CAN INVESTORS PROFIT FROM MEASURING STOCK LIQUIDITY WITH ORDERED FUZZY NUMBERS?

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1. Aim of the paper

The paper aims to analyse whether measuring stock liquidity based on ordered fuzzy numbers representation of a limit order book (Marszałek & Burczyński, 2024) yields some benefits to investors. In particular, we analyse whether liquidity measure based on ordered fuzzy numbers contains distinct or similar information to other commonly used liquidity measures. We also test if the stock liquidity measure based on the limit order book modelled with ordered fuzzy numbers captures a higher liquidity premium, which translates into a higher return to a zero-investment portfolio that goes long with the least liquid and shorts the most liquid stocks.

2. Data and methods

To check whether investors can profit from measuring stock liquidity with ordered fuzzy numbers, we aim to create an investment strategy based on one of the most pervasive asset pricing phenomena, i.e. liquidity premium. To this end, we conduct several types of examination of whether our LIQ^{OFN} generates positive liquidity premium: one-way (univariate) portfolio sorting, cross-sectional regressions and two-way (bivariate) dependent portfolio sorting. To carry out such tests we need to merge data gathered from several sources. Limit order book (LOB) data, in particular data on all buy and sell orders, are sourced directly from

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the Warsaw Stock Exchange. The time scope of the data covers the years from 2014 to 2021 and we have only the data on stocks included in three indices: WIG20, mWIG40 and sWIG80, which means in each period we dispose of the data on 140 stocks (out of approximately 450 listed in this period). Firms' accounting data and stock prices adjusted for corporate actions are sourced from the S&P Capital IQ database. We match LOB data with other data using issue ISIN.

To calculate our ordered fuzzy numbers liquidity measure we take LOB snapshots in 10-minute intervals. Taking into account that trading on the WSE take place between 9 a.m. and 5 p.m. and we discard opening and closing auctions due to different rules of trading, we have 47 LOB snapshots per day and about 235 snapshots per week. For each snapshot, we compute the OFN measure of a limit order book measure as in Marszałek and Burczyński (2024) and average its value for each week, which leads us to have weekly LIQ^{OFN} values.

We compare the results of the strategy based on our LIQ^{OFN} with other commonly used liquidity proxies, i.e. Amihud (2002) illiquidity ratio (LIQ^{Amihud}) and bid-ask spread (LIQ^{BAS}), and to another measure of stock liquidity based on the limit order book data, i.e. LOB slope (Næs & Skjeltorp, 2006). Amihud's (2002) illiquidity ratio is calculated as the absolute value of the price change between two snapshots divided by the trading volume in this 10-minute interval, averaged across the week. Næs and Skjeltorp's (2006) LOB slopes are computed for each snapshot from 10 ticks from the best quote ($LIQ^{LOBslope10}$) and from the full limit order book ($LIQ^{LOBslope}$) and also averaged across the week. Also, the bid-ask spread is calculated from best buy- and sell orders from each LOB snapshot and averaged across the week.

To perform the univariate portfolio sorts, at the end of each week *t-1*, we rank all the stocks in the sample according to the value of one of five liquidity measures – LIQ^{OFN}, LIQ^{Amihud}, LIQ^{BAS}, $LIQ^{LOBslope10}$, and $LIQ^{LOBslope}$ and form equal-weighted and value-weighted quintile portfolios. Also, we build a zero-investment portfolio, which serves as an ad hoc check of monotonicity in the cross-section of returns. This portfolio goes long the quintile of the least liquid and shorts the most liquid shares according to one of the five measures. We evaluate the performance of these portfolios with their week *t* raw log-return and risk-adjusted returns (αs) calculated from CAPM (α_{CAPM}), Fama and French's (1992) three-factor model (α_{FF3}) and Carhart's (1997) four-factor model ($\alpha_{Carhart}$). The factor returns are computed based on all firms listed on the Warsaw Stock Exchange and closely replicating the procedures in original papers by Fama and French (1992) and Carhart (1997). To compute the momentum factor we used the cumulative return from the previous 52 weeks (one year).

The second method employed to verify the potential profitability of our LIQ^{OFN} is cross-sectional regressions. Unlike Fama and MacBeth (1973), we use pooled cross-sectional time-series data to get the coefficient estimates. In this method, we regress week *t* stock returns on liquidity measures and other company characteristics in week *t-1*:

$$R_{it} = \alpha_i + \alpha_t + \beta_{LIQ} LIQ_{it-1} + \sum_{j=1}^{J} \beta_j X_{it}^{j} + \varepsilon_{it} \quad (1)$$

where R_{it} is the weekly excess return on stock *i* in week *t*, LIQ_{it-1} is one of the liquidity measures (LIQ^{OFN}, LIQ^{Amihud}, LIQ^{BAS}, $LIQ^{LOBslope10}$ or $LIQ^{LOBslope}$) and X_{it-1} refers to the control variables. The set of control variables includes the market value (*MV*) represented by the natural logarithm of total stock market capitalisation at the end of the preceding week (Banz, 1981), book-to-market ratio (*B-MV*) for week *t* calculated as the book value of equity half a year before week *t* over the most recent market capitalisation (Fama & French, 1992); momentum (*MOM*) is the 52-weeks average weekly log-return (Jegadeesh & Titman, 1993), stock return volatility (*VOL*) calculated as a standard deviation of weekly stock returns in recent 52 weeks, and turnover ratio (*TURN*) computed as a trading volume (in units of shares) scaled by the numbers of outstanding shares. To alleviate the effect of the outliers, we cross-sectionally winsorised all continuous variables at the 1 and 99 percentile of the distribution.

In the last test, we verify whether the magnitude of returns from our strategy is independent of the type of equities. To check this, we split our sample by different variables and checked the long-short portfolio performance within. In particular, we form quartile portfolios from two-way dependent sorts on additional control variables and a liquidity measure. In the first pass, we rank all the stocks in a given week on one of the control variables, i.e. *MV*, *B-MV*, *MOM*, *VOL* or *TURN*. In the second pass, within each quartile, we sort the stocks into four portfolios based on one of the liquidity measures, and we also form long-short quartile portfolios based on *LIQ* within each of the quartiles of the company characteristics. We form both, equal- and value-weighted portfolios.

3. OFN limit order book measure as a liquidity indicator

We begin our analysis of the profitability of measuring stock liquidity using ordered fuzzy numbers with a comparison of our measure, LIQ^{OFN}, to other commonly used liquidity proxies, i.e. Amihud (2002) illiquidity ratio (LIQ^{Amihud}) and bid-ask spread (LIQ^{BAS}), and another measure of stock liquidity based on the limit order book data, i.e. LOB slope (Næs & Skjeltorp, 2006). In particular, we compute the LOB slope from 10 ticks from the best quote ($LIQ^{LOBslope10}$) and the full limit order book ($LIQ^{LOBslope}$). Full details of computing each of these measures are presented in the previous section. We compare LIQ^{OFN} to other liquidity proxies in several aspects.

The first field of comparison is the distributional properties of the measures. To this end, we compare the means, standard deviations, coefficients of variations, skewnesses and kurtoses of the measures' distributions. Table 1 presents the results; Panel A contains time-series averages of cross-sectional means, standard deviations, skewnesses and kurtoses. Meanwhile, Panel B demonstrates cross-sectional averages of the time-series statistics. Both provide us with a different piece of information and allow us to infer about the measures' distributions for other purposes.

