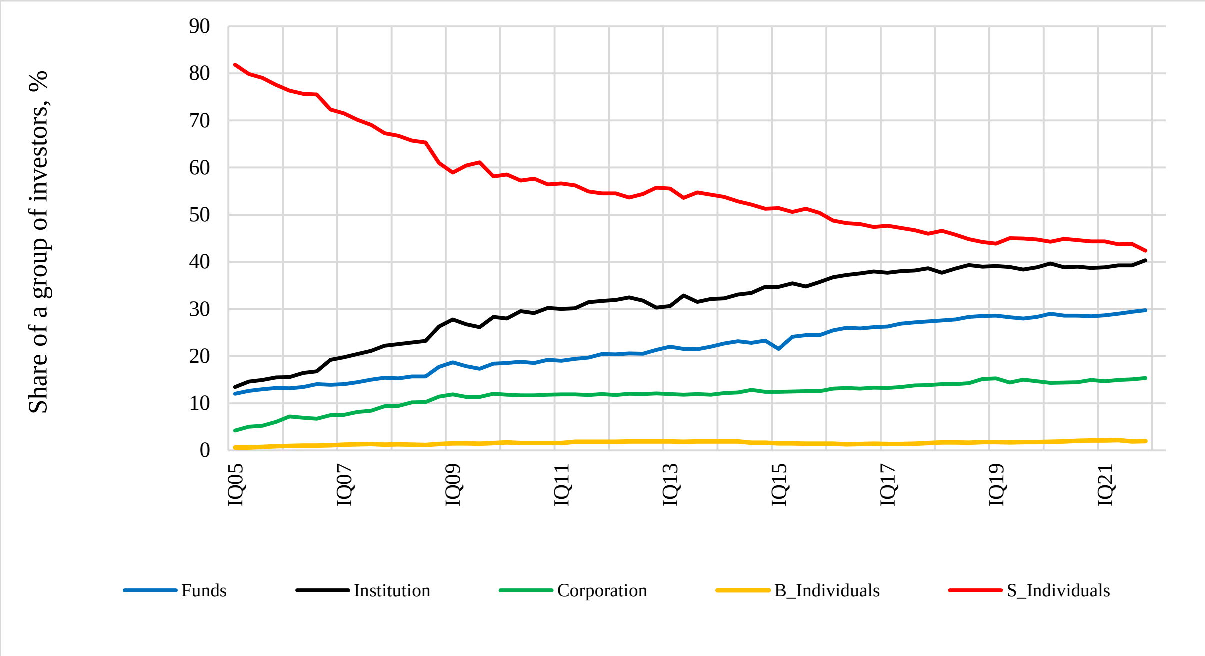


Fig. 17 b)

