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Modelling the implied volatility – a case of EUR/PLN currency options

Abstract

Implied volatility is quoted by market makers for OTC foreign exchange options. It builds a volatility surface that allows pricing of all vanilla contracts. The paper presents a model that explains implied volatility changes in three dimensions: as a volatility level, a slope of the volatility curve and a slope of the volatility smile. The long term time series from the EUR/PLN market are used to calibrate the model. The evidence shows a tight interdependence between prices of option strategies. The interdependence can be used to build error correction models based on cointegration of time series. In the models ATM volatility is explained by spot returns and historical standard deviation. Moreover the shape of volatility curve and volatility smile are explained by the level of ATM volatility. The models have significant forecasting value quantified as a directional quality measure. The error correction component improves a correctness of forecasting decisions.

Keywords: currency options, volatility surface, error correction models, directional quality measure

Introduction and literature review

Foreign exchange options are a dominant non-linear derivative in Polish zloty – in 2022 their share in local option market (including interest rate and equity contracts) amounted to 96% (NBP 2023). The turnover is generated on the OTC market with the prevailing share of non-residents (93% according to NBP 2023). The prices are quoted in implied volatility terms. The volatility is dependent on delta and maturity of the contract. The standard model used for premium calculations of European vanilla currency options is the Garman-Kohlhagen formula (1983).

The subject of volatility modelling is widespread in financial literature since the publication of first equity options models (Black, Scholes 1973, Merton 1976). Since then, the volatility is treated as the exogenous coefficient that is shaped directly by demand and supply on the options market.

EUR/PLN market is treated as an emerging market with the significant asymmetry of risk. The statistical evidence of such pattern is skewness and kurtosis of daily returns. The skewness for the whole sample (22.07.2010-24.10.2023) was 0,22 and the average annual skewness was +0,17 (with a range from -0,25 to +0,59). The excess kurtosis for the whole sample was 4,42 with the annual average 1,80. Non-normality of daily returns and fat tails of the density function have profound consequences in the FX option pricing.

First of all, the asymmetry of risk means high positive risk reversal prices (Zhang, Xiang 2008; Santa-Clara, Saretto 2009). Risk reversal is a simultaneous purchase of low delta call and a sale of low delta put (or otherwise). For EUR/PLN one observes a constant demand on high strikes that is covered with a supply on low strikes. A significant share of investors that hold positive carry exposure on zloty market (i.e. long PLN position) hedge against the possible currency crisis. In effect, the average price of risk reversal in the analysed period was +1,23 (a difference in volatility terms between 25-delta EUR call PLN put and 25-delta EUR put PLN call calculated as a daily average of four most liquid maturities – 1M, 3M, 6M and 1Y) with a maximum at +4,12 and minimum at -0,37.

Moreover, one observes an interdependence between the price of the underlying asset and its implied volatility. In EUR/PLN market an average correlation between daily changes of option and spot prices for the whole sample amounts to 30% (for details see table 3). It means that zloty depreciation brings hike of implied volatility for zloty options. It is related to the evidence of growing variance of the spot rate if it is recorded on higher levels. A correlation between log-returns of the spot level in a three-month window and changes of a standard deviation in such window amounts for a whole sample to 18%. However market makers add a premium for non-normality of daily returns as they use a pricing model that theoretical assumptions are not met (Lim et al 2006; Hood et al 2009; Simonato & Stentoft 2015). The statistical evidence of such behaviour is an average excess of implied volatility over realized volatility (from 0,47 for one month to 0,84 for one year in percentage points for the whole sample).

A positive correlation between changes of historical and implied volatility is slightly visible only for standard deviation calculated on past returns. Such correlation (whole sample average for all maturities) amounts to 13%. It is the evidence of moderate influence of historical performance on market makers. On contrary correlation with future returns (therefore comparison of realized and forecasted volatility) is negative and close to zero (for details see table 3). It might be explained by the fact, that implied volatility is not a forecast of a future standard deviation as the pricing model has embedded assumptions that are not met in the real financial market. The latter is a realisation of the Rebonato thesis (2004): "Implied volatility is the wrong number to put in the wrong formula to get the right price of plain-vanilla options."