[TABLE 1 ABOUT HERE]

Cross-sectional distributional properties are important e.g. for asset pricing studies to ensure sufficient variation among companies. As one can see in Panel A of Table 1, our LIQ^{OFN} is characterised by the second-highest coefficient of variation and only LIQ^{Amihud} provides higher cross-sectional variation. Bid-ask spread's and Næs and Skjeltorp's (2006) LOB slopes' standard deviations are of significantly lower orders of magnitude. All the measures are right-skewed, which means all the measures have higher means than medians. This is interesting since LIQ^{Amihud} and LIQ^{BAS} measure illiquidity, i.e. their higher values denote lower liquidity, and other proxies measure liquidity, which means liquidity increases with their values. Thus, mean liquidity as measured with bid-ask spread or Amihud's (2002) ratio is lower than the median, inversely to liquidity as measured with measures based on LOB data. All five measures have elevated cross-sectional kurtoses, which means a greater extremity of outliers. However, our LIQ^{OFN} has the second-lowest kurtosis from all five measures. Hence, our liquidity measure provides quite good cross-sectional variation, is right-skewed and has elevated kurtosis. The two latter statistics, it is better than at least one of the commonly accepted liquidity proxies, i.e. bid-ask spread and Amihud's (2002) ratio.

Taking the time-series distributional properties into consideration (Panel B of Table 1), our LIQ^{OFN} exhibits the second-highest volatility, which unfortunately is rather not a desired feature due to potential difficulties in forecasting liquidity. Similarly to cross-sectional variation, bid-ask spread and Næs and Skjeltorp's (2006) LOB slopes' time-series volatilities are of significantly smaller magnitude. One should note that these measures also exhibit lower skewnesses and kurtoses than LIQ^{OFN}. All this information suggests potentially more problems with forecasting liquidity with LIQ^{OFN} than with the other measures. Only LIQ^{Amihud}'s time-series distributions are "worse" than that of LIQ^{OFN}.

Second, we compare our ordered fuzzy numbers measure of liquidity to other proxies by analysing the correlations among the measures. To this end, we do similar to Goyenko et al. (2009), Corwin and Schultz (2012), Abdi and Ranaldo (2017) or Fong, Holden and Trzcinka (2017) and calculate the time-series average of cross-sectional Pearson correlation among liquidity proxies and the cross-sectional average of the time-series Pearson correlation among measures. To complement this analysis, we also consider the time-series average of cross-sectional Spearman rank correlation, similar to what Fong, Holden and Tobek (2017) did. The results are presented in Table 2; Panel A reports time-series averages of cross-sectional Pearson correlation and Panel B demonstrates cross-sectional average of time-series Pearson correlation among liquidity proxies. Panel C presents time-series averages of cross-sectional Spearman rank correlation and Panel D presents the absolute changes in liquidity rank from one week to another[3]. As LIQ^{BAS} and LIQ^{Amihud} measure illiquidity, only for correlation purposes we multiply their values by -1 to ensure that a positive value of the correlation coefficient denotes a positive correlation of liquidity.

[TABLE 2 ABOUT HERE]

As we compare our liquidity measure to other liquidity measures without indicating a benchmark, as done e.g. in Goyenko et al. (2009), Corwin and Schultz (2012), Abdi and Ranaldo (2017) or Fong, Holden and Trzcinka (2017), we can only claim whether LIQ^{OFN}

[3] The absolute change in a liquidity rank is calculated as follows: $AbsChange_{it} = |LIQ_{it}^{R} - LIQ_{it-1}^{R}|$, where LIQ_{it}^{R} is the rank of *i*th company in week *t* based on the given *LIQ* measure.

brings the same or different information about liquidity than other metrics under scrutiny. As one can see from Panel A of Table 2, LIQ^{OFN} has the highest cross-sectional correlation with Næs and Skjeltorp's (2006) LOB slope calculated from 10 ticks from best quotes. However, the correlation is about 0.7 which means that LIQ^{OFN} yields some information not contained in $LIQ^{LOBslope10}$. The correlation of LIQ^{OFN} with other considered proxies is less than 0.5 and with LIQ^{Amihud} and $LIQ^{LOBslope}$ it equals only several percent. Thus, the cross-sectional Pearson correlation suggests that our liquidity measure behaves similarly to another measure based on LOB data, but potentially captures a different liquidity dimension than Amihud's (2002) illiquidity ratio and the bid-ask spread.

A bit weaker correlation is observed between LIQ^{OFN} and $LIQ^{LOBslope10}$ when one considers the time-series Pearson correlation. Inversely, there is a stronger (than cross-sectional) time-series correlation of LIQ^{OFN} with LIQ^{BAS}, LIQ^{Amihud} and $LIQ^{LOBslope}$. However, still one can claim our liquidity measure based on ordered fuzzy numbers contains some piece of information that is not captured by other proxies.

Though the cross-sectional Pearson correlation among the considered liquidity proxies is rather weak, all except $LIQ^{LOBslope}$ seem to rank stocks according to their liquidity similarly, which is evidenced by Panel C of Table 2. Much higher Spearman rank correlations than Pearson correlations suggest possible non-linearity in the relationship among the measures under scrutiny. One should note that for asset pricing purposes and portfolio formation based on percentiles of some companies' characteristics, Pearson correlation does not matter as these issues are focused on a rank, not the specific value. This means that the use of LIQ^{OFN} for the purposes of asset pricing and portfolio formation should yield quite similar results to the use of other considered proxies.

Nevertheless, which is also very important in creating a profitable investment strategy, one should notice that LIQ^{OFN} provides more stable ranks, which is evidenced by mean absolute change in liquidity rank (Panel D of Table 2). Surprisingly, though OFN^{LIQ} exhibits relatively high time-series volatility and quite well correlates with other liquidity proxies, each week it ranks analysed stocks according to their liquidity in an order very similar to that of the previous week. This gives a premise that the turnover of a long-short portfolio based on this measure will be relatively lower than that of long-short portfolios based on other measures. Lower portfolio turnover would result in lower transaction costs and make LIQ^{OFN} superior to other proxies in this term.

Finally, we compare our liquidity measure to other proxies in terms of a forecasting error. Since investors are interested not only in liquidity-related costs they incur at the moment of purchase but also at the moment of sales (Amihud et al., 2005; Eleswarapu, 1997), it is reasonable to expect liquidity to be predictable. In case of high prediction errors, the usefulness of a liquidity measure may be questionable. In order to verify the liquidity predictability with a specific measure, we compare them in terms of the average relative errors[4] (ARE), mean absolute errors[5] (MAE) and root mean squared errors[6] (RMSE) of the forecasts based on an AR(1) model. Such an approach mimics those of e.g. Li et al. (2018), but we do not take the differences between estimated and "true" spread, but between the predicted and true value of a liquidity measure.

To do forecasts of liquidity measures, we utilise a simple AR(1) model. Specifically, to predict a value of a given liquidity measure for week *t*, we estimate an AR(1) model using the values of this liquidity from weeks *t-27* to *t-1* (26 weeks gives about half a year of data). Then we predict the week *t* liquidity measure value based on the week *t-1* value and calculate the error as a difference between liquidity estimated for week *t* ($E_{t-1}[LIQ_t]$) and the observed one

[4] Average relative error is computed as $ARE = E[(E_{t-1}[LIQ_t] - LIQ_t)/LIQ_t]$.
[5] Mean absolute error is calculated as $MAE = E[|E_{t-1}[LIQ_t] - LIQ_t|/LIQ_t = |RE|]$.
[6] Root mean squared error is calculated as $RMSE = \sqrt{E[(E_{t-1}[LIQ_t] - LIQ_t)/LIQ_t]^2}$.

(LIQ_t), scaled by the observed value of a liquidity measure (LIQ_t). Table 3 presents the values of forecasting errors for all five liquidity measures used in our study.