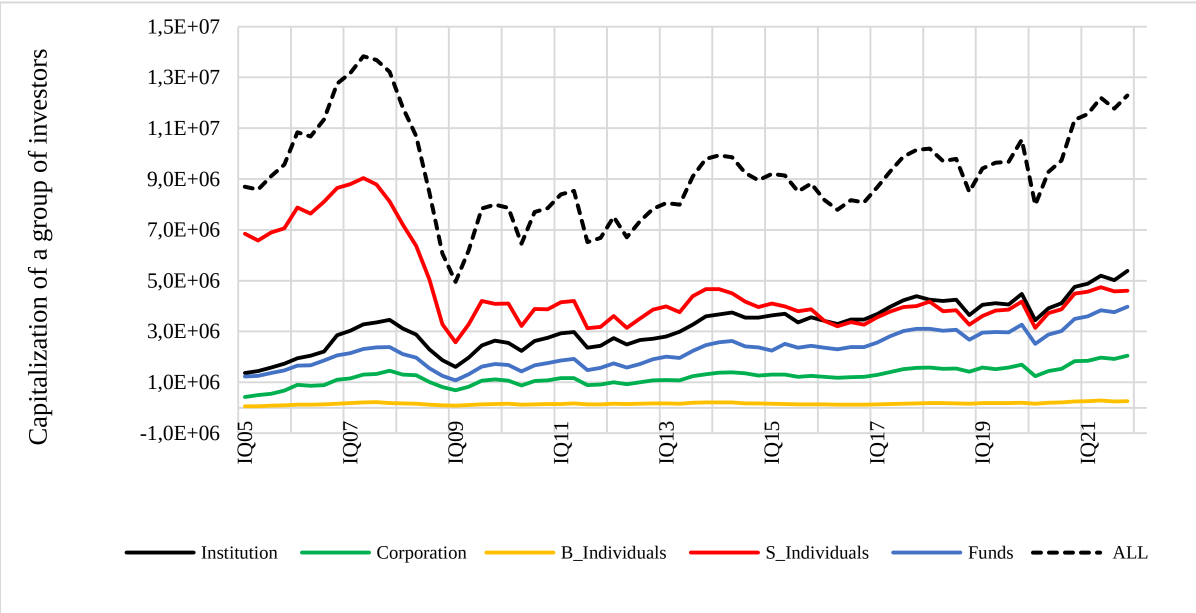

Fig. 17. The structure of individual investor groups investing in *BE500* index companies; a) share of the group of investors at the end of the quarter periods; b) capitalization of the investor groups.

*Notes*: The share of the examined group of investors is calculated as the ratio of the capitalization of stocks held by a given group to the capitalization of all groups. 'Institution' includes public and private investment firms. 'Funds' includes private and public funds. The Funds are included in the Institution group. 'Corporation' includes all company types excluding the public and private investment firms. 'B_Individuals' includes individuals listed by the stock exchange as shareholders. 'S_Individuals' incudes small individuals investors. 'ALL' includes all firms of *BE500* index. Source: own elaboration using the EquityRT database.

Fig. 18 a)

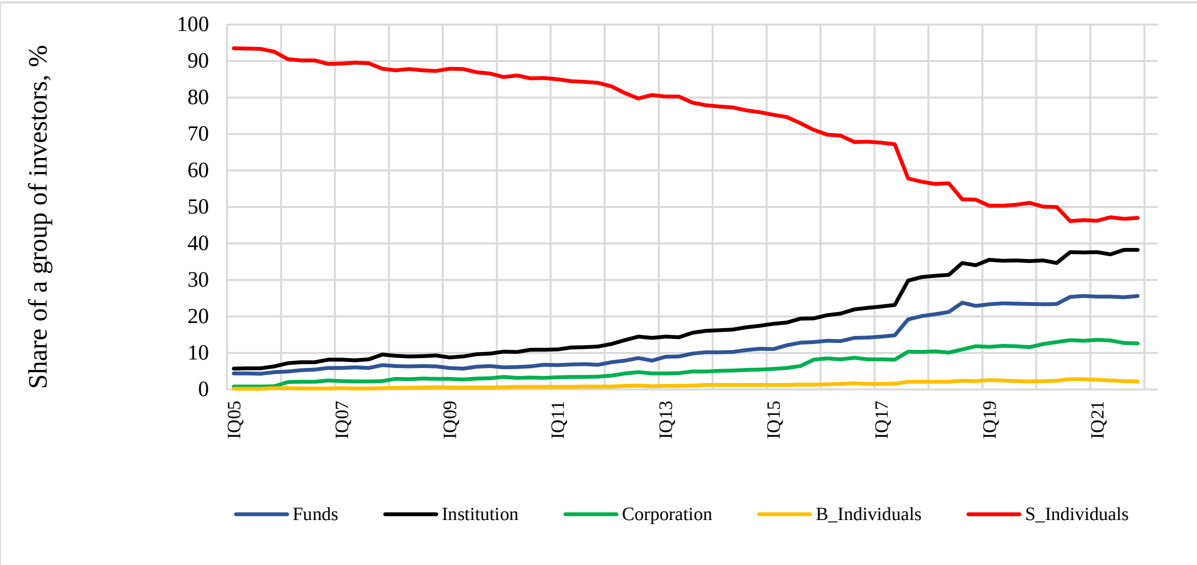


Fig. 18 b)