A complete volatility surface is a matrix consisting of market prices for options with various strike prices (i.e. deltas) and expiry dates (i.e. maturities). In order to build such surface one should know both a volatility curve and a volatility smile (Cont et al 2002). The volatility curve is a dependence of implied volatility on maturity of the contract. The volatility smile is a dependence on strike (generalized in delta terms). The central points of the volatility surface are generated with prices for at-the-money options (ATM) that are quotes for so called zero-delta straddles (Ahoniemi 2009).

A shape and a level of the volatility curve takes into account the dynamics of implied volatilities for various maturities. They envisage a mean reversion phenomenon (Bali & Demitras 2008; Goudarzi 2013; Ahmed et al 2018). The evidence of such pattern is a comparison of a standard deviation of daily changes of implied volatility for various maturities. For the whole sample a 1M instability is two times higher than for 1Y. If we take into account the standard deviation of historical volatility these divergence is even stronger (nine times). It means that short term volatility is much more sensitive for market changes than long term volatility. The latter is closer to long term mean. In practice one observes inverted volatility curve if a general level of volatility is high and a normal curve if volatility is low. The correlation between changes of the level and changes of the slope amounts to -58% for the whole sample. The volatility curve is built on the basis of so called calendar spreads (simultaneous trade on options with different maturities).

In turn, the volatility smile encompasses an expected skewness and kurtosis of daily returns (Hafner, Schmidt 2005). Market makers quote option strategies that directly forecast higher moments of the density function. These strategies are aforementioned risk reversal for the skewness and butterfly for the kurtosis. The latter has limited liquidity and low sensitiveness of prices for market impulses, therefore is not analysed in this paper. On contrary the risk reversal is very liquid and its price shapes a slope of volatility smile. The slope is directly dependent on the level of risk reversal prices. However the statistical evidence exhibits a reliance of risk reversal on a level of implied volatility for ATM options. The correlation between 3M risk reversal and 3M straddle amounts to 31% for a whole sample.

Taking into account the abovementioned considerations, one can build three models: one referring to the level of implied volatility for ATM options, the second describing the slope of the volatility curve and the third explaining the slope of volatility smile. The paper presents such models including error correction mechanism and assess they forecasting power as far as direction of volatility changes is concerned.

Methodology and results

The research encompasses the time series taken from EUR/PLN option market for a period 22/07/2010-31/07/2023 (3289 daily observations). The data source is Refinitv. Three month period was taken as a basic maturity due to its adequate liquidity and its location in the geometric middle of the liquid volatility curve. The list of variables presents table 1.

The survey was conducted twofold: in the first step the models were calibrated for the full sample and the models' efficiency was estimated on the same sample. Such approach allowed to assess the quality of the model for annual sub-periods within the sample. In the second step, the models were calibrated for the training period only (2010-2021) and forecasting was performed for the validating period (2022-2023). This approach was a stability check of the applied methodology.

The models were constructed in two forms: as a simple ordinary least square (OLS) on daily returns and as error correction model (ECM) based on cointegration tests. Due to that one can estimate the added value of the error correction component used in ECM models.

In order to build error correction models one shall perform the following procedure:

1. stationarity assessment of variables (see table 2)
2. Granger causality analysis (see table 4)
3. construction of cointegration equations and evaluation of the stationarity of its residuals (see table 5a and 5b)
4. estimation of error correction model (see table 6a and 6b)

The procedure is based on Engle-Granger cointegration test (1987) and Charemza & Deadman (1997).

The ECM has the following construction:

$$\Delta y_t = \alpha_1 \times \Delta x_t + \alpha_2 \times uhat_{t-1} + \varepsilon_t$$

where:

$uhat_{t-1} = \mathrm{y}_{t-1} - \beta_0 - \beta_1 \times \mathrm{x}_{t-1}$

$y_t \sim \mathrm{I}(1)$

$x_t \sim \mathrm{I}(1)$

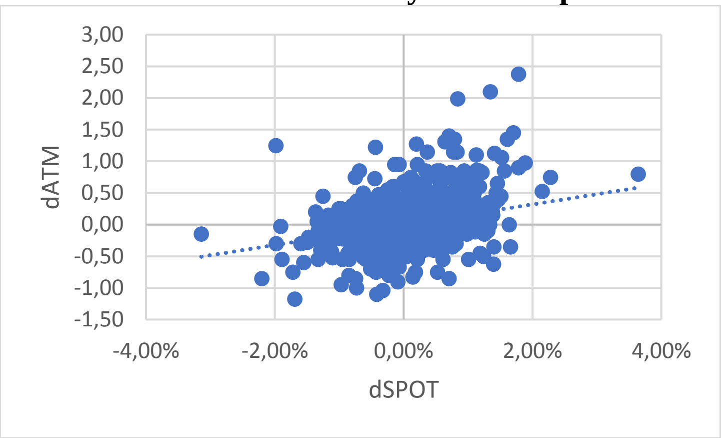
$uhat_t \sim \mathrm{I}(0)$

Having in mind correlation and causality analysis the following models are constructed:

1. changes of ATM Implied volatility explained by spot returns (model 1)
2. changes of ATM Implied volatility explained by changes historical standard deviation (model 2)
3. changes of volatility curve slope explained by changes of ATM implied volatility (model 3)
4. changes of risk reversal explained by changes of ATM implied volatility (model 4)

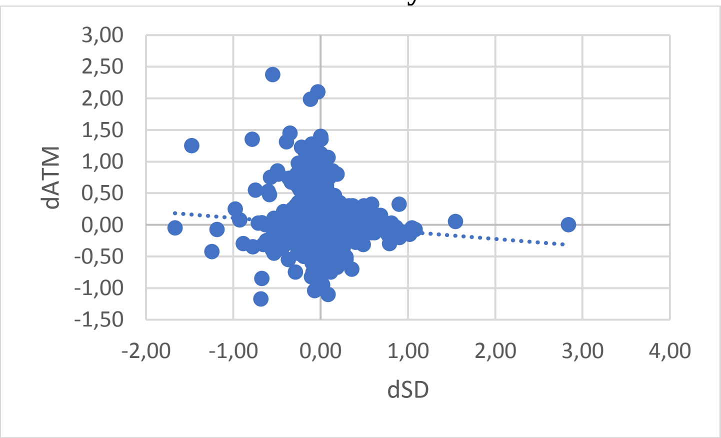
A plot of daily returns in all abovementioned models is presented in charts 1-4.

Chart 1. Plot of ATM volatility and FX Spot returns



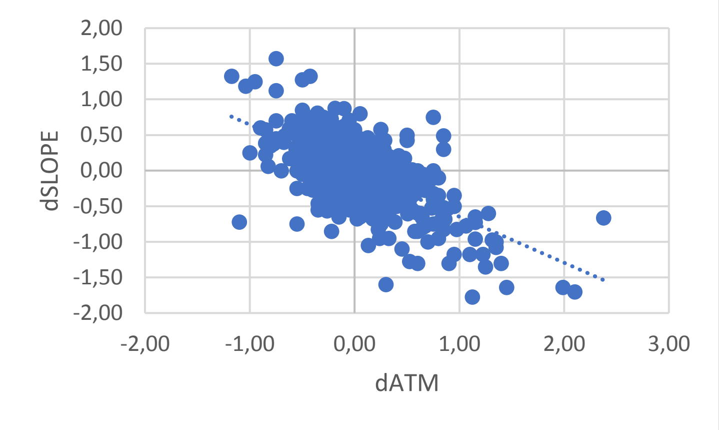
Source: own calculations

Chart 2. Plot of ATM volatility and historical standard deviation



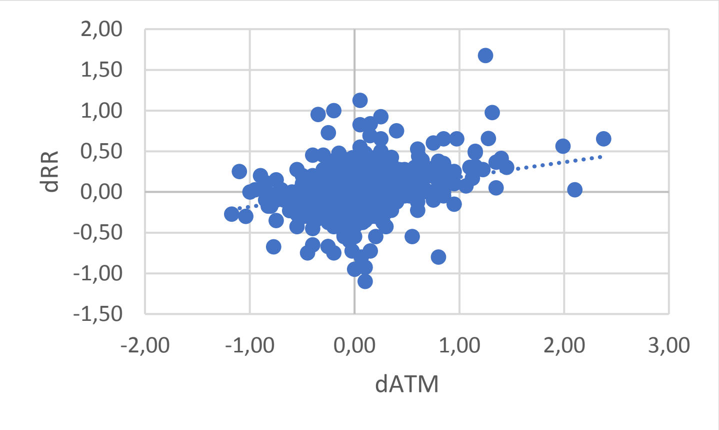
Source: own calculations

Chart 3. Plot of volatility curve slope and ATM volatility



Source: own calculations

Chart 4. Plot of Risk Reversal and ATM volatility



Source: own calculations

The rationale of such models is as follows:

1. On emerging currency markets depreciation of local currency brings higher risk that is presented in more expensive FX options. It means higher volatility in high FX rate environment. Therefore one can explain changes in implied volatility with spot returns.
2. Changes in implied volatility are connected with changes in historical volatility referring to the same number of days. However due to psychological and statistical reasons described in the Introduction, the link is observed between implied and past standard deviations (and not with standard deviation realized during the life of the contract).
3. Due to mean reversion, a shape of volatility curve is strictly dependent on a level of implied volatility. Therefore a change in the shape may be caused by parallel shifts in volatility.
4. Volatility smile is mostly determined by risk reversal. As higher skewness is observed in volatile environment, it links risk reversal prices with ATM levels. Risk aversion causes a demand both in straddles (i.e. ATM) and risk reversals as market players hedge against both spot and volatility rise (due to the phenomenon described in the point 1).

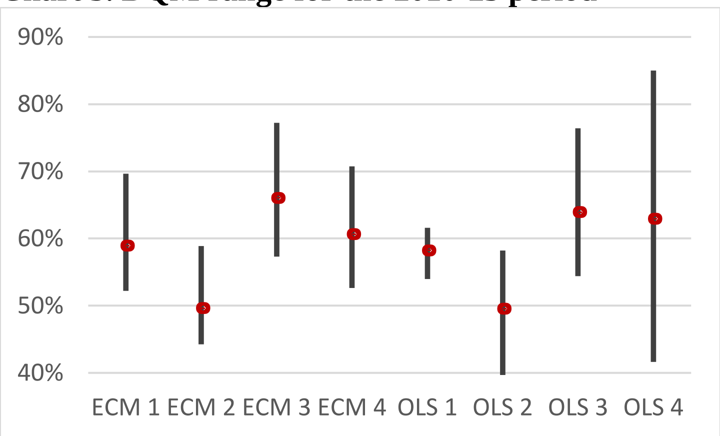
ECM can be constructed if both time series are I(1) and the residual time series is I(0). The stationarity tests give ambiguous results (see tables 2 and 5). Nevertheless the models have significant parameters and a sign for the error correction component α_2 is always negative (see table 6a and 6b).

In order to check serviceableness of the models, the robustness check was performed. The practical value of the model can be confirmed if the model forecasts the changes of the explanatory variable. According to Levich (2001) the quality of the forecast in financial market can be related to direction and not to the size of the change (Chan-Lau, Mendez-Morales 2003; Campbell et al. 2014). In order to verify this feature, the Direction Quality Measure (DQM) was calculated. The results (presented in table 7a and 7b) indicate that models 1, 3 and 4 presents significant added value in forecasting a direction of the variable change. Model 2 based on historical volatility has random forecasting power but it performed well in the validating period. In general, DQM values for the validating period are much better than the average coefficient for the full sample. It is an evidence of a significant usefulness of the contemporary forecasts based on the calibrated models.

Finally, the overperformance of ECM models was measured. The ordinary least squares models (OLS) were estimated and both mean square error (MSE) and DQM were calculated. The comparison of the results is presented in table 8a and 8b. It is visible that - for the full sample - apart from DQM for the model 4 all ECM overperform OLS In both measures. For the validating period the results are less convincing – DQM for models 3 and 4 and MSE for model 4 are better for OLS model.