[TABLE 3 ABOUT HERE]

The results from Table 3 should not be surprising. Since LIQ^{OFN} exhibits the second-highest time-series volatility one should expect this measure to be hardly predictable. Results from Table 3 somehow mimic those from Panel B of Table 1, i.e. Amihud's (2002) illiquidity ratio has the highest time-series volatility and so do the prediction errors. Other liquidity measures, i.e. the bid-ask spread and Næs and Skjeltorp's (2006) LOB slopes are both less volatile over time and are thus easier to predict.

Overall, we believe our ordered fuzzy numbers limit order book measure is likely to reflect stock liquidity and possibly could give some profits to investors using it. First, it provides a sufficient cross-sectional variation to differentiate stocks of low and high liquidity. Second, it correlates with another LOB measure in the cross-section, with other liquidity measures in the time series, and ranks stocks according to their liquidity similarly to other proxies. Third, since LIQ^{OFN} provides quite stable cross-sectional stock rankings according to their liquidity, it gives a premise of a lower turnover of long-short portfolios based on this metric. This issue is likely to make our liquidity measure superior to other proxies when creating an efficient investment strategy, even taking into account its relatively high prediction errors.

4. Basic results

4.1. Univariate portfolio sorting

We start our analyses with the examination of the performance of quintile portfolios sorted by liquidity, measured both with our LIQ^{OFN} proxy and other liquidity measures for comparison. This will make it possible to indicate whether sorting stocks into portfolios based on their liquidity translates into a cross-sectional return pattern, which would make it possible to create a profitable investment strategy. The summary of the results of one-way sorted portfolios is displayed in Table 4 and Figure 1 presents the cumulative return on zero-investment long-short portfolios created by shorting the most liquid stocks and going long the least liquid ones using five different liquidity proxies.

[TABLE 4 ABOUT HERE]

[FIGURE 1 ABOUT HERE]

Unlike recent evidence from developed (Amihud & Noh, 2021; Chiang & Zheng, 2015; Huh, 2014; Koch et al., 2016), other emerging markets (Amihud et al., 2015; Bekaert et al., 2007; Ben-Rephael et al., 2015), and also from the Warsaw Stock Exchange (Stereńczak, 2021, 2022), the zero-investment long-short portfolio based on stock liquidity yields negative returns. Regardless of the liquidity measure used, the average weekly return on an equal-weighted long-short portfolio equals from -0.183% to 0.288%, which accounts for from about -9.52% to -14.98% p.a. These negative returns are statistically significant and are not due to the higher or lower risk exposure of portfolios as risk-adjusted returns on these long-short portfolios are also significantly negative. Both returns and alphas from value-weighted zero-investment portfolios also yield negative results, though statistically insignificant.

Our liquidity measure based on the ordered fuzzy numbers places in the middle of a horserace regarding the return on a long-short liquidity portfolio. A portfolio based on the indication of this measure yields on average -0.207% per week, which is a better result than a portfolio based on LIQ^{Amihud} and $LIQ^{LOBslope}$ and worse than a portfolio based on LIQ^{BAS} and $LIQ^{LOBslope10}$.

Overall, our results of univariate portfolio sorting resemble those of Marshall and Young (2003) and Brennan and Subrahmanyam (1996) who both found a negative and significant relationship between the bid-ask spread and stock returns. However, they traced this relationship to the fact that the bid-ask spread is likely acting as a proxy for a risk variable related to the reciprocal of the stock's price due to an inaccurate beta estimation. This, in turn, suggests that the performances of our one-way sorted portfolios may be driven by other companies' characteristics somehow captured by our liquidity measures. We resemble this concern by carrying out cross-sectional regressions in the following section.

4.2. Cross-sectional regressions

Table 5 presents the results of the cross-sectional regressions. To test the robustness of the inferences, we use several models' specifications. In each specification, however, we use robust standard errors clustered by a firm and by a week to take potential heteroskedasticity and autocorrelation of residuals into account.

Panel A of Table 5 reports the slope coefficients for univariate models, with only one explanatory variable, i.e. stock liquidity. Thus, it resembles analyses presented in Table 4, though on a single stock, not a portfolio level. Interestingly, only coefficients on LIQ^{Amihud} and LIQ^{BAS} are significantly negative, confirming the results from Table 4. Coefficients on liquidity measures based on the LOB data, i.e. LIQ^{OFN}, $LIQ^{LOBslope10}$ and $LIQ^{LOBslope}$, are positive though insignificantly different from zero. Bearing in mind that negative returns on long-short liquidity portfolios are driven mostly by the negative return on the "long leg" of the portfolio, such results likely suggest that the relationship between stock liquidity and returns is non-linear or dependent on some stock features.

Coefficients reported in Panel B of Table 5 are from multivariate models, which beyond stock liquidity include other companies' characteristics likely affecting future stock returns, but without controlling for time-invariant elusive features (i.e. companies' fixed effects) and macroeconomic conditions (i.e. time-dummies). The results remain qualitatively unchanged, i.e. slope coefficients on LIQ^{Amihud} and LIQ^{BAS} are significantly negative, while on LIQ^{OFN}, $LIQ^{LOBslope10}$ and $LIQ^{LOBslope}$ they remain statistically insignificant.

Including companies' fixed effects (Panel C) and both companies' and time-fixed effects (Panel D) does not alter the conclusions. The only change is that the coefficient on LIQ^{Amihud} became statistically insignificant. The signs of the coefficients on control variables are mostly consistent with previous literature and expectations. A negative coefficient on *MV* suggests bigger firms yield lower future returns; a positive coefficient on *B-MV* implies higher returns on value stocks. A positive slope on *MOM* suggests that the momentum is likely to continue, while a positive coefficient on *VOL* means investors require higher returns on more risky stocks. Since *TURN* may serve as a proxy for both liquidity and investors' holding period, it is expected to negatively affect stock returns (Atkins & Dyl, 1997; Stereńczak, 2022), which is confirmed in our analyses.

To sum up, the results of the cross-sectional regressions slightly diverge from those of univariate portfolio sorts. All the results presented so far suggest there may be some non-linearities in the relationship between stock liquidity and returns which may alter the profitability of the long-short liquidity portfolios. To check this, we subsequently turn to the subsample analysis by forming portfolios from two-way dependent sorts.

4.3. Subsample analysis – bivariate portfolio sorting

For the subsample analysis, we form both equal- and value-weighted zero investment liquidity portfolios. The returns on those portfolios, sorted by various companies' characteristics and alternative liquidity measures are presented in Table 6 (equal-weighted portfolios) and in Table 7 (value-weighted portfolios). The results reported therein provide some interesting insights.

The most interesting insight is that long-short liquidity portfolios sorted by LIQ^{OFN} yield significantly positive returns in the subsample of the largest companies listed on the WSE. The weekly return of 0.184%, which accounts for 9.568% p.a, is significant at the 0.1 level. Risk-adjusted return on that portfolio ($\alpha_{Carhart}$) is even higher, which means the positive return on that portfolio is not due to higher exposure to risk. The above applies to the equal-weighted portfolio. Returns on value-weighted long-short liquidity portfolios for the subsample of the largest companies are positive when one uses LIQ^{OFN}, LIQ^{BAS} and $LIQ^{LOBslope10}$. All these returns are statistically significant, even after adjusting for risk. However, the LIQ^{OFN} portfolio yields the highest raw return.

The above inferences somehow contradict the findings of Cakici and Zaremba (2021), who found that liquidity premium exists only among microcap stocks. In their study, the average market value of a microcap stock is 0.2 USD billion. The average capitalisation of the subsample of the largest companies in our study equals roughly 3 USD billion, with a mean median value of roughly 1.8 USD billion. This, in turn, proves that the average size of companies among which we detected liquidity premium is roughly 3.8 times higher than those of Cakici and Zaremba (2021).