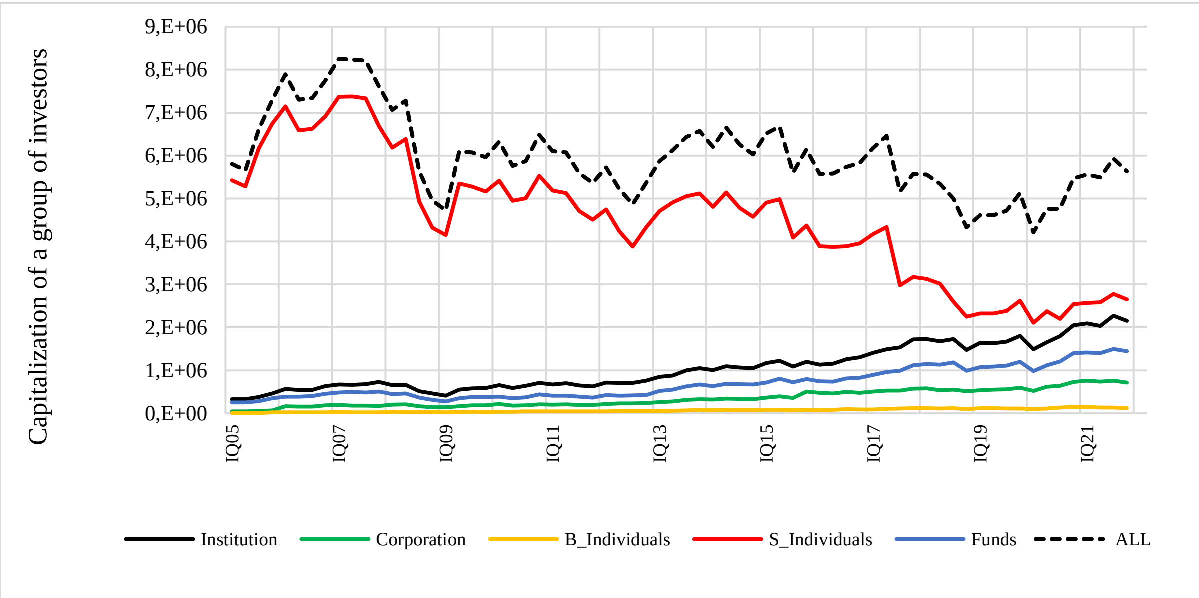

Fig. 18. The structure of individual investor groups investing in *TOPIX500* index companies; a) share of the group of investors at the end of the quarter periods; b) capitalization of the investor groups.

*Notes*: The share of the examined group of investors is calculated as the ratio of the capitalization of stocks held by a given group to the capitalization of all groups. 'Institution' includes public and private investment firms. 'Funds' includes private and public funds. The Funds are included in the Institution group. 'Corporation' includes all company types excluding the public and private investment firms. 'B_Individuals' includes individuals listed by the stock exchange as shareholders. 'S_Individuals' incudes small individuals investors. 'ALL' includes all firms of *TOPIX500* index. Source: own elaboration using the EquityRT database.

Fig. 19 a)

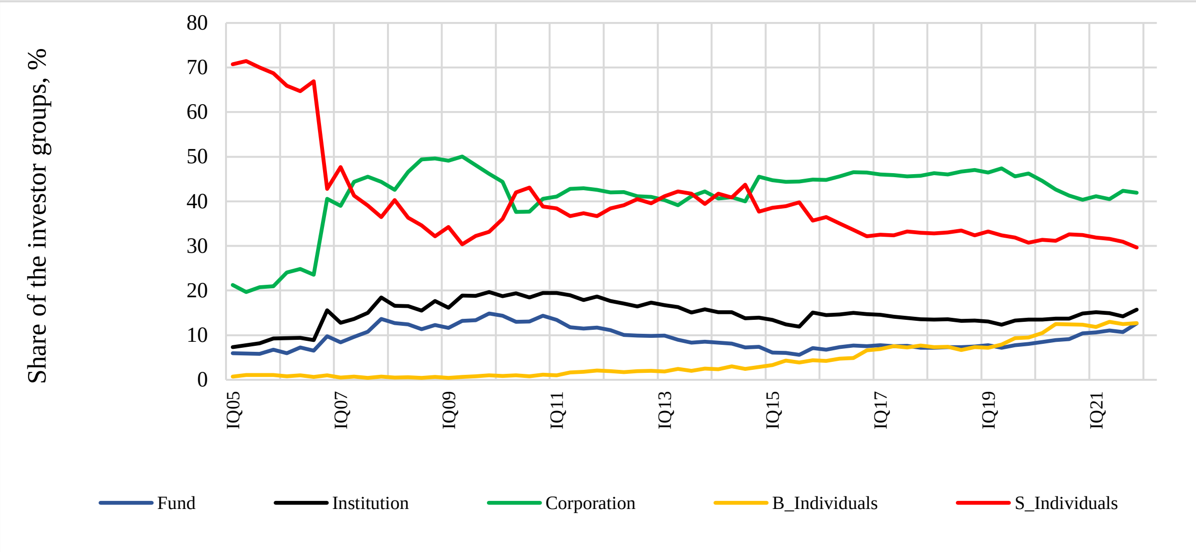


Fig. 19 b)