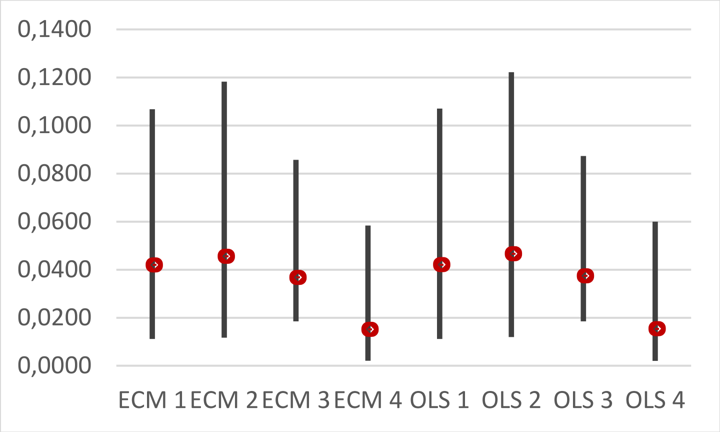
The stability of DQM and MSE measures can be verified in 14 annual sub-periods. The results present charts 5 and 6. The MSE ranges are similar for both types of equations. However for DQM one can notice much wider ranger for OLS. It is an evidence of the higher stability of a forecasting power for ECM models. Therefore, the added value of error correction component in forecasting a direction of daily returns is verified.

Chart 5. DQM range for the 2010-23 period



Red dot: average value, black bar: min-max for 14 annual periods
Source: own calculations

Chart 6. MSE range for the 2010-23 period



Red dot: average value, black bar: min-max for 14 annual periods
Source: own calculations

<u>Conclusions</u>

FX options are a popular hedging and investment tool, especially on emerging markets with small open economy having its own currency, like Poland. Prices of FX options are entirely shaped by implied volatilities feeding the Garman-Kohlhagen model. The volatilities quoted by market makers are collected in matrices known volatility surfaces. The surfaces are built on the basis of the prices of option strategies. The general level of the surface is created by zero-delta straddles, the maturity axis is shaped by calendar spreads and the delta axis by risk reversals.

Implied volatilities are connected endogenously – risk reversals and calendar spreads are correlated with straddles. Moreover one observes exogenous influence generated by an underlying price and its historical variance. For FX options it means correlation of volatility changes with spot returns and shifts in a historical standard deviation.

In order to verify such interdependence, the OLS and ECM models were calibrated. The models were built on the basis of long term EUR/PLN options time series. Such approach allowed to assess the added value of the error correction component based on cointegration of the time series. Moreover, the forecasting performance and stability of the models were evaluated.

The analysis of coefficients and a forecasting power of the models allowed to verify the determinants of the level and shape of the EUR/PLN volatility surface. The models confirm the sign and strength of correlation and causality estimated on the basis of historical time series. Moreover the models express a significant forecasting power as far as a direction of the price change is concerned. The direction quality measure for majority of the models is significantly over 60%.

Attachments.

Table 1. Description of variables

variable	description
SPOT	EUR/PLN spot exchange rate (EoD)
3M ATM	3-month EUR/PLN implied volatility for ATM option (zero-delta straddle)
3M RR	3-month EUR/PLN implied volatility for 25-delta risk reversal (in euro call terms)
3M SD	3-month EUR/PLN historical volatility (SD on log-returns p.a.)
SLOPE	difference between 1Y and 1M implied volatility for ATM options (calendar spread)

Source: own elaboration

Table 2. Stationarity tests

variable	*p-value* for ADF test	*p-value* for KPSS test	ADF decision - H0: I(1)	KPSS decision - H0: I(0)
SPOT	0,1401	value p < .01	I(1)	I(1)
3M ATM	0,0168	value p < .01	-	I(1)
3M RR	0,0010	value p < .01	I(0)	I(1)
3M SD	0,0000	value p < .01	I(0)	I(1)
SLOPE	0,0000	value p < .01	I(0)	I(1)

Source: own calculations

Table 3. Pearson correlation matrix (for daily returns)

	1M ATM	3M ATM	6M ATM	1Y ATM
SPOT	32%	32%	29%	25%
RR	33%	31%	24%	11%
SD1	11%	15%	14%	12%
SD2	-7%	-9%	-10%	-6%

Source: own calculations

Table 4. Granger causality tests (for daily returns)

	SPOT	3M ATM	3M RR	3M SD
SPOT		10,031 [0,0000]*	33,117 [0,0000]*	3,6827 [0,0025]*
3M ATM	0,5771 [0,7176]		16,916 [0,0000]*	7,9984 [0,0000]*
3M RR	1,5067 [0,1842]	4,1901 [0,0008]*		5,4973 [0,0000]*
3M SD	0,9866 [0,4243]	1,8946 [0,0919]*	0,2716 [0,9288]	