The abovementioned differences in the existence of liquidity premium among companies of different sizes may result from a different time horizon. Unlike Cakici and Zaremba (2021), who have studied the effect of stock liquidity on monthly returns, our study utilises weekly returns and weekly liquidity measures. Such divergence in the results of studies on different horizons would likely suggest the varying importance of liquidity according to the investors' horizons of portfolio rebalancing. However, one should recall the study of Cakici and Zaremba (2021) has covered companies listed on 45 markets around the globe, while ours covers only some of the companies listed in only one market. This also is a possible reason for the differences in the results.

On the other hand, Stereńczak et al. (2020) in their study of frontier markets found significantly positive returns on long-short liquidity portfolios among companies, whose stock prices most vividly co-move with international equities, so the diversification benefits are the smallest due to their high integration with the global economy. Since big companies listed in the WSE are likely the most integrated with the global economy, our results may simply capture that effect, directly supporting the hypothesis by Batten and Vo (2014) that the lack of a liquidity premium in less developed markets may be linked to their low integration with the global economy, resulting in some diversification benefits that offset low stock liquidity.

Zero-investment liquidity portfolios do not yield significantly positive returns in other subsamples, in particular based on *B-MV*, *MOM*, *VOL* and *TURN*. The lone exception is the capitalisation-weighted portfolio of companies with a moderately high turnover ratio. This portfolio brings a significant return of 0.298% per week, which translates into an annual return of 15.496%. The return on that portfolio is also significantly positive after adjusting for risk.

5. Further analyses (planned to do or in progress)

- *Buy- and sell-side liquidity*
- *Long- or short-side of the portfolio*
- *The role of transaction costs*
- *Liquidity portfolios vs. buy-and-hold*

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Table 1. Distributions of liquidity measures

Measure	Mean	Standard deviation	Coefficient of variation	Skewness	Kurtosis
Panel A: Time-series averages of cross-sectional statistics					
LIQ^{OFN}	0.4183	0.9840	2.3498	4.0716	19.772
LIQ^{Amihud}	21.267	96.777	3.6981	6.9421	58.907
LIQ^{BAS}	0.0093	0.0086	0.9030	2.6862	13.441
$LIQ^{LOBslope10}$	503.62	622.84	1.2258	3.9704	23.387
$LIQ^{LOBslope}$	162.34	141.69	0.8306	5.6670	50.062
Panel B: Cross-sectional averages of time-series statistics					
Measure	**Mean**	**Standard deviation**	**Coefficient of variation**	**Skewness**	**Kurtosis**
LIQ^{OFN}	0.2862	0.1756	0.7947	2.0457	9.9871
LIQ^{Amihud}	54.551	89.097	1.6328	3.7388	23.564
LIQ^{BAS}	0.0117	0.0054	0.4300	1.2979	4.2768
$LIQ^{LOBslope10}$	400.73	170.57	0.4267	1.3209	3.9555
$LIQ^{LOBslope}$	157.31	74.579	0.4077	1.6333	7.3048

Note: The table presents the descriptive statistics of compared liquidity proxies. Panel A demonstrates time-series averages of cross-sectional statistics. Panel B reports cross-sectional averages of time-series statistics.

Table 2. Correlations among liquidity measures

Panel A: Time-series averages of cross-sectional Pearson correlations					
Measure	LIQ^{OFN}	LIQ^{Amihud}	LIQ^{BAS}	$LIQ^{LOBslope10}$	$LIQ^{LOBslope}$
LIQ^{OFN}	1	0.1133	0.3678	0.7242	0.1419
LIQ^{Amihud}	0.1133	1	0.2519	0.1490	0.0361
LIQ^{BAS}	0.3678	0.2519	1	0.4431	0.1629
$LIQ^{LOBslope10}$	0.7242	0.1490	0.4431	1	0.5619
$LIQ^{LOBslope}$	0.1419	0.0361	0.1629	0.5619	1
Panel B: Cross-sectional averages of time-series Pearson correlations					
Measure	LIQ^{OFN}	LIQ^{Amihud}	LIQ^{BAS}	$LIQ^{LOBslope10}$	$LIQ^{LOBslope}$
LIQ^{OFN}	1	0.2574	0.5428	0.5079	0.2274
LIQ^{Amihud}	0.2574	1	0.3617	0.2358	0.1107
LIQ^{BAS}	0.5428	0.3617	1	0.6269	0.3167
$LIQ^{LOBslope10}$	0.5079	0.2358	0.6269	1	0.6992
$LIQ^{LOBslope}$	0.2274	0.1107	0.3167	0.6992	1
Panel C: Time-series averages of cross-sectional Spearman rank correlations					
Measure	LIQ^{OFN}	LIQ^{Amihud}	LIQ^{BAS}	$LIQ^{LOBslope10}$	$LIQ^{LOBslope}$
LIQ^{OFN}	1	0.7027	0.8938	0.8493	0.3519
LIQ^{Amihud}	0.7027	1	0.7018	0.6833	0.2927
LIQ^{BAS}	0.8938	0.7018	1	0.8941	0.3635
$LIQ^{LOBslope10}$	0.8493	0.6833	0.8941	1	0.5830
$LIQ^{LOBslope}$	0.3519	0.2927	0.3635	0.5830	1
Panel D: Absolute changes in liquidity ranks					
Measure	LIQ^{OFN}	LIQ^{Amihud}	LIQ^{BAS}	$LIQ^{LOBslope10}$	$LIQ^{LOBslope}$
The time-series avg of cross-sectional means	7.2925	17.383	10.674	12.408	22.030
Cross-sectional avg of time-series means	8.0004	18.273	11.078	13.226	22.463
Pooled mean	7.2916	17.384	10.673	12.407	22.031

Note: The table presents the correlation coefficients among analysed liquidity measures. Panel A demonstrates time-series averages of cross-sectional Pearson correlations. Panel B reports cross-sectional averages of time-series Pearson correlations. Panel C presents time-series averages of cross-sectional Spearman rank correlations, and Panel D reports absolute changes in a weekly change of a liquidity rank.

Table 3. Prediction errors of liquidity measures

Measure	LIQ^{OFN}	LIQ^{Amihud}	LIQ^{BAS}	$LIQ^{LOBslope10}$	$LIQ^{LOBslope}$
ARE	-0.2247	-1.9254	-0.0788	-0.0965	-0.0991
MAE	0.4013	2.5553	0.2254	0.2368	0.2395
RMSE	0.7189	14.662	0.2418	0.1836	0.2045

Note: The table presents the values of prediction errors of compared liquidity proxies. Forecasts are done using a simple AR(1) model using the data on the previous 26 weeks (half a year).