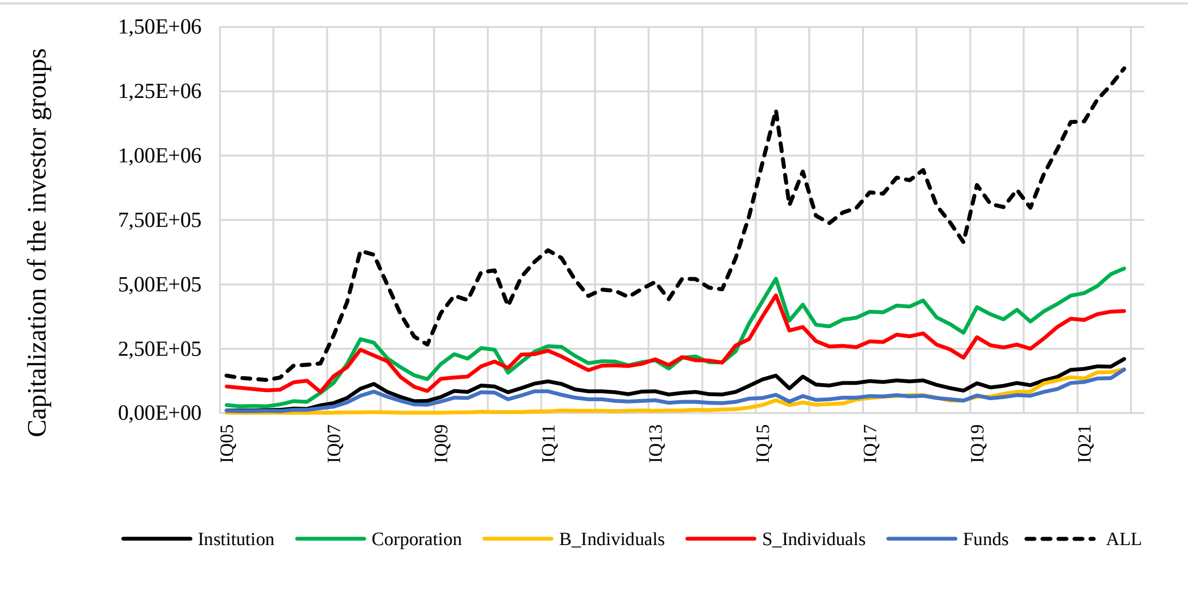

Fig. 19. The structure of individual investor groups investing in *SSE380* index companies; a) share of the group of investors at the end of the quarter periods; b) capitalization of the investor groups.

*Notes*: The share of the examined group of investors is calculated as the ratio of the capitalization of stocks held by a given group to the capitalization of all groups. ‘Institution’ includes public and private investment firms. Funds includes private and public funds. ‘Corporation’ includes all company types excluding the public and private investment firms. ‘B_Individuals’ includes individuals listed by the stock exchange as shareholders. ‘S_Individuals’ incudes small individuals investors. ‘ALL’ includes all firms of *SSE380* index.

Source: own elaboration using the EquityRT database.

Fig. 20 a)