* means rejection of the hypothesis about the lack of causality at 0,10 confidence level
The table presents F-statistics for a test that variables in a left-hand column Granger cause variables in a top row.
Source: own calculations

Table 5a. Residuals stationarity Engle-Granger test (full sample model)

Model	Residual variable	*p-value* for ADF test	ADF value for m=4, n=3284	MacKinnon (1994) decision	Blangiewicz-Charemza (1990) decision
1	uhat1	0,0004	-3,5194	I(0)	rejected
2	uhat2	0,0000	-4,9595	I(0)	I(0)
3	uhat3	0,0000	-5,5724	I(0)	I(0)
4	uhat4	0,0000	-4,6977	I(0)	I(0)

Extrapolated critical value according to Blangiewicz-Charemza (1990) amounts to 3,88-4,17.
Source: own calculations

Table 5b. Residuals stationarity Engle-Granger test (training sample model)

Model	Residual variable	*p-value* for ADF test	ADF value for m=4, n=2889	MacKinnon (1994) decision	Blangiewicz-Charemza (1990) decision
1	uhat1	0,0009	-3,3240	I(0)	rejected
2	uhat2	0,0001	-4,0312	I(0)	rejected
3	uhat3	0,0000	-5,0795	I(0)	I(0)
4	uhat4	0,0000	-5,2407	I(0)	I(0)

Extrapolated critical value according to Blangiewicz-Charemza (1990) amounts to 4,09-4,38.
Source: own calculations

Table 6a. Error correction models (full sample model)

Model	Response variable	Explanatory variable	α1	α2	β0	β1
1	3M VOL	EURPLN	15,9853***	−0,0048***	9,0466***	−0,5205***
2	3M VOL	3M SD	-0,1393***	-0,0180 ***	3,4137***	0,5325***
3	SLOPE	3M VOL	-0,6468***	-0,0226***	1,0460***	0,0453***
4	3M RR	3M VOL	0,1813***	−0,0399***	−0,7478***	0,2750***

Source: own calculations

Table 6b. Error correction models (training sample model)

Model	Response variable	Explanatory variable	α_1	α_2	β_0	β_1
1	3M VOL	EURPLN	14,7215***	-0,0045***	21,9061***	-3,5772***
2	3M VOL	3M SD	-0,1103***	-0,0172***	2,9705***	0,5878***
3	SLOPE	3M VOL	-0,6029***	-0,0248***	0,6832**	0,0245***
4	3M RR	3M VOL	0,1535***	-0,0601***	-0,6246***	0,2434***

Source: own calculations

Table 7a. Directional Quality Measure for ECM models (full sample model)

Model	Response variable	Explanatory variable	DQM (excluding zero returns)
1	3M VOL	EURPLN	58,9%
2	3M VOL	3M SD	49,6%
3	SLOPE	3M VOL	66,0%
4	3M RR	3M VOL	60,6%

Source: own calculations

Table 7a. Directional Quality Measure for ECM models (training sample model)

Model	Response variable	Explanatory variable	DQM (excluding zero returns)
1	3M VOL	EURPLN	60,7%
2	3M VOL	3M SD	60,3%
3	SLOPE	3M VOL	73,6%
4	3M RR	3M VOL	62,5%

Source: own calculations

Table 8a. Comparison of ECM vs OLS models (full sample model)

Measure	MSE		DQM	
Model	ECM	OLS	ECM	OLS
1	0,0420	0,0421	58,9%	58,2%
2	0,0456	0,0466	49,6%	49,5%
3	0,0368	0,0374	66,0%	63,9%
4	0,0151	0,0154	60,6%	62,9%

Source: own calculations

Table 8b. Comparison of ECM vs OLS models (training sample model)

Measure	MSE		DQM	
Model	ECM	OLS	ECM	OLS
1	0,0737	0,0736	60,7%	58,3%
2	0,0807	0,0835	60,3%	59,3%
3	0,0398	0,0404	73,6%	74,7%
4	0,0143	0,0123	62,5%	80,2%

Source: own calculations

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