Table 4. Returns on univariate portfolio sorts

Panel A: Equal-weighted portfolios

Measure	***Illiq***	***2***	***3***	***4***	***Liq***	***Illiq-Liq***	α_{CAPM}	α_{FF3}	$\alpha_{Carhart}$
LIQ^{OFN}	-0.183*	-0.086	-0.067	0.062	0.024	-0.207**	-0.185**	-0.244***	-0.234**
	(2.087)	(2.708)	(2.789)	(2.615)	(2.846)	(2.090)	(2.12)	(2.67)	(2.57)
LIQ^{Amihud}	-0.229*	-0.080	0.001	0.047	0.001	-0.235**	-0.221**	-0.267**	-0.247**
	(2.644)	(2.614)	(2.514)	(2.360)	(2.878)	(2.081)	(2.24)	(2.49)	(2.39)
LIQ^{BAS}	-0.174*	-0.127	-0.075	0.121	0.001	-0.183*	-0.162*	-0.216**	-0.208**
	(2.069)	(2.813)	(2.705)	(2.671)	(2.724)	(2.000)	(1.93)	(2.43)	(2.35)
$LIQ^{LOBslope10}$	-0.182*	-0.089	-0.037	0.059	0.001	-0.186*	-0.168*	-0.189*	-0.182*
	(2.240)	(2.670)	(2.781)	(2.674)	(2.690)	(2.078)	(1.81)	(1.91)	(1.84)
$LIQ^{LOBslope}$	-0.211*	-0.001	-0.070	-0.039	0.078	-0.288***	-0.283***	-0.264***	-0.263***
	(2.487)	(2.660)	(2.666)	(2.511)	(2.521)	(1.681)	(3.46)	(2.79)	(2.79)

Panel B: Value-weighted portfolios

Measure	***Illiq***	***2***	***3***	***4***	***Liq***	***Illiq-Liq***	α_{CAPM}	α_{FF3}	$\alpha_{Carhart}$
LIQ^{OFN}	-0.080	0.040	-0.026	0.059	-0.047	-0.033	-0.008	-0.021	-0.033
	(1.914)	(2.307)	(2.578)	(2.540)	(2.824)	(2.221)	(0.10)	(0.27)	(0.42)
LIQ^{Amihud}	-0.175	-0.001	0.077	0.017	-0.045	-0.129	-0.110	-0.094	-0.102
	(2.293)	(2.345)	(2.517)	(2.455)	(2.838)	(2.182)	(1.12)	(0.97)	(1.06)
LIQ^{BAS}	-0.062	-0.072	0.014	0.140	-0.062	-0.000	0.024	0.011	-0.004
	(2.013)	(2.499)	(2.537)	(2.651)	(2.794)	(2.307)	(0.27)	(0.12)	(0.04)
$LIQ^{LOBslope10}$	-0.092	0.000	0.058	0.110	-0.072	-0.020	0.002	0.002	-0.008
	(2.065)	(2.493)	(2.614)	(2.711)	(2.763)	(2.167)	(0.02)	(0.02)	(0.09)
$LIQ^{LOBslope}$	0.064	0.045	-0.074	-0.107	0.074	-0.010	-0.001	0.003	0.016
	(2.672)	(2.947)	(2.926)	(2.698)	(2.869)	(2.178)	(0.00)	(0.03)	(0.15)

Note: The table presents the returns on quintile portfolios sorted by stock liquidity alongside the return on a zero investment portfolio that goes short on most liquid stocks and long on least liquid ones. The table also reports risk-adjusted returns on these portfolios computed from CAPM (α_{CAPM}), Fama and French's (1992) three-factor model (α_{FF3}) and Carhart's (1997) four-factor model ($\alpha_{Carhart}$). Both returns and alphas are expressed in percentage terms. Panel A demonstrates returns on equal-weighted portfolios and Panel B reports returns on value-weighted portfolios. The values in brackets are the standard deviations of returns (for portfolio returns) and *t*-statistics based on Newey and West's (1987) adjusted standard errors (for alphas). The asterisks ***, ** and * denote statistical significance at 0.01, 0.05 and 0.1 levels respectively.

Table 5. Results of cross-sectional regressions

Panel A: Univariate tests					
Measure	LIQ^{OFN}	LIQ^{Amihud}	LIQ^{BAS}	$LIQ^{LOBslope10}$	$LIQ^{LOBslope}$
const	-0.044	-0.025	0.093	-0.054	-0.055
	(0.37)	(0.21)	(0.75)	(0.41)	(0.40)
LIQ	0.018	-0.001**	-14.12***	0.000	0.000
	(0.80)	(2.24)	(2.95)	(0.57)	(0.50)
R^2	0.0000	0.0003	0.0005	0.0000	0.0000
Number of obs.	57,935	57,935	57,935	57,935	57,935
Panel B: Multivariate tests					
Measure	LIQ^{OFN}	LIQ^{Amihud}	LIQ^{BAS}	$LIQ^{LOBslope10}$	$LIQ^{LOBslope}$
const	-1.18	-0.745	0.074	-1.05	-0.839
	(1.43)	(1.08)	(0.10)	(1.23)	(1.21)
LIQ	-0.041	-0.001*	-14.35***	-0.000	0.000
	(1.47)	(1.94)	(2.66)	(0.47)	(0.23)
MV	0.049	0.028	-0.004	0.044	0.032
	(1.38)	(0.96)	(0.12)	(1.15)	(1.07)
B-MV	0.068*	0.068*	0.064	0.066	0.066
	(1.66)	(1.67)	(1.60)	(1.63)	(1.64)
MOM	18.35**	17.79**	16.92**	18.33**	18.52**
	(2.08)	(2.02)	(1.93)	(2.07)	(2.11)
VOL	0.836	0.842	0.302	0.713	0.650
	(0.29)	(0.30)	(0.11)	(0.25)	(0.23)
TURN	0.020***	0.017***	0.012***	0.021***	0.017***
	(4.34)	(4.70)	(2.58)	(2.79)	(2.75)
Fixed effects	No	No	No	No	No
Time effects	No	No	No	No	No
R^2	0.0008	0.0010	0.0012	0.0008	0.0008
Number of obs.	55,361	55,361	55,361	55,361	55,361
Panel C: Multivariate tests with fixed effects					
Measure	LIQ^{OFN}	LIQ^{Amihud}	LIQ^{BAS}	$LIQ^{LOBslope10}$	$LIQ^{LOBslope}$
const	13.83***	13.93***	15.89***	13.94***	13.78***
	(4.49)	(4.49)	(4.94)	(4.48)	(4.39)
LIQ	-0.017	-0.000	-29.37***	0.000	0.000
	(0.363)	(0.30)	(5.20)	(0.78)	(1.16)
MV	-0.688***	-0.693***	-0.775***	-0.695***	-0.688***
	(4.69)	(4.69)	(5.07)	(4.69)	(4.61)
B-MV	0.137**	0.138**	0.149**	0.139**	0.139**
	(2.13)	(2.12)	(2.28)	(2.15)	(2.16)
MOM	20.72***	20.57***	18.39***	20.56***	20.54***
	(3.78)	(3.78)	(3.35)	(3.76)	(3.75)
VOL	4.92***	4.91***	4.69**	4.94***	4.99***
	(2.69)	(2.69)	(2.52)	(2.69)	(2.72)
TURN	-0.085***	-0.084***	-0.094***	-0.085***	-0.085***
	(4.85)	(4.94)	(5.32)	(4.95)	(4.95)
Fixed effects	Yes	Yes	Yes	Yes	Yes
Time effects	No	No	No	No	No
R^2	0.0119	0.0119	0.0126	0.0119	0.0119
Number of obs.	55,361	55,361	55,361	55,361	55,361
Panel D: Multivariate tests with fixed effects and time dummies					
Measure	LIQ^{OFN}	LIQ^{Amihud}	LIQ^{BAS}	$LIQ^{LOBslope10}$	$LIQ^{LOBslope}$
const	16.79***	16.98***	18.23***	16.92***	16.92***
	(6.03)	(6.01)	(6.28)	(6.00)	(5.98)
LIQ	-0.050	-0.000	-21.86***	-0.000	0.000
	(1.03)	(0.37)	(3.76)	(0.38)	(0.49)
MV	-0.726***	-0.736***	-0.790***	-0.733***	-0.734***
	(5.57)	(5.55)	(5.80)	(5.54)	(5.52)
B-MV	0.142***	0.143***	0.153***	0.142***	0.142***
	(2.64)	(2.65)	(2.82)	(2.63)	(2.65)