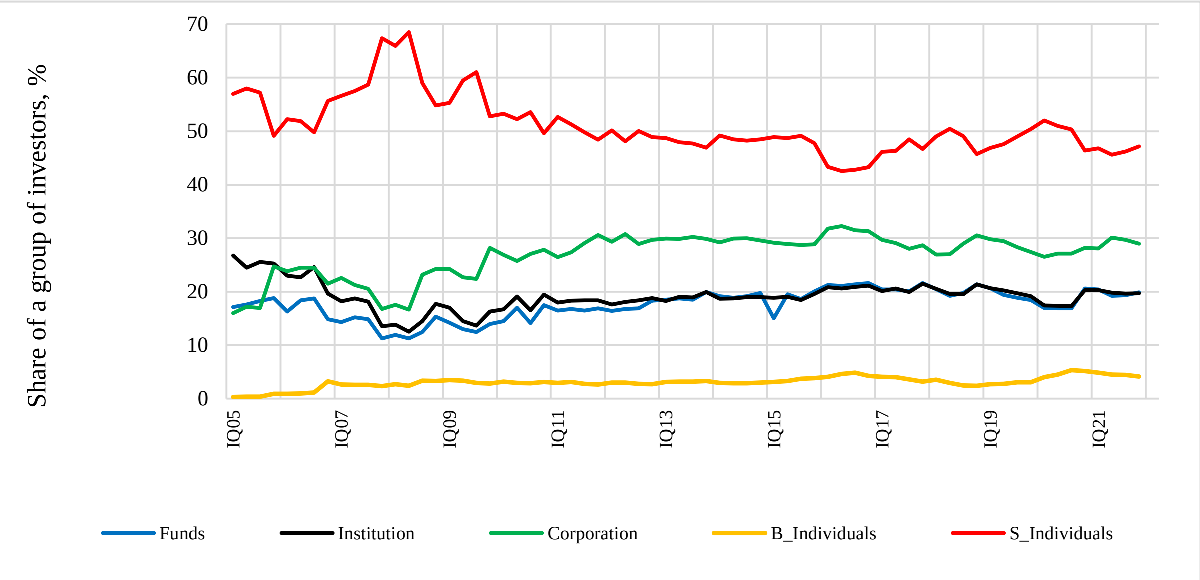


Fig. 20 b)

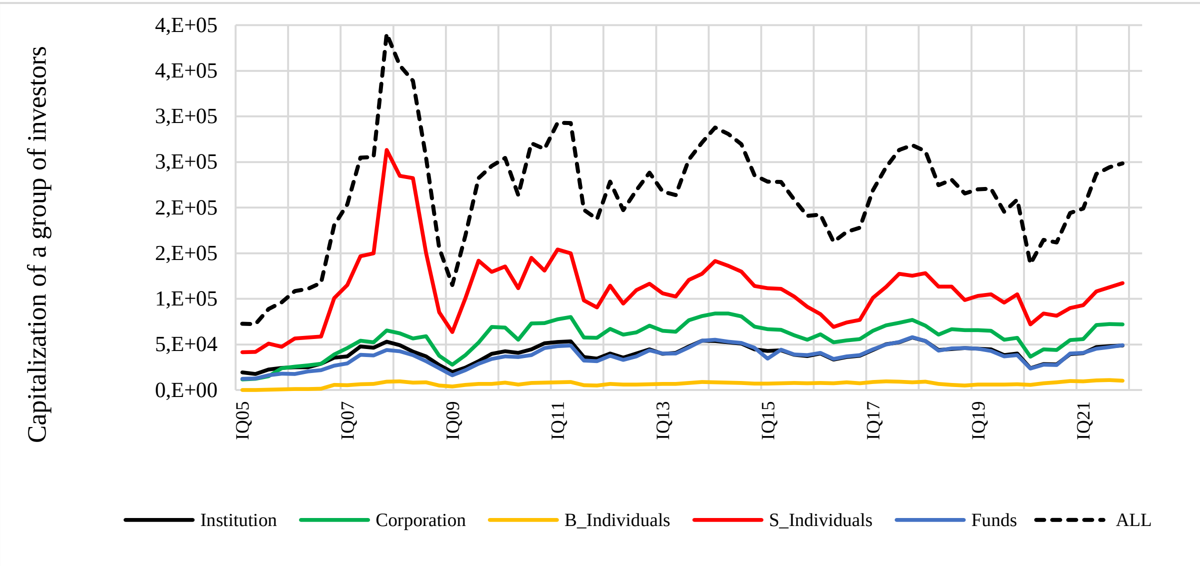

Fig. 20. The structure of individual investor groups investing in *WIG* index companies; a) share of the group of investors at the end of the quarter periods; b) capitalization of the investor groups.
*Notes*: The share of the examined group of investors is calculated as the ratio of the capitalization of stocks held by a given group to the capitalization of all groups. 'Institution' includes public and private investment firms. Funds includes private and public funds. 'Corporation' includes all company types excluding the public and private investment firms. 'B_Individuals' includes individuals listed by the stock exchange as shareholders. 'S_Individuals' incudes small individuals investors. 'ALL' includes all firms of *WIG* index.
Source: own elaboration using the EquityRT database.

A characteristic increase in the shares of institutional investors is observed on the U.S. and European markets since 2007 and 2005, respectively; 37% in 2007-2008, and 35% in 2009-2021 on the U.S. market, and 110% in 2005-2009, and 42% in 2009-2021 on the European market. This increase is accompanied by a delayed decrease in the stock capitalization of this group of investors in 2007-2008; 39% on the U.S. market, and 53% on the European market. In the following years 2009-2021, the capitalization increases by 760% on the U.S. market, and 235% on the European market.

The stock shares and capitalization structure of corporate investors and big individual investors are practically unchanged throughout the period under review. A slight increase in the capitalization of stocks held by the 'Corporation' group is observed on the European stock market after 2009. The shares of these investors are several times lower than the shares of institutional investors.

It can be concluded that the shares of small individual investors successively decrease. This is confirmed by the calculation results presented in Figures 16a and 17a. The greatest share decline gradients are observed in 2007-2008; 24% in 2007-2008, and 35% in 2009-2021 on the U.S. market, and 18% in 2007-2008, and 28% in 2009-2021 on the European market. Such decreases in shares of small individual investors are accompanied by decreases in capitalization in 2007-2009 (64%) and increases in capitalization in 2009-2021 on the U.S. market (318%). However, the increase in the stock capitalization in 2009-2021 is several times lower than the increase in the stock capitalization of institutional investors (760%) (see Fig. 16b). For the European market, we observe a decrease in small investors capitalization by approximately 72% in 2007-2009 and increase by 78% in 2009-2021 (see Fig. 17b).

The presented changes in the shares of individual groups of investors confirm that in the case of the examined stocks of the U.S. market, starting from 2008-2009, the representative investor is an institutional investor. If the investment perspective of an institutional investor, e.g., an investment fund, can be considered as a long-term perspective, then, in light of the research of Fama and French (1995), investment decisions of such an investor are based on changes in past long-term financial results. Making such decisions by the institutional investor, as a representative investor, explains the observed changes in returns in the market of U.S. stocks being tested (see Section III, Figs. 1-3).

In the case of the European stock market, the shares of small individual investors decrease as of 2005 (the largest decreases in 2005-2009), and the shares of institutional investors successively increase as of 2005 (the largest increases in 2005-2009). These changes allow for concluding that the position of the representative investor is successively taken over by the institutional investor. Investment decisions of the representative investor explain the changes in returns on the European market in the second decade of the 21st century (see Section III, Figs. 4-6).

Changes in the shares and capitalization of stocks institutional investors and small individual investors on the Japanese market under study are similar to those on the European market. The decrease in shares of small individual investors in the entire period of 2005-2021 is about 50%, and on the European market 48%. The increase in shares of institutional investors in the period 2005-2021 is about 576%, and on the European market 200%. At the same time, we observe a decrease in the capitalization of small investors by about 51%, and by about 28% on the European market. In the case of institutional investors on the Japanese market, we observe an increase in capitalization by about 557% in the entire period under study, and about 294% on the European market. However, the dynamics of studied changes on the Japanese market, in contrast to the European market, is lower in the first decade of the 21st century and higher in the second decade (see Figs. 17a, 17b, 18a, 18b). These changes allow us to conclude that the importance of the representative investor is taken over by the institutional investor. This is similar to the European market, but with greater dynamics in the second decade of the 21st century. Thus, it can be assumed that this results in wider forecasted confidence intervals of differences in average return and *SR* values for the European market, found in Section III (see

Figs 5a, 5b, 6a, 6b, 8a, 8b and 9a, 9b). Negative slopes of the trend lines of differences between returns and *SRs* in the Japanese market seem to be the effect of such changes (see Figs. 7a and 7c). This has to be compared with positive slopes of such differences in the European market (see Figs. 4a and 4c). The institutional investor's decisions explain the changes in returns on the Japanese market in the second decade of the 21st century (see Section III, Figs. 7-9).

Changes in the shares and capitalization of stocks of institutional investors and small individual investors on the analyzed Chinese market indicate that until 2007 a small individual investor is a representative investor. It can be assumed that this investor decides about market returns and makes decisions based on the indications of Fama and French (1995) ahead of the decisions of American investors. Starting from 2007, corporate investors begin to dominate over small individual investors, while institutional investors continue to be less significant on the market. The reason for these changes may be political regulations, not analyzed in this paper. Therefore, until 2007-2008, changes in returns on the Chinese market may visually confirm the changes presented in Figures 10a and 10c. However, the short data segment available for 2007-2008 is not sufficient to apply SARIMA or Holt-Winters model, thus making quantitative verification impossible. The subsequent periods of 2009-2022 indicate comparable values of shares and capitalization of corporate investors and small individual investors, and their higher values in relation to institutional investors. Another factor indicating the market decisions of Chinese investors in the years up to 2007 are the higher values of the capitalization ratio of the formed portfolio to the capitalization of the examined companies of the *SSE380* index (*CAP_P/CAP_SSE*). The decreasing value of the *CAP_P/CAP_SSE* ratio from 0.13 in 2003 to approximately 0.03 in 2007 and subsequent years indicates that in this period the returns of formed portfolios depend mainly on large capitalization companies. The downward trend of *CAP_P/CAP_SSE* is an additional factor of similarity between returns and *SRs* of index and formed portfolios.