MOM	17.66***	17.45***	16.66**	17.59***	17.52***
	(2.75)	(2.73)	(2.59)	(2.77)	(2.75)
VOL	-2.80	-2.84	-2.82	-2.85	-2.88
	(1.58)	(1.61)	(1.52)	(1.60)	(1.62)
TURN	-0.061***	-0.060***	-0.066***	-0.059***	-0.060***
	(3.49)	(3.56)	(3.81)	(3.54)	(3.55)
Fixed effects	Yes	Yes	Yes	Yes	Yes
Time effects	Yes	Yes	Yes	Yes	Yes
R^2	0.1907	0.1907	0.1911	0.1907	0.1907
Number of obs.	55,361	55,361	55,361	55,361	55,361

Note: The table reports the slope coefficients (βs, multiplied by 100) of the pooled cross-sectional time-series regressions. The raw returns are regressed on liquidity measures (Panel A) and additional control variables (Panels B, C and D). Panel B reports slope coefficients for models without any effects; Panel C demonstrates the slopes for the models with fixed effects and Panel D – for the models with both fixed effects and time dummies. The control variables are: market value (*MV*), book-to-market ratio (*B-MV*), momentum (*MOM*), return volatility (*VOL*), and stock turnover (*TURN*). The numbers in brackets are *t*-statistics with robust standard errors clustered by a firm and by week (Petersen, 2009). The asterisks ***, **, and * denote statistical significance at 0.01, 0.05 and 0.1 levels respectively.

Table 6. Returns on equal-weighted bivariate portfolio sorts

Panel A: Portfolios sorted on MV and LIQ

	Raw returns				$\alpha_{Carhart}$			
Measure	***Low MV***	***2***	***3***	***High MV***	***Low MV***	***2***	***3***	***High MV***
LIQ^{OFN}	-0.343**	-0.232*	-0.139	0.184*	-0.514***	-0.330**	-0.175	0.226**
	(2.07)	(1.66)	(1.18)	(1.64)	(2.72)	(2.42)	(1.50)	(1.87)
LIQ^{Amihud}	-0.306	-0.273**	-0.160	0.091	-0.370*	-0.350***	-0.237**	0.099
	(1.52)	(2.16)	(1.41)	(0.86)	(1.72)	(2.62)	(2.09)	(0.83)
LIQ^{BAS}	-0.271	-0.310**	-0.110	0.079	-0.481***	-0.487***	-0.096	0.055
	(1.58)	(2.25)	(0.99)	(0.79)	(2.58)	(3.56)	(0.85)	(0.51)
$LIQ^{LOBslope10}$	-0.255	-0.177	-0.034	0.085	-0.417**	-0.268*	-0.059	0.155
	(1.55)	(1.28)	(0.31)	(0.84)	(2.09)	(1.83)	(0.52)	(1.51)
$LIQ^{LOBslope}$	-0.376*	-0.304***	-0.078	-0.087	-0.258	-0.267**	-0.087	-0.033
	(1.96)	(2.81)	(0.68)	(0.81)	(1.13)	(2.18)	(0.75)	(0.29)

Panel B: Portfolios sorted on B-MV and LIQ

	Raw returns				$\alpha_{Carhart}$			
Measure	***Low B-MV***	***2***	***3***	***High B-MV***	***Low B-MV***	***2***	***3***	***High B-MV***
LIQ^{OFN}	-0.257*	0.033	-0.120	-0.168	-0.394***	0.049	-0.100	-0.081
	(1.84)	(0.27)	(0.92)	(1.05)	(2.65)	(0.39)	(0.82)	(0.46)
LIQ^{Amihud}	-0.322**	0.010	-0.202	-0.117	-0.457***	0.161	-0.208*	0.008
	(2.29)	(0.09)	(1.48)	(0.68)	(2.95)	(1.34)	(1.61)	(0.04)
LIQ^{BAS}	-0.209*	0.070	-0.172	-0.317*	-0.277**	0.050	-0.171	-0.306*
	(1.63)	(0.57)	(1.38)	(1.88)	(2.02)	(0.38)	(1.46)	(1.64)
$LIQ^{LOBslope10}$	-0.220*	0.022	-0.099	-0.329**	-0.316**	-0.005	-0.083	-0.301*
	(1.73)	(0.19)	(0.81)	(2.09)	(2.44)	(0.04)	(0.69)	(1.70)
$LIQ^{LOBslope}$	-0.184	-0.017	-0.143	-0.113	-0.265**	0.057	-0.123	-0.060
	(1.49)	(0.15)	(1.24)	(0.85)	(1.96)	(0.47)	(1.01)	(0.38)

Panel C: Portfolios sorted on MOM and LIQ

	Raw returns				$\alpha_{Carhart}$			
Measure	***Low MOM***	***2***	***3***	***High MOM***	***Low MOM***	***2***	***3***	***High MOM***
LIQ^{OFN}	-0.068	-0.044	-0.019	-0.242*	-0.045	-0.011	-0.044	-0.301**
	(0.40)	(0.35)	(0.18)	(1.76)	(0.24)	(0.09)	(0.41)	(2.14)
LIQ^{Amihud}	-0.161	-0.070	-0.035	-0.281**	-0.139	-0.087	-0.059	-0.299**
	(0.87)	(0.54)	(0.34)	(2.14)	(0.64)	(0.64)	(0.52)	(2.22)
LIQ^{BAS}	-0.224	0.025	-0.003	-0.447***	-0.272	0.007	-0.034	-0.514***
	(1.25)	(0.20)	(0.03)	(3.52)	(1.34)	(0.06)	(0.32)	(3.93)
$LIQ^{LOBslope10}$	-0.082	-0.009	-0.058	-0.323**	-0.097	-0.013	-0.100	-0.381***
	(0.48)	(0.08)	(0.56)	(2.49)	(0.49)	(0.10)	(0.89)	(2.97)
$LIQ^{LOBslope}$	-0.137	-0.032	-0.150	-0.317**	-0.138	-0.046	-0.111	-0.325**
	(0.84)	(0.30)	(1.53)	(2.34)	(0.70)	(0.40)	(1.02)	(2.09)

Panel D: Portfolios sorted on VOL and LIQ

	Raw returns				$\alpha_{Carhart}$			
Measure	***Low VOL***	***2***	***3***	***High VOL***	***Low VOL***	***2***	***3***	***High VOL***
LIQ^{OFN}	0.023	0.081	-0.144	-0.507***	0.057	0.069	-0.112	-0.539***
	(0.24)	(0.67)	(1.00)	(3.16)	(0.57)	(0.57)	(0.75)	(3.21)
LIQ^{Amihud}	0.044	-0.006	-0.132	-0.432**	0.046	0.037	-0.067	-0.480**
	(0.48)	(0.05)	(0.95)	(2.49)	(0.50)	(0.32)	(0.46)	(2.51)
LIQ^{BAS}	-0.003	0.112	-0.203	-0.395**	-0.008	0.107	-0.110	-0.437**
	(0.03)	(0.92)	(1.46)	(2.44)	(0.08)	(0.89)	(0.77)	(2.45)
$LIQ^{LOBslope10}$	0.045	0.000	-0.138	-0.537***	0.104	0.012	-0.064	-0.627***
	(0.47)	(0.01)	(0.96)	(3.48)	(1.04)	(0.10)	(0.41)	(3.70)
$LIQ^{LOBslope}$	-0.164*	-0.096	-0.110	-0.263	-0.158	-0.090	-0.062	-0.214
	(1.87)	(0.98)	(0.85)	(1.55)	(1.59)	(0.87)	(0.41)	(1.09)

Panel E: Portfolios sorted on TURN and LIQ

	Raw returns				$\alpha_{Carhart}$			
Measure	***Low TURN***	***2***	***3***	***High TURN***	***Low TURN***	***2***	***3***	***High TURN***
LIQ^{OFN}	-0.163*	-0.219*	0.096	-0.278*	-0.208**	-0.200	0.175	-0.162
	(1.61)	(1.78)	(0.70)	(1.73)	(1.97)	(1.59)	(1.20)	(0.96)

LIQ^{Amihud}	-0.292***	-0.099	0.030	-0.197	-0.274**	-0.086	0.076	-0.013
	(3.02)	(0.90)	(0.22)	(1.18)	(2.54)	(0.72)	(0.50)	(0.07)
LIQ^{BAS}	-0.089	-0.126	-0.048	-0.359**	-0.143	-0.129	0.030	-0.232
	(0.84)	(1.17)	(0.36)	(2.12)	(1.29)	(1.27)	(0.20)	(1.23)
$LIQ^{LOBslope10}$	-0.127	-0.148	0.017	-0.445***	-0.177	-0.089	0.092	-0.284
	(1.28)	(1.27)	(0.13)	(2.59)	(1.60)	(0.75)	(0.61)	(1.46)
$LIQ^{LOBslope}$	-0.214**	-0.053	-0.257**	-0.378**	-0.223**	-0.014	-0.201*	-0.259
	(2.24)	(0.49)	(2.29)	(2.41)	(1.99)	(0.13)	(1.65)	(1.43)

Note: The table presents the returns on equal-weighted two-way sorted quartile zero investment portfolios that goes short on most liquid stocks and long on least liquid ones. In the first pass, stocks are ranked according to the value of one of the variables, and then, within each quartile, a long-short quartile portfolio based on LIQ is formed. Left side of the table reports raw returns on long-short portfolios and the right side demonstrates risk-adjusted returns on these portfolios computed from Carhart's (1997) four-factor model ($\alpha_{Carhart}$). Both returns and alphas are expressed in percentage terms. Panel A demonstrates the results for portfolios sorted on *MV* and *LIQ*; Panel B reports the returns on portfolios sorted by *B-MV* and *LIQ*; Panel C is devoted to portfolios sorted on *MOM* and *LIQ*; Panel D reports returns on portfolios sorted on *VOL* and *LIQ*; and Panel E demonstrates the results for portfolios sorted on *TURN* and *LIQ*. The values in brackets are the *t*-statistics based on Newey and West's (1987) adjusted standard errors. The asterisks ***, ** and * denote statistical significance at 0.01, 0.05 and 0.1 levels respectively.

Table 7. Returns on value-weighted bivariate portfolio sorts

Panel A: Portfolios sorted on MV and LIQ

	Raw returns				$\alpha_{Carhart}$			
Measure	***Low MV***	***2***	***3***	***High MV***	***Low MV***	***2***	***3***	***High MV***
LIQ^{OFN}	-0.212	-0.212	-0.172	0.199*	-0.277*	-0.248*	-0.173	0.187*
	(1.35)	(1.53)	(1.42)	(1.80)	(1.77)	(1.96)	(1.59)	(1.74)
LIQ^{Amihud}	-0.316*	-0.249**	-0.197*	0.164	-0.311*	-0.269**	-0.178	0.145
	(1.92)	(2.03)	(1.66)	(1.51)	(1.87)	(2.20)	(1.60)	(1.32)
LIQ^{BAS}	-0.227	-0.278**	-0.121	0.192*	-0.291*	-0.329***	-0.108	0.202*
	(1.47)	(2.08)	(1.05)	(1.83)	(1.88)	(2.69)	(0.99)	(1.90)
$LIQ^{LOBslope10}$	-0.214	-0.150	-0.045	0.175*	-0.260*	-0.157	-0.015	0.200*
	(1.40)	(1.13)	(0.37)	(1.69)	(1.71)	(1.25)	(0.14)	(1.93)
$LIQ^{LOBslope}$	-0.346**	-0.278***	-0.004	-0.036	-0.313*	-0.247**	0.029	-0.003
	(2.21)	(2.63)	(0.03)	(0.29)	(1.91)	(2.25)	(0.26)	(0.02)

Panel B: Portfolios sorted on B-MV and LIQ

	Raw returns				$\alpha_{Carhart}$			
Measure	***Low B-MV***	***2***	***3***	***High B-MV***	***Low B-MV***	***2***	***3***	***High B-MV***
LIQ^{OFN}	-0.016	0.037	-0.003	-0.029	-0.119	0.116	0.008	0.019
	(0.11)	(0.27)	(0.02)	(0.17)	(0.91)	(0.91)	(0.06)	(0.12)
LIQ^{Amihud}	-0.106	0.055	-0.163	-0.051	-0.170	0.155	-0.120	0.037
	(0.73)	(0.37)	(1.15)	(0.26)	(1.28)	(1.08)	(0.88)	(0.20)
LIQ^{BAS}	-0.027	0.130	-0.096	-0.126	-0.118	0.197	-0.089	-0.078
	(0.18)	(0.92)	(0.70)	(0.71)	(0.90)	(1.54)	(0.70)	(0.45)
$LIQ^{LOBslope10}$	-0.068	0.142	-0.032	-0.072	-0.157	0.215	-0.036	-0.035
	(0.47)	(0.98)	(0.24)	(0.43)	(1.18)	(1.57)	(0.29)	(0.21)
$LIQ^{LOBslope}$	0.147	0.223	-0.117	-0.129	0.101	0.290*	-0.099	-0.133
	(0.96)	(1.50)	(0.72)	(0.74)	(0.66)	(1.96)	(0.65)	(0.74)

Panel C: Portfolios sorted on MOM and LIQ

	Raw returns				$\alpha_{Carhart}$			
Measure	***Low MOM***	***2***	***3***	***High MOM***	***Low MOM***	***2***	***3***	***High MOM***
LIQ^{OFN}	-0.117	-0.005	0.093	-0.150	-0.139	0.019	0.076	-0.134
	(0.68)	(0.03)	(0.80)	(1.00)	(0.81)	(0.14)	(0.69)	(0.89)
LIQ^{Amihud}	-0.315*	0.010	0.133	-0.217	-0.300	0.031	0.142	-0.189
	(1.63)	(0.07)	(1.04)	(1.54)	(1.53)	(0.23)	(1.11)	(1.30)
LIQ^{BAS}	-0.310*	0.027	0.071	-0.349**	-0.313*	0.036	0.045	-0.316**
	(1.70)	(0.18)	(0.59)	(2.28)	(1.72)	(0.28)	(0.39)	(2.06)
$LIQ^{LOBslope10}$	-0.166	0.036	-0.000	-0.164	-0.189	0.071	-0.017	-0.131
	(0.92)	(0.25)	(0.00)	(1.05)	(1.02)	(0.55)	(0.14)	(0.83)
$LIQ^{LOBslope}$	-0.285*	0.046	-0.128	-0.163	-0.338*	0.121	-0.104	-0.074
	(1.72)	(0.30)	(0.99)	(0.89)	(1.95)	(0.78)	(0.80)	(0.42)