Changes in the shares and capitalization of stocks of the *WIG* index of small individual investors in relation to investors from other groups indicate that a small individual investor is a representative investor throughout the analyzed period. The largest differences in the shares and capitalization of small individual investors and investors from other groups occur in 2007-2009, and decrease in subsequent years. However, transactions made by small individual investors on the Polish market are under a strong and growing influence of individual foreign investors, who pay their attention mainly to the U.S. market as the world's leading market. The share of foreign investors in trading on the main market of the Warsaw Stock Exchange increased from 36% in 2009 to 64% in 2022.[7] Such behavior of investors on the Polish market explains similar changes in returns and *SRs* to changes in the U.S. market (see Figs. 13-15).

## V. Conclusion

In our work we investigate whether the average stock returns of the U.S., European, Japanese, Chinese and Polish markets are correctly described by Merton's ICAPM theory, and that the examined markets are efficient.

We reduce the market efficiency study to checking whether the formed test portfolios generate average 96 monthly returns and risks insignificantly different from the average returns and risks of stock indexes. We form portfolios in accordance with the statements of Fama and French (1995) that future returns are generated by the structure of past long-term financial results. Our research leads to the following conclusions:

1) The conducted research on the U.S. market indicates that there are no grounds to conclude that the average returns and *SR* values of formed portfolios are different from the average returns and *SR* values of the *S&P500* index in 2007-2022. This means that there are no grounds to conclude that the U.S. stock market is not efficient in 2007-2022. The main reason for such changes seems to be the influence of investment funds constituting the dominant share among all institutional investors. Investment funds can be seen as a representative investor making decisions in line with ICAPM guidelines. This can be a significant indication for managers of investment funds.
2) The conducted research on the European market indicates that there are no grounds to conclude that the average returns and *SR* values of formed portfolios are different from the average returns and *SR* values of the *BE500* index in 2010-2021. This means that there are no grounds to conclude that the European stock market is not efficient in 2010-2021. The reason for such changes (similarly to the American market) seems to be the dominant role of investment funds in the second decade of the 21st century.

[7] www.gpw.pl/analizy

3) Research on the Japanese market shows large differences between the values of returns and *SRs* of formed portfolios and the *TOPIX500* index over the entire analyzed period 2004-2021. However, differences in average monthly returns and *SRs* in the final periods reduce the values from 1% to 0.7%, and from 0.22 to 0.05, respectively. Time-series statistical studies of the differences in average returns and *SRs* indicate the ineffectiveness of the Japanese stock market in 2004-2022. The reason for such changes seems to be the competitive influence of a small individual investor in relation to the institutional investor as a representative investor in the final years of the second decade of the 21st century.
4) Research on the Chinese market allows us to distinguish two different periods of changes in returns and *SRs*. In the period up to 2007, the returns and *SR* values of formed portfolios are close to those of the *SSE380* index. The representative investor is a small individual investor, whose decisions generate a structure of returns similar to the U.S. market in the years from 2007. Starting from 2007, the differences in the returns of formed portfolios and the *SSE380* index increase. The corporate investor becomes the representative investor, and the role of the institutional investor is negligible, which distinguishes the Chinese market from the other tested markets. Time-series statistical studies of differences in average returns and *SRs* from formed portfolios and the *SSE380* index indicate the lack of efficiency of the Chinese stock market in 2005-2022.
5) The conducted research on the Polish market indicates that, similarly to the U.S. market, there are no grounds to conclude that the average returns and *SR* values of formed portfolios are different from the average returns and *SR* values of the *WIG* index in 2007-2022. This means that there are no grounds to conclude that the Polish stock market is not efficient in 2007-2022. Although the representative investor is a small individual investor, the reason for such changes seems to be the strong influence of individual foreign investors, who pay their attention mainly to the U.S. market as the world's leading market.

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