Panel D: Portfolios sorted on VOL and LIQ

	Raw returns				$\alpha_{Carhart}$			
Measure	***Low VOL***	***2***	***3***	***High VOL***	***Low VOL***	***2***	***3***	***High VOL***
LIQ^{OFN}	0.080	0.121	-0.049	-0.233	0.084	0.159	-0.016	-0.214
	(0.74)	(0.84)	(0.29)	(1.13)	(0.85)	(1.32)	(0.10)	(1.08)
LIQ^{Amihud}	0.021	0.044	-0.165	-0.388**	0.026	0.092	-0.103	-0.345*
	(0.18)	(0.33)	(0.96)	(2.00)	(0.24)	(0.76)	(0.58)	(1.78)
LIQ^{BAS}	0.073	0.126	-0.125	-0.265	0.064	0.177	-0.080	-0.226
	(0.65)	(0.84)	(0.71)	(1.27)	(0.63)	(1.37)	(0.46)	(1.11)
$LIQ^{LOBslope10}$	0.066	-0.001	-0.134	-0.182	0.062	0.076	-0.068	-0.136
	(0.58)	(0.01)	(0.77)	(0.88)	(0.58)	(0.61)	(0.38)	(0.66)
$LIQ^{LOBslope}$	-0.142	0.050	-0.188	-0.008	-0.143	0.141	-0.142	0.007
	(1.15)	(0.32)	(1.14)	(0.03)	(1.10)	(0.96)	(0.85)	(0.03)

Panel E: Portfolios sorted on TURN and LIQ

	Raw returns				$\alpha_{Carhart}$			
Measure	***Low TURN***	***2***	***3***	***High TURN***	***Low TURN***	***2***	***3***	***High TURN***
LIQ^{OFN}	-0.184	-0.088	0.298**	0.130	-0.157	-0.072	0.361***	0.120
	(1.46)	(0.57)	(2.15)	(0.77)	(1.34)	(0.53)	(2.73)	(0.75)

LIQ^{Amihud}	-0.266**	0.097	0.174	0.038	-0.235*	0.142	0.219*	0.060
	(2.08)	(0.67)	(1.26)	(0.23)	(1.82)	(1.06)	(1.63)	(0.38)
LIQ^{BAS}	-0.185	0.101	-0.021	-0.001	-0.189	0.147	0.046	-0.021
	(1.40)	(0.72)	(0.14)	(0.01)	(1.51)	(1.20)	(0.32)	(0.13)
$LIQ^{LOBslope10}$	-0.211*	-0.008	0.144	0.021	-0.180	0.039	0.219	0.036
	(1.68)	(0.05)	(1.01)	(0.12)	(1.50)	(0.28)	(1.56)	(0.21)
$LIQ^{LOBslope}$	-0.270**	0.232	-0.225	-0.019	-0.267**	0.265*	-0.199	0.024
	(2.23)	(1.53)	(1.41)	(0.08)	(2.10)	(1.85)	(1.26)	(0.11)

Note: The table presents the returns on capitalisation-weighted two-way sorted quartile zero investment portfolios that goes short on most liquid stocks and long on least liquid ones. In the first pass, stocks are ranked according to the value of one of the variables, and then, within each quartile, a long-short quartile portfolio based on LIQ is formed. Left side of the table reports raw returns on long-short portfolios and the right side demonstrates risk-adjusted returns on these portfolios computed from Carhart’s (1997) four-factor model ($\alpha_{Carhart}$). Both returns and alphas are expressed in percentage terms. Panel A demonstrates the results for portfolios sorted on *MV* and *LIQ*; Panel B reports the returns on portfolios sorted by *B-MV* and *LIQ*; Panel C is devoted to portfolios sorted on *MOM* and *LIQ*; Panel D reports returns on portfolios sorted on *VOL* and *LIQ*; and Panel E demonstrates the results for portfolios sorted on *TURN* and *LIQ*. The values in brackets are the *t*-statistics based on Newey and West's (1987) adjusted standard errors. The asterisks ***, ** and * denote statistical significance at 0.01, 0.05 and 0.1 levels respectively.

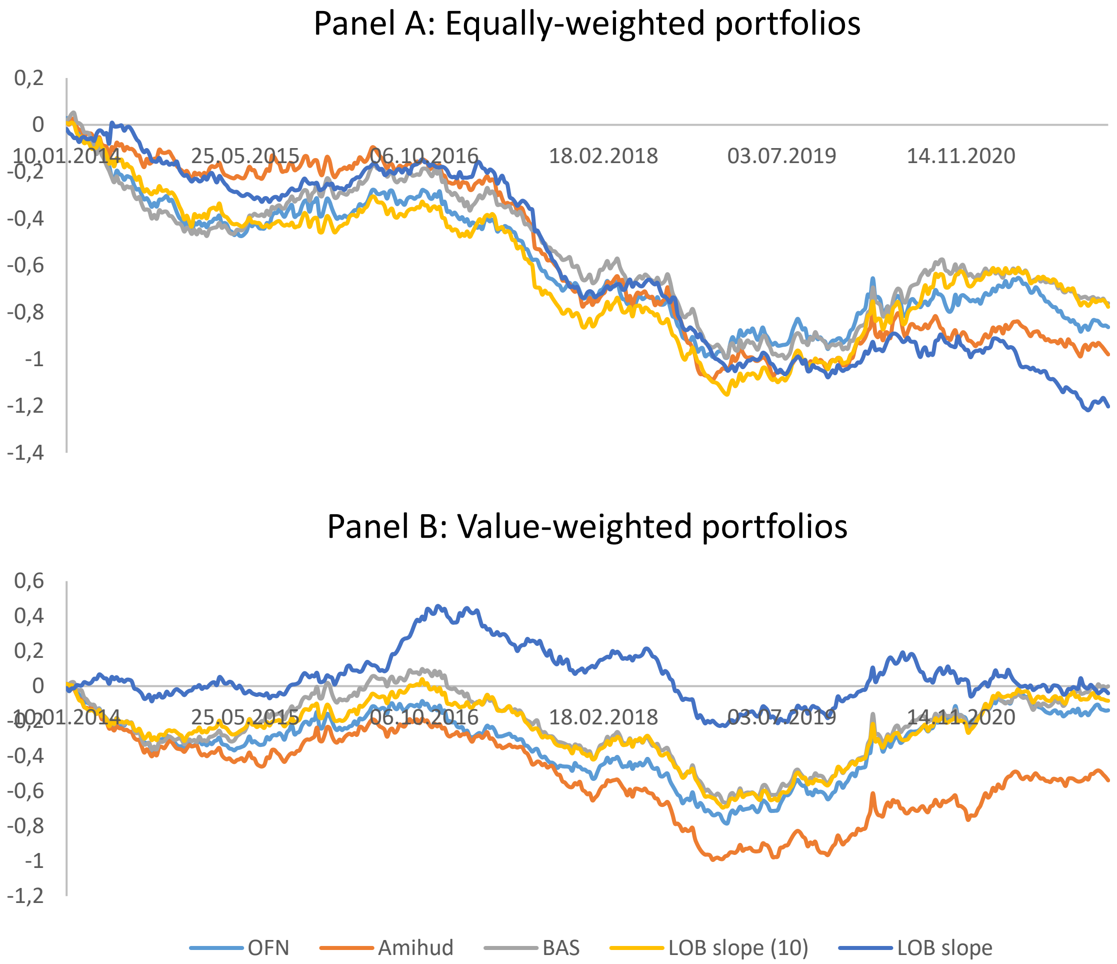

Figure 1. Cumulative long-short returns on long-short portfolios formed on stock liquidity
Note: The figure presents the cumulative returns on zero-investment equal-weighted (Panel A) and value-weighted (Panel B) portfolios formed on various liquidity measures. The portfolios go long the quintile of the least liquid stocks and short the quintile of the most liquid ones. The returns are expressed in percentage terms.